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THE BALLISTICS OF A ROCKET. CERTAIN PROBLEMS
OF BALLISTICS OF LONG-RANGE ROCKETS

by

A. A. Lebedev and N. F. Gerasyuta

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13. ABSTRACT

Methods for solving some problems in rocket ballistics of long-range are examined. Attention is given to the interaction of the various parts of the rocket and the control system. Examined are flight conditions for rockets, peculiarities of rockets as a controlled mechanical system, general equations of rocket movement, adjustment data for firing, ultimate range, dynamics of start and processes of division, dynamics of the uncontrolled head part, scattering of rockets, selection of nominal trajectory. [AM1115776]

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U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH

DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

Missile ballistics, Lebedev, A. A., Gerasyuta, N. F., "Mechanical engineering," 1970, p. 244.

This book is devoted to an examination of the methods of solving certain problems of the ballistics of long-range missiles. Considerable attention is allotted to questions of the interaction of various parts of the rocket and the control system, and also to the interdependent solution of problems of ballistics, dynamics, control and firing, to the subordinate requirement of optimizing the basic characteristics of the missile – maximum range and firing accuracy. The flight conditions of the missile, the characteristics of the missile as a guided mechanical system, the general equations of motion of the rocket, setting data for firing, maximum range, launching dynamics and separation processes, the dynamics of the unguided nose section, missile-deflection,[1] the selection of the optimum trajectory are examined.

This book is intended for engineers and scientists who are involved with questions of missile design and research and missile control systems. It will also be useful to students pursuing advanced college courses in corresponding specialities.

96 illustrations. 9 tables. 31 bibliographic entries.

Reviewed by Doctor of Physico-Mathematical Sciences G. S. Narimanov

[1][Translator's note: The Russian term is more at dispersion].

FOREWORD

This book examines some problems of the ballistics of long-range guided missiles. The selection of these problems was influenced by the monograph "Ballistics of Long-Range Guided Missiles" by R. F. Appazov, S. S. Lavrov and V. P. Mishin. The authors of this book have first of all attempted to develop those sections of ballistics which were only covered briefly or completely omitted in the mentioned monograph.

In the practical work of engineers involved in designing any products, the necessity of solving and correlating numerous interdependent problems and questions for the purpose of ensuring diverse and usually inconsistent technical specifications, imposed on a product is characteristic. In this case the solution of any design problem is begun, as a rule, by the engineer by compiling or substantiating a rational mathematical model of the product, sufficiently complex for obtaining the correct answer to the posed question, and at the same time sufficiently simple, so that the expenditures of labor and time on the calculations are not excessively great.

In connection with the introduction of contemporary computer technology into engineering practice making it possible to solve very complex technical problems, the importance of operations in the compiling and substantiating mathematical models has increased and the expenditures of labor by engineers on these types of operations have become greater. Considering what has been stated, in stating the main problems of ballistics (investigating the motion of a missile

in the transitional phases of the trajectory, investigating the motion of the nose section, calculating the dispersion of the nose section impact points, selecting of the type of missile trajectory, determining the setting data for missile launching, ensuring the maximum firing range) the authors intend:

1) to show the characteristics of a long-range guided ballistic missile as a very complex object of dynamic design and to show the connections between ballistics, dynamics, control, firing, strength, construction and operational questions;

2) to examine the mathematical models employed in solving various ballistic problems, to show the dependence of the relationships considered in the models on the purpose of the investigation and the design specifications imposed on a missile, and also to show the necessity of taking numerous random perturbing factors into consideration.

The calculation methods and the results of actual solutions of ballistic problems are not examined in this book It is assumed that the basic method of obtaining the numerical results are by calculations on digital computers.

The presentation of the above enumerated problems and questions is carried out using long-range ballistic missiles with liquid-propellant engines as examples whose thrust is governed. By examining long-range missiles, it is possible to very graphically show the effect of various factors on the solution of ballistic problems. Since liquid-propellant missiles, with controlled thrust have comparatively simple control systems, many interrelations between various questions of dynamic design are substantially simplified, which in turn makes it possible to simplify the presentation and the study of the material in this book.

In their work on this book the authors constantly obtained friendly cooperation from their many comrades, to whom they wish to express their deep appreciation. The authors are also grateful to the reviewer, Doctor of Physico-Mathematical Sciences G. S. Narimanov

for a number of useful remarks, which made it possible to improve the contents of their book.

It is requested that opinions and suggestions concerning this book be sent to the following address: Moscow, B-66, No. 1 Basmannyy Alley, 3, "Mechanical engineering" publishing house.

CONVENTIONAL DESIGNATIONS
(for missile and nose section)

a - speed of sound in the atmosphere; semimajor axis of terrestrial ellipsoid;

b - semiminor axis of terrestrial ellipsoid;

c_x, c_y, c_z - coefficients of drag, lift and side force respectively;

c_{x1} and c_τ - coefficients of axial aerodynamic force;

c_{y1} and c_n - coefficients of normal aerodynamic force;

c_{z1} - coefficient of transverse aerodynamic force;

D - diameter of maximum cross section;

e - eccentricity of terrestrial ellipsoid;

F - resultant of complete aerodynamic force and the attractive force;

f - gravitational constant;

G - force of gravity (weight of the object);

G_0 - launch weight of the missile;

G_T - attractive force of the earth;

$G_{\text{топл}}$ - weight of fuel;

$\dot{G}$ - weight per-second rate of fuel consumption $\left(\dot{G}=\left|\frac{dG}{dt}\right|\right)$;

g - acceleration due to gravity;

g_0 - acceleration due to gravity on the surface of the earth;

g_T - acceleration due to attractive force (gravity);

h - height of the center of mass of the object over the surface of the terrestrial ellipsoid;

J - controlling functional;

J_{x1}, J_{y1}, J_{z1} - moments of inertia relative to the body axes Ox_1, Oy_1, Oz_1, coinciding with the main central axes;

j - acceleration of the center of mass of an object in a relative (terrestrial) coordinate system;

j_a - absolute acceleration of the center of mass of an object;

j_u - centrifugal acceleration;

j_c - coriolis acceleration;

K - ratio of oxidizer weight to fuel weight;

L - missile flight range;

l - length of the object;

M - Mach number;

M - moment of force; mass of the earth;

M_{x1}, M_{y1}, M_{z1} - moments of bank, yaw and pitch respectively;

m - mass of the object;

$\dot{m}$ - mass flow rate per second through the nozzle exit cross section;

m_{x1}, m_{y1}, m_{z1} - coefficients of bank, yaw, pitch moments respectively;

P - rocket engine thrust;

$P_{уд}$ - the specific rocket engine thrust;

p - air or gas pressure;

q - dynamic pressure;

q_i - generalized coordinates of a missile;

q_W - dynamic pressure taking wind velocity into account;

R - radius of the terrestrial sphere; complete aerodynamic force;

Re - Reynolds number;

r - distance between the center of mass of the object and the center of the earth;

S - area of maximum cross section;

s - apparent path;

T - absolute air or fuel temperature in °K;

t - time;

t_h - moment of the termination of powered-flight phase (separation of nose section);

V - ground speed of the center of mass of the object (when there is no wind, it coincides with airspeed); the volume of fuel or fuel system;

V_W - velocity of the center of mass of the object relative to the atmosphere (and also relative to the earth when there is no wind);

W - wind velocity relative to the earth;

w - apparent missile velocity;

X - drag;

X_1 - axial force;

x_3 - coordinate of the center of mass of the object along terrestrial axis Ox_3;

$x_{дв}$ - distance from the apex of the object to the controlling engines or other control elements;

$x_д$ - distance from the apex of the object to the center of pressure;

$x_т$ - distance from the apex of the object to the center of mass (center of gravity);

Y - lift;

Y_1 - normal force;

y_3 - coordinate of the center of mass of the object along terrestrial axis Oy_3;

Z - side force;

Z_1 - lateral force;

z_3 - coordinate of the center of mass of the object along terrestrial axis Oz_3;

A - geodetic azimuth of the direction of firing (Chap. VII and VIII);

α - angle of attack; polar compression of terrestrial ellipsoid;

α_W - angle of attack taking wind velocity into account;

β - angle of sideslip;

γ - specific gravity; angle of bank;

δ - angle of deflection of the control elements;

δ_ϕ, δ_ξ, δ_η - angle of deflection of the control elements by pitch, yaw and bank respectively;

δ_1, δ_2, δ_3, δ_4 - angles of deflection of the control elements;

ζ - coordinate of the center of mass of the missile along inertial axis $O\zeta$;

η - angle of bank of missile relative to initial launch coordinate system; coordinate of the center of mass of the missile along inertial axis $O\eta$;

Θ - angle between the velocity vector and the local horizon;

θ_W - angle between airspeed vector V_W and the local horizon;

ϑ - angle of pitch;

λ - geocentric and geodetic longitude;

ξ - angle of yaw of a missile relative to the initial launch coordinate system; coordinate of the center of mass of a missile along inertial axis $O\xi$;

ρ - air density;

ϕ - angle of pitch of a missile relative to initial launch coordinate system;

$\phi_г$ - geodetic latitude;

$\phi_ц$ - geocentric latitude;

Ψ - azimuth of the projection of the velocity vector on the horizonal plane;

ψ - angle of yaw firing azimuth (Chap. II);

ψ_W - azimuth of wind direction;

ω_{x1}, ω_{y1}, ω_{z1} - projections of the angular velocity vector of the object on the body axes Ox_1, Oy_1, Oz_1;

$\omega_з$ - angular spin rate of the earth.

INTRODUCTION

By *ballistic missile* is customarily understood a guided flight vehicle with a rocket engine, intended for the delivery of a payload over long distances along an assigned open flight path, the greater part of which is an unpowered flight path.

Characteristics of ballistic missile trajectories. Depending on the forces acting on a ballistic missile, its flight path can be divided into three sections (Fig. 0.1):

A - *powered-flight phase*, i.e., the flight phase with the engine system operating , in which, as a rule, missile flight control is being carried out;

B - *unpowered-flight phase*, in which the missile moves as a free body. Usually this phase occurs at a comparatively high altitude, where the aerodynamic forces are very low and the missile moves practically only under the effect of gravity;

C - *descent phase* in the dense layers of the atmosphere, in which the aerodynamic forces have a significant effect on missile flight.

Since in phases B and C the engine system is not working and the rocket is affected only by gravity and aerodynamic forces, the corresponding flight path is ballistic, but the B-C flight phase itself of a ballistic missile is called *the passive*, or *ballistic* phase.

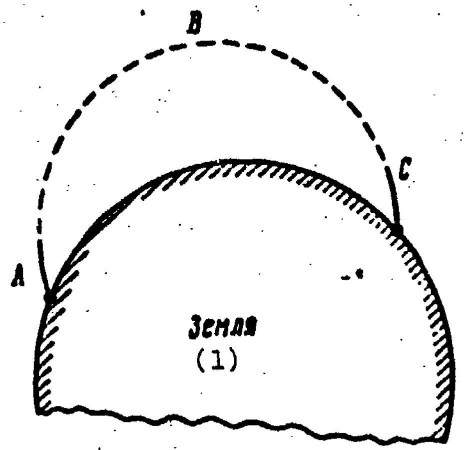


Fig. 0.1. Trajectory of a ballistic missile.
KEY: (1) earth.

The unpowered-flight phase is determined by the parameters of missile motion at the beginning of unpowered flight (i.e. at the end of the powered phase): by the coordinates of the center of mass of the missile and by the projections of its velocity. In particular, the coordinates of the point of impact of a rocket on the surface of the earth depend on the parameters of the motion of a missile at the end of the powered-flight phase and on a number of other factors, for example, on the condition of the atmosphere in the descent phase, on anomalies in the gravitational field of the earth, etc. Thus, in order that a missile carry out the mission assigned to it, it is necessary at the end of the powered-flight phase that it have completely definite values of the parameters of motion of the center of mass of the missile.

Rocket complex. The preparation of ballistic missile for launching is performed by *a missile complex* which ensures the carrying out of the prelaunch cycle of operations and the launching of missiles. Included in a missile complex, besides the missiles which are the means for delivering warheads to a target area, are:

1) launching installations with aggregates of operational equipment, servicing and communications systems;

2) the launch control system with the control center and communications;

3) the aiming system and the outboard [that which is not onboard] flight guidance equipment.

The characteristics of missile complex in many respects are determined by the type of launch positions. Depending on the conditions of missile application and the requirements, imposed for protecting the launch positions, various types of launches can be employed; from mobile ground-based launchers, from silo-launching structures, etc.

The basic characteristics of a missile complex are range and firing accuracy, warhead effectiveness, capability of overcoming antimissile systems, combat readiness, reliability, service life, production and operational economy, ease of servicing. These characteristics are intimately connected with each other and, as a rule, are conflicting.

A missile complex is a complex system consisting of a large number of elements connected with each other. However the complexity of a missile complex is due not only to the large number of interconnections. It is very significant and characteristic that the connections between the individual elements of a complex are qualitatively different and each of them has an individual importance. The latter means that the malfunction of only one connection can disturb the launch or the normal flight of the missile.

In creating a missile complex its elements are examined as parts of an entirety and each of the elements is developed so as to ensure the required characteristics of the complex as a whole.

A missile and its component systems. Contemporary ballistic missiles are distinguished by the diversity of their structural shapes. They can be single- and multistage with sequential ("tandem" configuration) and parallel ("packet" configuration) stage arrangement, used as a liquid or solid fuel working body, etc. The difference in the designs depends upon the purpose of the missile and the requirements, imposed on it, and also on the level of the development of technology. These conditions also determine the selection of the structural layout

of a missile, type of propellant and engine, method of launch, etc.

According to the type of propellant employed ballistic missiles are divided into liquid- and solid-propellant rockets. With respect to specific thrust impulse liquid rocket propellants have advantages as compared with the existing solid propellants. A significant advantage of liquid-propellant rocket engines is the possibility of multiple starting and stopping and also the possibility of controlling the thrust magnitude in flight for reducing the flight path deflection of a missile during the powered-flight phase.

Solid-propellant rockets as compared with liquid-propellant rockets, as a rule, have a simpler design. However as a result of the large deviations in the basic characteristics of solid-propellant engines (thrust and weight flow rate per second) great deflection in missile flight path in the powered-flight phase from their optimum values occurs. This gives rise to complication of the flight control system. Subsequently we will examine the individual questions connected with missile design, as illustrated by liquid-propellant rockets.

Intercontinental ballistic missiles, as a rule, are made multi-staged, most frequently two-staged with sequential stage arrangement. Such missiles consist of three sequentially positioned parts: the separable first stage part, the second stage housing and the nose section (Fig. 0.2). In flight after a missile has attained the assigned velocity (or upon burnout) the control system issues the command for the shut-down of the first-stage engines, stage separation and the starting of the second-stage engine. At the end of the powered-flight phase upon a command which is shaped by the control system on the basis of information about the parameters of missile motion, the second-stage engines are shut down and the nose section is separated from the second-stage housing.

Each of the stages of a liquid-propellant rocket, as a rule, consists of a fuel compartment, a compartment for positioning of instruments and the control system apparatus and a tail section for accomodation of the engine system. To ensure the operation of the

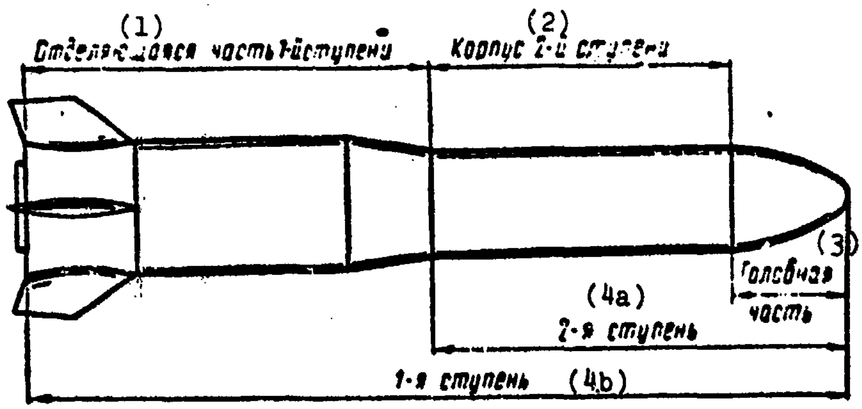


Fig. 0.2. Diagram of a two-stage liquid-propellant ICBM.
KEY: (1) 1st stage separating part; (2) 2nd stage housing; (3) nose section; (4a) 2nd stage (4b) 1st stage.

engine system a number of special systems is included in this stage: pressurization of the fuel tanks, synchronous emptying of tanks, checking of the fuel levels and others.

<u>The fuel tank pressurizing system</u> is intended for creating pressure excesses in the tanks before starting the engines and during their operation.

<u>The system for the synchronous emptying of the tanks</u> is installed to regulate the volumetric expenditure of the fuel components. This system shapes command signals in such a way, so as at the moment of engine shut-down during firing for maximum possible range to ensure complete consumption of both propellant components. In the case of the absence of this system as a result of an unavoidably arising deviation in the ratio of the weight consumption rate per second of the propellant components toward the end of the operation of the engine systems in one of the rocket tanks the unused working propellant supplies remain, which lead to a reduction in the maximum firing range.

The level monitoring system serves for the remote monitoring of the oxidizer and fuel levels in the tanks during servicing and for giving commands to the servicing units at the end of servicing. The level monitoring system also makes it possible to periodically monitor the levels of the propellant constituents in the tanks during the prolonged storage of a fueled rocket.

Each stage of a missile has *control elements* whose deflection for the creation of controlling moments is accomplished by control actuators. At the present time a large number of diverse ballistic missile control elements is known, among which the most frequently used are turning combustion chambers.

The flight control system is the totality of instruments and devices which ensure in accordance with the executed aiming the controlled flight of a missile and the impact of its nose section in the vicinity of the target with the required accuracy.

The basic stages in the development of a missile complex.[1] Missile complex is developed on the basis of the tactical-technical specifications, which define its purpose, technical and operational characteristics, and the interaction of its component parts. Among the basic characteristics of a missile it is possible to include range and firing accuracy, type of payload and its weight, the possibility of its overcoming the means of antimissile defense, launch weight, type of launch, engine model and propellant components, the number of rocket stages, the type of flight control system, combat readiness, and reliability.

The process of developing a missile complex includes a number of phases. In the first phase various types of exploratory procedures and preliminary investigations are carried out, a large number of diverse variants of missile complex layouts is examined. The problems of this phase are evaluating the possibilities of creating a missile complex, satisfying the assigned tactico-technical requirements, the

[1]See, for example, book [20].

selecting of an optimum variant (or variants) and the obtaining of original data for basic planning, which are absent in the tactico-technical requirements, and also estimating the cost for carrying out all the operations and their execution times.

Of the large number of questions, usually examined during preliminary planning, let us take note, for example, the following: selection of the missile layout, evaluation of the weight and centering [c.g] characteristics, selection of the propellant components, selection of the main and control engines.

The result of the first phase operations is a pre-draft design of the missile. In the pre-draft design, preliminary materials on ballistics (flight-path, calculations), aerodynamic characteristics, strength, controllability, missile stability, etc., are presented. Furthermore, materials on the possibility of using existing ground equipment, and also the existing production capacities are presented.

The second development phase is the *basic designing* which by tradition is frequently called *sketch designing*. Before the beginning of this phase the individual tactical and technical data of the missile complex, the composition and the characteristics of its basic parts are made more precise.

In the basic designing phase in-depth studies of all questions connected with the creating of a missile and the ground equipment of the complex are carried out. For this, besides calculations, laboratory investigations and experimental adjustment of various units, subassemblies and systems are employed. Thus, for instance, laboratory investigations and adjustment of the instruments and subassemblies of the control system, ground equipment, strength testing of the housing, tanks and the individual units of the missile, bench testing of the engines, experimental investigations of the vibrations of fluid in the tanks, etc., are carried out.

The problem of this phase of operations is the preparation of valid materials for the turning out of technical-drawing documentation

and the manufacture of prototypes of a missile and the ground equipment.

The next phase in the creation of a missile complex is the *working out of technical-drawing documentation and the manufacture of prototypes*. It is difficult to separate this phase in time from the basic designing phase because the turning out of the technical-drawing documentation and the manufacture of individual units and systems is frequently carried out in the basic design period. In particular this pertains to equipment and units having a prolonged technological manufacturing cycle.

The next phase of operations is the ground adjustment of prototypes of the materiel (individual elements and systems) using test stands, etc. This phase to a greater or less extent can also coincide in time with the previous phases.

The final phase is the final adjustment and evaluation of the prototypes by flight testing. This phase of operations is preceded by the preparation of the documentation, necessary for carrying out the flight testing and, especially, by the turning out of instructions for all the types of operations carried out on the test range. It is expecially necessary to note the working out of questions of ballistic ensuring of the flight tests (the selection of the test range, the firing routes, the impact area of the separating elements of the missile and of the nose sections, the selection and validation of the flight control programs, flight-trajectory calculations and control system adjusting data).

The flight testing of prototypes is intended for checking conformity of the actual and assigned technical-flight characteristics of a missile, control equipment, ground equipment, for finding ways of improving them, etc. This phase plays an important role in the creation of a missile complex. On the basis of the test results the necessary changes are introduced into the design of the complex. Thus this phase is usually called the *flight-design testing phase*. In the course of testing operational questions are defined more precisely, the operational reliability of all systems and units is evaluated,

an evaluation of the operation of the missile complex as a whole is carried out, operational documentation is prepared.

The place of dynamic planning in developing a missile complex. In creating a missile complex and, especially, the missile itself a very large role is played by dynamic planning which mainly involves the solution of problems of ballistics, dynamics, control and firing. The basic characteristics of the missile and its layout are determined from the results of dynamic planning.

In solving a question concerning the possibility of creating a missile which satisfies the assigned tactico-technical specifications, a large number of ballistic calculations is carried out, on the basis of which the most rational variants of the layout scheme and the basic design parameters of the missile, the weight and c.g. characteristics, optimum flight paths are determined.

Questions about the possibility of ensuring the controllability and stability of a missile are solved by means of research on its dynamic layout. The latter is described by differential equations of perturbed motion the coefficients of which are determined by the layout scheme and by the design parameters of the missile, and also by the parameters of the motion of the missile along the optimum flight path.

By examining the diverse variants of the solutions, the most rational dynamic and therefore, layout scheme of the missile is selected. In this case it is necessary to overcome a number of inconsistencies. It is possible, that the layout scheme of a missile which satisfies the ballistic, technological-design and operational specifications, will not satisfy the controllability and stability specifications. For instance, a decrease in the rigidity of a missile for the purpose reducing its weight leads to a reduction in the frequencies of the elastic vibrations of a missile, which creates significant difficulties in ensuring flight stability. The use of sufficiently effective control elements (control combustion chambers, jet vanes, etc.) always gives rise to a reduction in the specific thrust of an engine system or to an increase in the "dry" weight and

thus, negatively affects the energetic possibilities of the missile. In connection with this the determination of the most rational missile variant, its layout scheme, servicing methods, cost, development time, etc., to a great extent depends on how correctly the problems of the dynamic planning of a missile are solved.

In the basic design phase the role of dynamic planning is still greater. In this phase it is necessary to give a comprehensive answer to the question of the sufficiency of the accepted solutions with respect to ensuring the assigned range and firing accuracy, controllability and stability under all possible operating conditions for the missile being planned, i.e., under all geophysical launching conditions, in diverse meteorological conditions, deviations in the parameters of the missile and guidance equipment from the rated values, variations in the missile assembly, etc.

The presence of a large number of perturbing factors (variance in the atmospheric parameters, propellant and design parameter characteristics, errors in the operation of assembly units and systems, etc.) causes deviations in the parameters of missile motion from the rated values. This fact predetermines the use of probability and statistical methods in solving many problems of the dynamic planning of a missile (for instance, in ensuring assigned maximum firing range, in evaluating nose section impact point dispersion, etc.).

In solving the problems of the dynamic planning of a missile complex the method of complex development manifests itself brightly, in which a complex is considered as a single unit. In dynamic planning it is necessary to find rational compromise solutions for numerous interdependent questions. Thus, for instance, the selection of the control method affects the layout and the power engineering of a missile; the selection of flight paths is connected with the energy characteristics of the missile, the temperature and strength limitations, the firing accuracy requirements, the type of control system (autonomous or electronic), and also the keep-away areas intended for the falling of the separating parts of the first stages, and by many other factors; the selection of the site for the mounting of gyroscopic instruments is connected with the question of ensuring the

stability of an elastic missile; the selection of the method of stage separation and the separation of the nose section - with the specifications imposed on the characteristics of the engine systems; the selection of the number and site for installing devices damping the oscillations of the liquid propellant in the tanks, - with the layout of the missile and its energetic characteristics.

The role of dynamic planning in developing a silo launching structure is great. In this case an analysis of the diverse variants of missile motion in a silo structure (free motion and motion along guides) can be carried out, the necessary diameter of the silo shaft, inside which the missile moves, the sizes of the gas flow passage cross-sectionals areas and other data, necessary for the planning of a silo structure, are determined.

In selecting a variant of a silo structure for use, besides these data, the characteristics of the reliability of the exit of a missile from the silo, the cost of the silo structure and other factors are taken into account.

The intimate interrelationship between the various questions of the dynamic planning of a missile complex makes it necessary to carry out the planning in several phases, correlating the obtained results for each of them with all the co-operators. It is necessary to approach the selection of the command instruments and the other equipment of the control system with great care because of the great complexity of their manufacture and their relatively high cost.

Computer technology is broadly employed in solving dynamic planning problems. Specifically, the calculations of the powered and unpowered flight paths of nose sections and the separating parts of missile stages and other ballistic calculations are conducted with the aid of electronic digital computers (ЦВМ = EDC).

In analyzing the stability of motion, the basic method of investigation is the simulation of perturbed missile motion on electronic analog computers (АВМ = EAC) using real onboard control equipment. This method makes it possible to obtain a rather complete

picture of the actual processes occurring in flight.

Recently for investigating the stability of missile motion analog-digital complexes (АЦК = ADC) are beginning to be widely used which are a combination of analog and digital computers with the actual equipment of a flight control system. Such a complex makes it possible to much more efficiently, comprehensively and at a high technical level solve the problems of the dynamics of missile motion. One of the problems solved with the aid of an analog-digital complex, is the problem of determining the worst combinations of parameters of missile and control system equipment and the checking of the reliability of ensuring the stable motion of a missile under various adverse conditions.

The use of electronic computer technology makes it possible to carry out missile flight simulation, taking the majority of random factors into account, i.e., in other words, for a given model of a random process (missile flight) to obtain a number of executions of this process - the "electronic launching" of a missile and to evaluate, for example, the nose section impact point dispersion.

Besides the calculations on digital computers and simulations on analog computers, graphical-analytical methods of investigation are broadly employed in dynamic planning, especially in the preliminary design stage. The use of graphical-analytical methods requires a significant simplification of the dynamic layout of a missile. From the number of necessary numerous simplifications it is necessary to indicate linearization of the equations of motion of the missile and the replacement of the variable coefficients of these equations with constant coefficients (the method of "freezing" coefficients). Thus, for instance, in the preliminary investigation of the stability of motion of the missile the noted simplifications are assumed, in order to then use the frequency method or the root-locus technique. The linearization of the equations during the investigation of firing accuracy makes it possible to use the appropriate methods of the probability theory. When it is not possible to disregard the nonlinear properties of a missile or control system, such methods of

approximation, as the method of harmonic balance or the method of statistical linearization are employed. The graphical-analytical methods make it possible for the engineer to penetrate deeply into the essence of the phenomenon being investigated, which facilitates a more successful subsequent solution of the problems of dynamic planning with the aid of more precise methods using digital and analog computers and digital-analog complexes.

The basic problems of missile ballistics. Missile ballistics solves the following basic problems.

1. The investigation of the dependence of the flight characteristics of a missile, and primarily of its flight range, design parameters for the purpose of selecting the most advantageous combination of these parameters (ballistic design).

2. The determination of flight path and other basic characteristics of the motion of a missile with known design parameters and control system with assigned aiming data (ballistic test calculations).

3. Determining the initial data for the nose section design and investigating nose section dispersion (the problem of nose section ballistics).

4. Ensuring maximum aiming firing range under conditions of the effect of various perturbing factors - variance in design parameters, variations in the ambient flight conditions and others (ensuring maximum firing range).

5. Investigating the effect of various perturbing factors on the powered-flight phase and, especially, the errors of the control system elements on nose section impact point dispersion (investigating missile deflection).

6. Determining aiming data from the given coordinates of the launch point and target (compilation of flight mission).

7. The selecting of the optimum flight path which ensures the best use of the missile's capabilities (selecting the control program).

8. Determining the initial data for the flight-design testing of missiles and analyzing the results of these tests.

All these problems are intimately connected with the solution of a number of other questions relating, especially, to:

– aerodynamics (determining aerodynamic forces and aerodynamic heating of the surface and structural elements of a missile or nose section);

– structural dynamics (calculating the elastic vibrations and the vibrations of the liquid in the fuel tanks);

– missile control (ensuring the stability of motion and controllability of a missile taking into account the elastic vibrations and the vibrations of fluid; selecting the design and the basic parameters of the control system);

– the dynamics of non-steady-state modes – launch and the processes of stage separation and separation of the nose section (ensuring the separation and the controllability of the missile during these phases);

– calculating of the missile design for strength (determining structural loads for various flight paths).

Ballistic design plays a very large role in the development of a missile when selecting the design layout of the missile, its arrangement and the values of its structural and energetic characteristics, in the very best manner conforming to the specifications, imposed on the missile. At the present time ballistic design has developed into an independent discipline. In connection with this, this book does not include ballistic design. The basic questions of ballistic design are examined, for example, in book [2].

Ballistic test calculations and, especially, the calculation of the nose section unpowered flight phase, is not of direct interest for the present book dedicated to the complex solution of the basic problems of ballistics, especially since the methods and the characteristics of these calculations are also presented in book [2].

Ballistic calculations for flight-design testing are inseparably connected with these testing methods. The latter are a separate discipline, requiring independent exposition.

Thus, included within the scope of this book are such ballistics problems, as nose section ballistics, ensuring maximum firing range, investigating missile deflection, determining the setting data for a rocket launching, selection of optimum trajectory. Furthermore, this book touches upon certain other related questions.

CHAPTER I

MISSILE FLIGHT CONDITIONS, PECULIARITIES OF A MISSILE AS A GUIDED MECHANICAL SYSTEM

The solution to any ballistics problem begins with the compilation of a mathematical model (dynamic layout) of missile flight which is described more or less by the complex equations of missile motion. The mathematical model is determined, in the first place, by the posed problem, depending on which model of flight conditions the investigator selects, the mechanical model of the missile itself, the model of the forces and moments, applied to the missile, etc. The success of the investigation depends on how rationally the mathematical model of missile flight is composed. The basic information about the flight conditions of the missile and the characteristics of the missile as a guided mechanical system, which must be kept in mind when compiling the mathematical model of missile flight in ballistics problems are presented below.

1.1. MOTION, SHAPE AND GRAVITATIONAL FIELD OF THE EARTH

The Motion of the Earth

The earth carries out complex motion which mainly consists in the following components.

1. Rotation around its axis from west to east with a period of 23 h 56 min 4.091 s = 86164.091 s of mean solar time, or 24 h = = 86400 s of sidereal time; the angular velocity of rotation in this

case is respectively equal to

$$\omega_3 = \frac{2\pi}{86164,091} = 7,2921 \cdot 10^{-5} \text{ rad/s.}$$

The vector of the angular velocity of the earth $\bar{\omega}_3$ is directed along the axis of rotation from the south pole to the north pole in accordance with the rules of signs for right-handed coordinate systems.

2. Annual revolution around the sun with an average orbital velocity of 29.893 km/s.

3. Nutational oscillations of the terrestrial axis with a period of about 18.6 and with an amplitude, not exceeding 9.2".

4. Precessional motion relative to the axis of the ecliptic with a period of 25,800 years.

5. Motion together with the solar system relative to the other stars.

In investigating the flight of a ballistic missile all these components of terrestrial motion, except diurnal rotation, are not taken into account because their effect on flight path is extremely small. It is assumed that the center of mass of the earth moves rectilinearly and uniformly and the earth rotates uniformly around its axis whose direction does not vary. The phenomena connected with the rotation of the earth, play an extremely large role in missile dynamics. Thus, in calculating the flight paths of missiles it is necessary to consider the forces of inertia caused by the diurnal rotation of the earth.

As a result of its rotation the earth is an oblate spheroid, in which the distance between the poles is less than the diameter of the equator. This fact together with other deviations in the shape of the earth from spherical shape and the non-uniform distribution of masses inside the earth make it difficult to determine the magnitude and the direction of the attractive force of the earth acting on a missile.

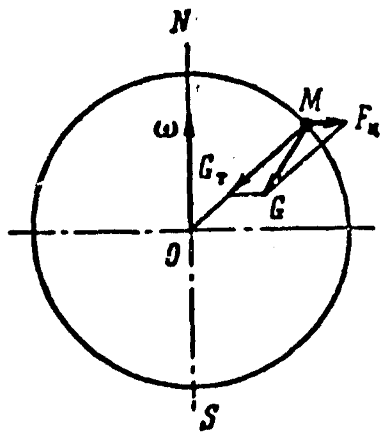
The Shape of the Earth

The earth is a body of complex shape. The surface of the earth with all its irregularities is called the physical surface of the earth. All kinds of geodetic measurements are being carried out on it for the purpose of obtaining initial data for solving various geodetic problems. The physical surface of the earth is practically impossible to describe mathematically, because it cannot be used as a surface for processing the results of the measurements. As such a surface it is necessary to use a body surface which most closely approaches the earth as a whole in shape and dimensions, and whose surface is expressed by a mathematical dependence suitable for practical use. Of the geometric bodies which describe the shape of the earth, the body which has received the name geoid most closely approximates the actual earth. In order to define this body let us recall the concept of equigravitational potential surface [level or equipotential surface of the earth's gravity].

As is known, the diurnal rotation of the earth creates centrifugal inertia which acts on a body located on the surface of the earth. Thus, it is not possible by experimental means to separate centrifugal inertia from the force of terrestrial attraction. The resultant vector of these forces is the vector of the force of gravity (Fig. 1.1) whose direction in space can be determined with the aid of a plumb line or level.

Equigravitational potential surface is a surface, at each point of which the normal to the surface is collinear to the direction of the force of gravity. A geoid is a body, limited by an equigravitational potential surface which coincides with the surface of oceans (undisturbed by tides and waves) and extending under the continents (Fig. 1.2). The surface of a geoid is continuous, closed and does not have sharp creases and folds. Since the direction of the force of gravity depends on the attracting action of masses non-uniformly distributed inside the earth, then the surface of a geoid is extremely complex and cannot be described mathematically. For this reason the geoid is replaced by a simpler body in such a way that its surface differs as little as possible from a geoid, and the carrying out of

calculations on this surface does not present significant difficulties.



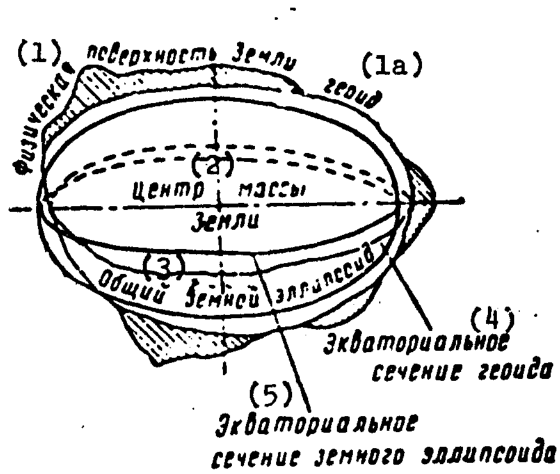


Fig. 1.1. Diagram of the application of attractive force, centrifugal force and gravity.

Fig. 1.2. The physical surface of the earth, a geoid and a general terrestrial ellipsoid.
KEY: (1) the physical surface of the earth; (1a) geoid; (2) the center of mass of the earth; (3) General terrestrial ellipsoid; (4) the Equatorial cross section of a geoid; (5) Equatorial cross section of a terrestrial ellipsoid.

As a first approximation it is possible to consider the earth a sphere whose volume is equal to the volume of the earth. The radius of such a sphere is R = 6,371,110 m. In some ballistics problems this approximation satisfies the required calculational accuracy, in others, for example, in preparing flight tests and in analyzing results of a launch such an approximation introduces a large error in determining nose section impact points.

In most cases a geoid is replaced with sufficient practical accuracy by an ellipsoid of revolution obtained by revolving an ellipse around its minor axis. Such a properly oriented ellipsoid, which in the very best manner approximates the surface of a real geoid, is called *a general terrestrial ellipsoid* (see Fig. 1.2).

A general terrestrial ellipsoid is defined on the basis of the following conditions:

1) the center of the ellipsoid coincides with the center of mass of the earth, and the plane of its equator is parallel to the equatorial plane of the earth;

2) the volumes of the ellipsoid and the geoid are equal;

3) the sum of the squares of the deviations (with respect to height) of the surface of a general terrestrial ellipsoid from the surface of a geoid should be minimum.

The determining of the dimensions of a general terrestrial ellipsoid is one of the basic problems of geodesy. At the present time this problem is still not completely resolved because the appropriate measurements (geodetic, astronomical and gravimetric), being employed as the initial material for the solution to the indicated problem, have still not been carried out on all the continents. All the available dimensions of the general terrestrial ellipsoid are approximate and to one or another degree differ from the dimensions of the real general terrestrial ellipsoid. Subsequently we will proceed on the basis of the following approximate values of the parameters determining the dimensions of the general terrestrial ellipsoid:

– semimajor axis (radius of the equator) $a = 6,378,137$ m;

– compression $\alpha = \frac{a-b}{a} = \frac{1}{298.25}$, where b – the semiminor axis of the general terrestrial ellipsoid.

The surface of even an accurate (with respect to dimensions) general terrestrial ellipsoid, correctly oriented with respect to the earth, can deviate from the surface of the geoid with respect to height by tens of meters. In the opinion of a number of scientists, the greatest values of these deviations are located within the limits of ±150 m. In certain cases for the purpose the reducing the errors in the replacement of a geoid by the general terrestrial ellipsoid the concept of reference-ellipsoid is introduced.

A reference-ellipsoid is an ellipsoid of revolution with appropriate dimensions, oriented in a definite manner relative to the earth and to whose surface the results of geodetic operations on an investigated part of the terrestrial surface (in a given country) pertain. The following conditions are imposed on the orientation of a reference-ellipsoid:

a) the greatest proximity of the surface of the reference-ellipsoid to the surface of the geoid only on the examined part of the terrestrial surface;

b) the parallelness of the axis of revolution of the reference-ellipsoid and the axis of rotation of the earth (coincidence of its center of mass with the center of mass of the earth is not mandatory).

On the territory of the USSR for the dimensions of the reference-ellipsoid it is possible to use the dimensions of the Krassowski ellipsoid, namely: the semimajor axis a = 6,378,245 m; compression $\alpha = 1/298.3$. The center of the Krassowski ellipsoid is removed a certain distance from the center of mass of the earth. Clarke, Kayford, Everest ellipsoids are also used as reference ellipsoids.

Coordinate Systems Defining the Position of a Point on the Terrestrial Surface

The following coordinate systems are used for defining the position of a point on the terrestrial surface, a mathematical description of the gravitational field of the earth and a number of other problems.

The geocentric coordinate system (Fig. 1.3). The position of point M on the surface of the Krassowski ellipsoid is determined by the two coordinates λ and $\phi_{ц}$.

Longitude λ – the dihedral angle between the planes of the prime (Greenwich) meridian and the local meridian, passing through point M. East longitudes, i.e., the longitudes of the points located to the east of the Greenwich meridian, are considered positive, and the western longitudes – negative.

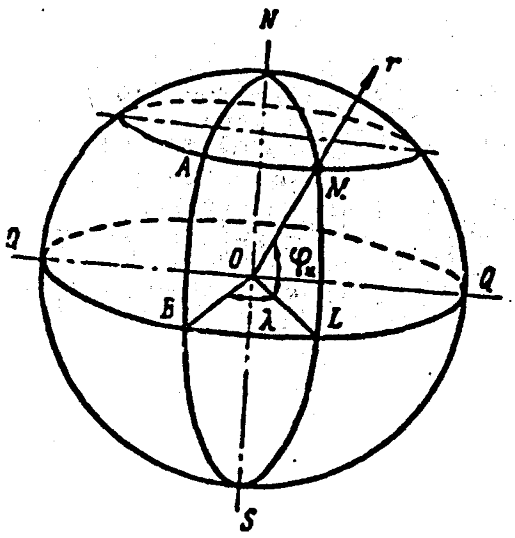


Fig. 1.3. The geocentric coordinate system: NABS - the prime (Greenwich) meridian; NMLS - local meridian; QBLQ - equator; $-180° \leq \lambda \leq 180°$, $-90° \leq \phi_ц \leq 90°$

Geocentric latitude $\phi_ц$ - the angle included between the equatorial plane and radius-vector $\bar{r}$, drawn from the center of the ellipsoid through point M. North latitudes, i.e., the latitudes of the points located to the north of the equator, are customarily considered positive, the south latitudes - negative.

The geodetic coordinate system (Fig. 1.4). In this system point M on the surface of the Krassowski ellipsoid has the following two coordinates: *geodetic longitude* λ which is defined in the same way as in the geocentric coordinate system, and *geodetic latitude* $\phi_г$ which is the angle included between the equatorial plane and the normal to the surface of the ellipsoid at point M. The geodetic azimuth of direction is the angle ψ computed clockwise from the northern direction $\bar{p}$ of the geodetic meridian of the given point to assigned direction $\bar{l}$. The geodetic coordinate system has found extensive application in ballistics for determining the launch and target coordinates.

Geocentric and geodetic latitudes are connected with each other by the relationship

$$\sin(\varphi_г - \varphi_ц) = e^2 \sin\varphi_г \cos\varphi_ц,$$

where e - the eccentricity of the meridional ellipse of the general terrestrial ellipsoid.

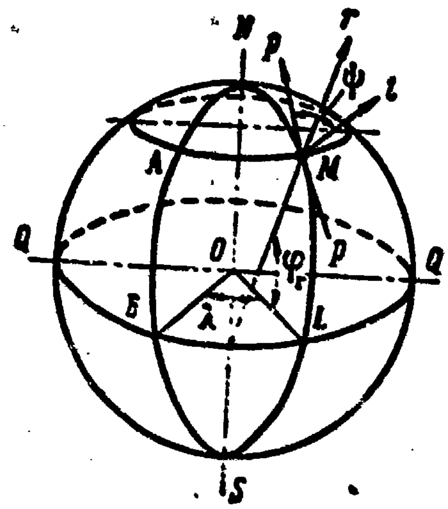


Fig. 1.4. The geodetic coordinate system: NABS - the prime (Greenwich) meridian; NMLS - local meridian; QBLQ - equator; pp - a tangent to the local meridian of the Krassowski ellipsoid at point M; $-180° \leq \lambda \leq 180°$, $-90° \leq \phi_r \leq 90°$.

<u>The astronomical (geographic) coordinate system.</u> In contrast to geodetic coordinates determined on the basis of geodetic measurements and pertaining to the surface of the ellipsoid of revolution, astronomical coordinates are determined on the basis of astronomical observations and pertain to the surface of the geoid.

In this coordinate system astronomical (geographic) latitude is defined as the angle ϕ_A between the plumb line at a given point and the equatorial plane. Astronomical (geographic) longitude is the angle λ_A between the plane of the prime meridian and the plane of the astronomical meridian, passing through the plumb line at a given point. The reference directions and the signs of astronomical latitude and longitude are determined in the same way as for geocentric and geodetic coordinates.

Astronomical latitude and longitude do not coincide with the corresponding geodetic values, since in the general case a normal to the geoid and to the ellipsoid do not coincide with each other. The angle included between a normal to the ellipsoid and the plumb line at the point being examined is called complete plumb-line deflection. The mean plumb-line deflection with respect to the surface of the earth is about 5", the maximum about 1'.

The Gravitational Field of the Earth

According to Newton's law of gravitation every particle with mass M attracts another particle with mass m with a force of gravitational attraction (gravity) G_T, determined by the dependence

$$G_T = -f\frac{Mm}{r^2}, \qquad (1.1)$$

where $f = 65.41 \cdot 10^{-11} \frac{m^4}{kgf \cdot s^4}$ - gravitational constant; r - the distance between the particles.

During the flight of a missile the attractive forces of the earth and the other celestial bodies act on it. For ballistic missiles whose flight paths lie in the immediate proximity of the earth, the attractive forces of the celestial bodies are extremely small (thus, the attractive forces of the moon and sun give rise to an insignificant variation in the acceleration due to the attractive force and plumb-line deflection; the effect of the remaining celestial bodies is still less). In connection with this we will subsequently examine only the gravitational field of the earth.

Attractive force is conservative, i.e., having a force function. The force function of a material particle with mass M is called Newtonian potential and is equal to

$$U = f\frac{M}{r}, \qquad (1.2)$$

where r - the distance from the material particle to the point in space being examined.

The Newtonian potential of an arbitrary body with mass M can be written in the form

$$U = f\int_M \frac{dm}{r}, \qquad (1.3)$$

where r - the distance from the particle having mass dm, to the point in space being examined.

As a first approximation, if it is considered that the mass of the earth is concentrated at a point or distributed inside the sphere so that the density at all points, equidistant from the center of the sphere, is identical, the potential function of the earth is written in the form of (1.2). In this case value r is the distance from the center of the earth.

Using the property of force function, it is possible to determine the projections of the attractive force of a particle of unit mass on the axis of a certain coordinate system Oxyz:

$$g_{\tau x}=\frac{\partial U}{\partial x};\quad g_{\tau y}=\frac{\partial U}{\partial y};\quad g_{\tau z}=\frac{\partial U}{\partial z}. \qquad (1.4)$$

In particular, the projection of the attractive force on radius-vector r is determined by the expression

$$g_{\tau r}=\frac{\partial U}{\partial r}=-f\frac{M}{r^2}. \qquad (1.5)$$

In this case the acceleration imparted to the particle of unit mass by a spherical earth, is directed to the center of the earth, and is equal to

$$-g_{\tau r}=-\frac{fM}{r^2}. \qquad (1.6)$$

The product of the gravitational constant f and the mass of the earth m is constant and for approximate calculations can be taken equal to: $fM = 3.986004\cdot10^{14}\ m^3/s^2$.

The normal potential of the earth. In general form the problem of determining potential function U for the real earth having a complex shape and non-uniform distribution of mass, is extremely difficult. In gravimetry it is customary to represent the potential of the earth in the form of an infinite series

$$U(r,\varphi_{u})=\frac{a_{00}}{r}+\frac{a_{20}}{r^3}P_{20}(\sin\varphi_{u})+\frac{a_{40}}{r^5}P_{40}(\sin\varphi_{u})+\ldots, \qquad (1.7)$$

in which the associated Legendre polynomials are determined by the expressions:

$$P_{20}(\sin\varphi_{ц}) = \frac{3}{2}\sin^2\varphi_{ц} - \frac{1}{2};$$

$$P_{40}(\sin\varphi_{ц}) = \frac{35}{8}\sin^4\varphi_{ц} - \frac{15}{4}\sin^2\varphi_{ц} + \frac{3}{8}$$

etc.

Being limited in expression (1.7) by the terms which are the main spherical functions of the zero, second and fourth orders, a convenient formula for attractive potential is obtained called the normal potential of the earth:

$$U_0 = \frac{a_{00}}{r} + \frac{3}{2}\frac{a_{20}}{r^3}\left(\sin^2\varphi_{ц} - \frac{1}{3}\right) + \frac{35}{8}\frac{a_{40}}{r^5}\left(\sin^4\varphi_{ц} - \frac{6}{7}\sin^2\varphi_{ц} + \frac{3}{35}\right), \quad (1.8)$$

where a_{00}, a_{20}, a_{40} - the constant coefficients dependent on the angular velocity of rotation $\omega_з$ and the parameters of the accepted model of the earth:

$$a_{00} = fM;$$

$$a_{20} = -g_э a^4\left(\frac{2}{3}\alpha - \alpha^2 - \frac{1}{3}\mu + \frac{10}{7}\mu\alpha\right);$$

$$a_{40} = \frac{8}{35} g_э a^6\left(\frac{7}{2}\alpha^2 - \frac{5}{2}\mu\alpha\right);$$

$$\mu = \frac{\omega_з^2 a}{g_э}; \qquad \alpha = \frac{a-b}{a};$$

$g_э$ - the gravitational constant at the equator.

The normal potential of the earth corresponds to the potential of a certain spheroid which represents an idealized earth, and differs somewhat from the potential of the earth. This difference is expressed in the form of an anomaly in gravitational field and is taken into account in accurate calculations.

The normal potential of the earth's attraction depends only on the distance r to the point in question and the geocentric latitude $\phi_ц$. The intensity vector of the normal gravitational field is always located in the plane of the meridian, passing through the axis of

rotation of the earth and the point in space being considered. This vector $g_т$ of the acceleration due to the forces of normal attraction can be assigned two components: $\bar{g}_{т1}$ and $\bar{g}_{т2}$ lying in the plane of the meridian (Fig. 1.5), in this case

$$g_{т1}=-\frac{\partial U_0}{\partial r};\; g_{т2}=\frac{1}{r}\frac{\partial U_0}{\partial \varphi_ц}, \tag{1.9}$$

or

$$\left.\begin{aligned} g_{т1}&=\frac{a_{00}}{r^2}+\frac{3}{2}\frac{a_{20}}{r^4}(3\sin^2\varphi_ц-1)+\\ &+\frac{35}{8}\frac{a_{40}}{r^6}\left(5\sin^4\varphi_ц-\frac{30}{7}\sin^2\varphi_ц+\frac{3}{7}\right);\\ g_{т2}&=\frac{3}{2}\frac{a_{20}}{r^4}\sin 2\varphi_ц+\frac{5}{2}\frac{a_{40}}{r^6}\cos\varphi_ц(7\sin^3\varphi_ц-3\sin\varphi_ц). \end{aligned}\right\} \tag{1.10}$$

The relative error in these formulas is comparatively small (it does not exceed $3\cdot 10^{-5}$) and is entirely permissible in solving the majority of ballistics problems connected with flight-trajectory calculation and with preparing aiming data for firing.

In deriving the equations of motion of a missile it is convenient to examine the following two components of acceleration due to attractive force (see Fig. 1.5): $g_{тr}$, directed toward the center of the earth; $g_{т\omega}$ directed parallel to the rotation axis of the earth.

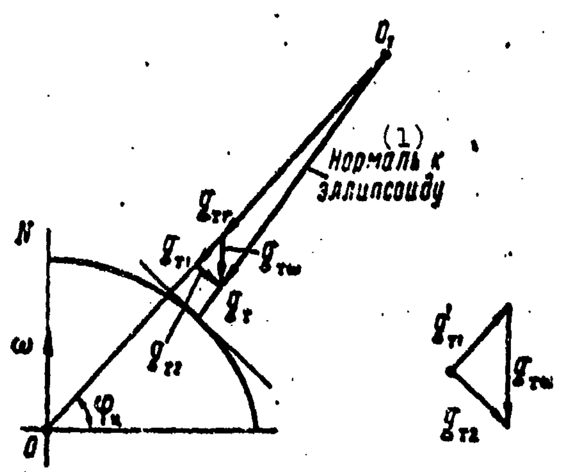


Fig. 1.5. The components of acceleration due to gravity.
KEY: (1) normal to the ellipsoid.

In order to find them, it is necessary to separate, in turn, the meridional component of acceleration due to attractive force g_{T2} into two components in the direction of radius-vector r and the rotation axis of the earth:

$$\left.\begin{aligned} g'_{T1} &= g_{T2}\,\mathrm{tg}\,\varphi_{\text{ц}}; \\ g_{T\omega} &= \frac{g_{T2}}{\cos\varphi_{\text{ц}}}. \end{aligned}\right\} \qquad (1.11)$$

Component $\bar{g}'_{T1}$ is directed opposite to component $\bar{g}_{T1}$. Thus

$$g_{Tr} = g_{T1} - g'_{T1}. \qquad (1.12)$$

We will finally obtain

$$g_{Tr} = \frac{a_{00}}{r^2} - \frac{3}{2}\,\frac{a_{20}}{r^4}(5\sin^2\varphi_{\text{ц}} - 1) + \frac{15}{8}\,\frac{a_{40}}{r^6}(21\sin^4\varphi_{\text{ц}} - 14\sin^2\varphi_{\text{ц}} + 1); \qquad (1.13)$$

$$g_{T\omega} = 3\,\frac{a_{20}}{r^4}\sin\varphi_{\text{ц}} - \frac{5}{2}\,\frac{a_{40}}{r^6}\sin\varphi_{\text{ц}}(7\sin^2\varphi_{\text{ц}} - 3). \qquad (1.14)$$

If especially high calculational accuracy is not required, then it is possible to be limited to the first terms of the expanfion in the series, i.e., to take

$$g_{Tr} = \frac{a_{00}}{r^2} - \frac{3}{2}\,\frac{a_{20}}{r^4}(5\sin^2\varphi_{\text{ц}} - 1); \qquad (1.15)$$

$$g_{T\omega} = 3\,\frac{a_{20}}{r^4}\sin\varphi_{\text{ц}}, \qquad (1.16)$$

where

$$a_{00} = 3{,}9861679 \cdot 10^{14}\ \text{м}^3/\text{сек}^2; \quad \mathrm{m}^3/\mathrm{s}^2$$

$$\frac{3}{2}\,a_{20} = 26{,}32785 \cdot 10^{24}\ \text{м}^5/\text{сек}^2. \quad \mathrm{m}^5/\mathrm{s}^2$$

1.2. THE ATMOSPHERE.

The flight of a ballistic missile in the initial and final phases of its trajectory occurs in the atmosphere. Aerodynamic forces arising here substantially depend upon the parameters of the atmosphere – density, pressure and air temperature. These parameters, in turn, depend upon the flight altitude, the geographic latitude of the site, the season, time of day, and a number of other factors, for example on the degree of solar activity.

For determining the design parameters of a missile, calculating trajectories and other investigations carried out in dynamic designing, the tables of standard atmosphere (SA) are usually used which give certain mean values of the parameters of static atmospheric conditions depending on altitude. Deviations in the atmospheric parameters from standard values, and also wind are atmospheric perturbances which affect missile flight and, especially, the dispersion of the impact points of its nose section.

The standard atmosphere SA-64 has been accepted in the USSR for altitudes up to +200,000 m (GOST 4401-64). For altitudes of 200,000-300,000 m the atmospheric characteristics, recommended by the co-ordination commission of the Academy of Sciences of the USSR on the compilation of the GOST for standard atmosphere, are given in this same GOST.

For solving problems of dynamic design, besides the standard values of atmospheric parameters, it is also necessary to know the ranges of the possible deviations in these parameters, which correspond to a definite level of probability, and for various conditions both not allowing for the season and the site on the terrestrial sphere and also taking them into account. Furthermore, for more precise investigations it is necessary to know the statistical dependences between the random deviations of each parameter at different altitudes, between the deviations in different parameters at a given altitude, etc.

Various methods of describing the perturbations in atmosphere parameters are possible. Let us examine one of them. Temperature T and atmospheric density ρ can be represented in the form

$$T(h)=T_{CT}(h)+\Delta T(h); \tag{1.17}$$

$$\rho(h)=\rho_{CT}(h)\left[1+\frac{\Delta\rho}{\rho_{CT}}(h)\right], \tag{1.18}$$

where $T_{CT}(h)$ and $\rho_{CT}(h)$ – the standard values of temperature and density; $\Delta T(h)$ – the deviation in temperature from the standard temperature; $\frac{\Delta\rho}{\rho_{CT}}(h)$ – the relative deviation in air density from the standard air density.

For assigning random functions ΔT and $\Delta\rho/\rho_{CT}$ it is possible to use the method of canonical expansions [5].

With respect to the case in question atmospheric parameters as random functions of the altitude of a point above the surface of the earth are represented in the form of a canonical expansion in the following manner:

$$\Delta T(h)=\overline{\Delta T}(h)+\sum_{i=1}^{m}\Delta T_i(h)\,b_i; \tag{1.19}$$

$$\frac{\Delta\rho}{\rho_{CT}}(h)=\overline{\frac{\Delta\rho}{\rho_{CT}}}(h)+\sum_{i=1}^{m}\frac{\Delta\rho_i}{\rho_{CT}}(h)\,c_i, \tag{1.20}$$

where $\overline{\Delta T}(h)$, $\overline{\frac{\Delta\rho}{\rho_{CT}}}(h)$ – average deviations SA values corresponding to the point in question; $\Delta T_i(h)$, $\frac{\Delta\rho_i}{\rho_{CT}}(h)$ – certain nonrandom deviation from the mean deviations $\overline{\Delta T}(h)$ and $\overline{\frac{\Delta\rho}{\rho_{CT}}}(h)$.

Such a recording of the parameters of a "random atmosphere" corresponds to its representation in the form of the sum of a certain number of m "atmospheres" with the random coefficients b_i and c_i.

These coefficients and the coordinate functions $\Delta T_i(h)$ and $\frac{\Delta\rho_i}{\rho_{CT}}(h)$ are determined on the basis of cumulative statistical data which characterize the state of the atmosphere. A rather accurate representation of the random parameters of the atmosphere is given by an expansion, including 10-11 terms.

The use of the method of canonical expansions of random atmospheric parameters makes it possible to solve various problems which arise during the designing of missiles. One of most frequently encountered problems is the problem of evaluating the statistical characteristics of atmospheric parameters taking into account the random character of the variation in the coordinates and the flight time of the missile (the geographic coordinates of the motion of the missile and the flight time were unknown earlier). A typical example of this type of problem is the problem of missile deflection. The structure of the canonical expansion in this case reduces to determining the unknown random variables and the coordinate functions for a rather extensive area from the data obtained by meteorological sounding of the atmosphere.

In missile design another group of calculations (for instance, when evaluating the strength of an apparatus) is encountered, the purpose of which is the study of the characteristics of an object for the worst (extreme) flight conditions and an evaluation of the effect of maximum deviations. The most important of the calculations of this type are the calculations at points which correspond to the greatest (in value) deviations in thermodynamic parameters. Since there can be different combinations of large deviations, it is possible to recommend two sets of functions: one corresponds to typical adverse winter conditions, and the second - to summer conditions. The lowest temperatures and the greatest densities near the earth and the lowest densities of high altitudes are characteristic of an adverse point in winter. In summer the adverse point is characterized by very high temperatures and low densities near the earth and high densities in the stratosphere.

In certain cases for the purpose of simplifying calculations instead of using canonical expansions for extreme conditions it is possible to use the maximum values of atmospheric temperature with respect to altitude. As maximum temperature distributions in this case temperatures for the so-called "standard days" – the maximum temperatures of a warm day and the minimum temperatures of a cold day – are taken.

The corresponding maximum values of relative air density are determined by the equation of state and the differential equation of equilibrium.

Wind characteristics are determined by analogy with the determination of random atmospheric parameters. In solving the first type of problems a systematic wind of constant direction [prevailing wind] (from west to east) and a random wind component are distinguished. In calculating the controllability and the strength of a missile an envelope of wind speeds with respect to height, which corresponds to maximum values, is used.

1.3. AERODYNAMIC FORCES AND MOMENTS.

The aerodynamic forces arising during the motion of a missile in the atmosphere, can be reduced to one resultant force $\bar{R}$, passing through the center of mass of the missile and the so-called *total aerodynamic force*, and resultant moment $\bar{M}$, acting relative to the center of mass of the missile and called *total aerodynamic moment*. The value and the direction of vectors $\bar{R}$ and $\bar{M}$ depend on a number of factors, including the orientation of the missile relative to the velocity vector of the airstream, incident on the missile, air density, etc.

In flight vehicle dynamics for determining the orientation of a vehicle relative to the airspeed vector and expanding force $\bar{R}$ and moment $\bar{M}$ along the coordinate axes wind and body systems of coordinate axes are usually used.

The body system of coordinate axes $Ox_1y_1z_1$ is a Cartesian, rectangular, right-hand system of coordinate axes, fixed relative to the missile or nose section (Fig. 1.6). The axes of this system are called *body axes* for short.

The origin of the coordinates of a body system is placed at the center of mass of the missile; axis Ox_1 is directed along the longitudinal axis of the missile in the direction of the nose section; axis Oy_1 is placed in that plane of symmetry of the missile which at the moment of launch coincides with the plane of firing – with the plane Ox_0y_0 of the initial launch coordinate system[1]. If aiming is accomplished without turning the missile on the launching device, then for plane Ox_1y_1 any plane of symmetry of the missile can be taken.

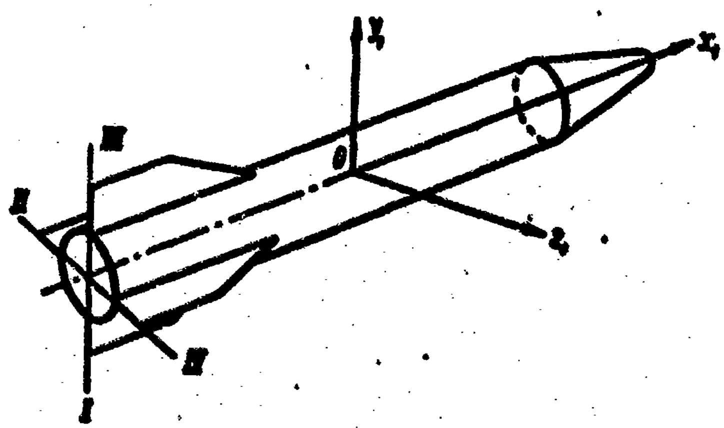


Fig. 1.6. Missile with body coordinate axis system.

For the nose section the directions of the axes Oy_1 and Oz_1 are conveniently selected in such a way that on the nose section connected with the rocket, axis Oy_1 is oriented in the direction of stabilizer III, and axis Oz_1 – in the direction of stabilizer IV (Fig. 1.7).

[1]see Sect 2.1.

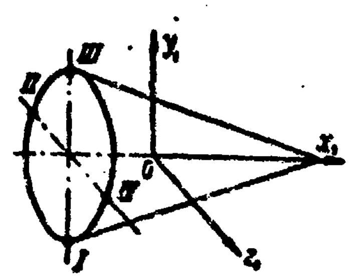


Fig. 1.7. Nose section with a body coordinate axis system.

The wind coordinate axis system $Ox_{II}y_{II}z_{II}$ – Cartesian, rectangular right-handed (Fig. 1.8). The origin of the coordinates of this system coincides with the center of mass of the missile; axis Ox_{II} is directed along velocity vector $\bar{V}_W$ of the missile relative to the air medium; axis Oy_{II} lies in the plane of symmetry of the missile Ox_1y_1.

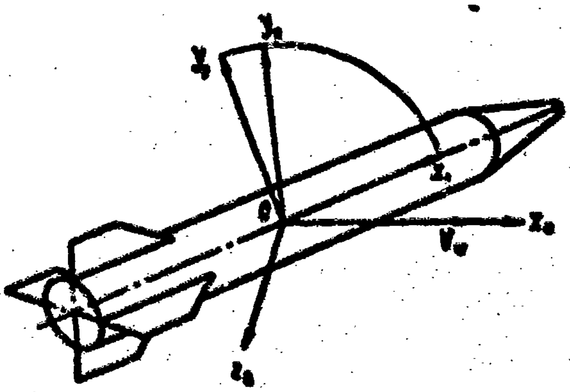


Fig. 1.8. A wind coordinate axis system.

The orientation of the missile relative to the airspeed vector $\bar{V}_W$ in the general case is determined by the angles of attack α and sideslip β, i.e., by angle β – between the velocity vector $\bar{V}_W$ and the plane of symmetry of the missile Ox_1y_1 and by angle α – between the projection of the velocity vector $\bar{V}_W$ on the plane of symmetry of the missile Ox_1y_1 and the longitudinal axis of the missile Ox_1.

The transformation to the arbitrary position of the body axes relative to the wind axes is accomplished by means of two rotations – by turning the body axes relative to axis Oy_1 by angle of sideslip β and then relative to axis Oz_1 by angle α (Fig. 1.9).

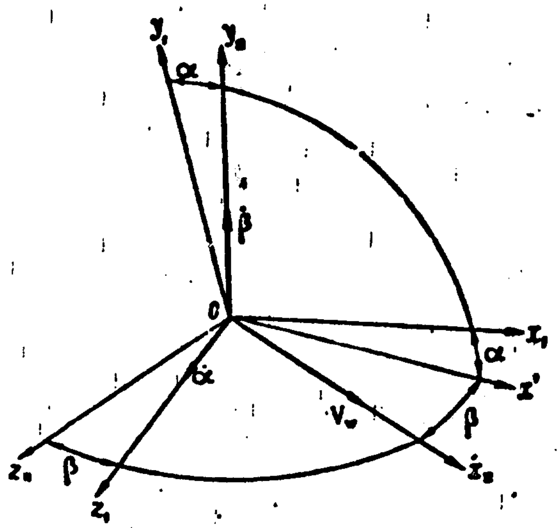


Fig. 1.9. Transformation from wind to body axes.

The cosines of the angles between the body and wind axes are given in Table 1.1.

Table 1.1. The cosines of angles between body and wind axes.

(1) Оси	Ox_{II}	Oy_{II}	Oz_{II}
Ox_1	$\cos\alpha\cos\beta$	$\sin\alpha$	$-\cos\alpha\sin\beta$
Oy_1	$-\sin\alpha\cos\beta$	$\cos\alpha$	$\sin\alpha\sin\beta$
Oz_1	$\sin\beta$	0	$\cos\beta$

KEY: (1) Axes.

The total aerodynamic force $\bar{R}$, dependent on angles α and β, is usually broken down into the components X, Y, Z for wind coordinate axes or into components X_1, Y_1, Z_1 for body axes:

$$R=\bar{X}+\bar{Y}+\bar{Z}=\bar{X}_1+\bar{Y}_1+\bar{Z}_1. \tag{1.21}$$

Since the projection of force $\bar{R}$ on the wind axis Ox_{II} is always negative, it is customary to examine the components X and X_1 of force R with respect to negative directions of axes Ox_{II} and Ox_1. Thus, the projections of force R on these axes – the drag and axial force are respectively equal to:

$$R_{x_{II}}=-X;\ R_{x_1}=-X_1.$$

A ballistic missile (without fin stabilization or with a cruciform fin assembly) is practically an aerodynamically axisymmetric body. If the axis of a missile is directed along the airspeed vector ($\alpha = \beta = 0$), then the flow of the missile will be symmetrical relative to its axis and therefore, forces Y and Z (or Y_1 and Z_1) will be equal to zero.

If the axis of a missile forms a certain angle with the airspeed vector, then the flow will be symmetrical relative to the plane, passing through the axis of the missile and the airspeed vector. In this case the total aerodynamic force, and consequently its component also, for example lift Y or normal force Y_1, will be located in this plane. Hence it follows that for an aerodynamically axisymmetric missile the dependences of force Z and Z_1 on angle β are analogous to dependences Y and Y_1 on α. Moreover, it is generally possible not to examine the angle of sideslip β, if the angle included between the longitudinal axis of the missile and the airspeed vector is taken for the angle of attack and the position in space of the plan, passing through the axis of the missile and the velocity vector is determined. Subsequently we will proceed precisely in this way.

On the basis of the theory of aerodynamic similarity aerodynamic forces are usually expressed in the following manner:

$$\left.\begin{aligned} X&=c_x qS; & X_1&=c_\tau qS; \\ Y&=c_y qS; & Y_1&=c_n qS, \end{aligned}\right\} \tag{1.22}$$

where $q=\varrho\frac{V_W^2}{2}$ – dynamic head; ρ – air density; S – characteristic area of the missile, usually the area of the maximum cross section; c_x and c_y; c_τ and c_n – *dimensionless aerodynamic coefficients.*

Aerodynamic coefficients depend on the shape of a missile, the orientation of a missile relative to the airspeed vector (i.e., on angle α) and on the criteria of aerodynamic similarity – Mach number $M=\frac{V_W}{a}$ and Reynolds number $Re=\frac{V_W l}{\nu}$,
where a – sound propagation velocity in air; l – the characteristic dimension of a missile, usually its length; ν – the kinematic

coefficient of air viscosity.

Using Table 1.1 for the case $\beta = 0$ and taking into account that the negative directions of the axes Ox_{II} and Ox_1 correspond to the positive values x and x_1, we obtain

$$\left.\begin{array}{l} c_\tau = c_x \cos\alpha - c_y \sin\alpha; \\ c_n = c_x \sin\alpha + c_y \cos\alpha. \end{array}\right\} \qquad (1.23)$$

Since during the flight of a missile in the atmosphere angle α is small — of the order of a few degrees, it is possible to consider that $\cos\alpha \simeq 1$, and $\sin\alpha \simeq \alpha$. Then we will obtain the expression of (1.23) in the approximate form

$$\left.\begin{array}{l} c_\tau \approx c_x - c_y\alpha; \\ c_n \approx c_x\alpha + c_y. \end{array}\right\} \qquad (1.24)$$

The aerodynamic characteristics of the missile are studied in detail in specialized literature and thus we will not dwell on them here in any great detail. Let us only note the basic features of the aerodynamic characteristics of a missile.

Aerodynamic investigations show that at small angles of attack ($\alpha \leq 10°$) the coefficient of axial force c_τ depends little on the angle of attack, and the coefficients of lift c_y and normal c_n forces are proportional to the angle of attack:

$$c_y = c_y^\alpha \alpha; \qquad c_n = c_n^\alpha \alpha, \qquad (1.25)$$

where c_y^α and c_n^α — partial derivatives depending on the corresponding coefficients for angle of attack.

The derivative of the coefficient of normal force c_n^α depends, mainly, on M number (Fig. 1.10). In the transonic speed range for a missile ($M \simeq 1$) this coefficient has a peak value, and with a further increase in M numbers diminishes, tending toward a certain constant value.

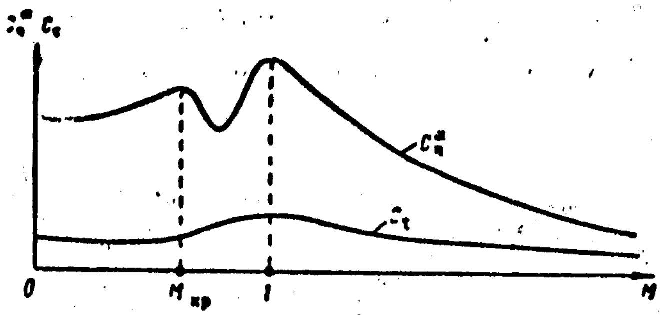


Fig. 1.10. The approximate dependence of aerodynamic coefficients c_τ and c_n^α on M number.

The coefficient of axial force c_τ depends substantially on the shape of the missile, the angle of attack, M and Re numbers. The approximate form of the dependence $c_\tau(M)$ is shown in Fig. 1.10. Since the speed of sound and the kinematic coefficient of viscosity ν depend on altitude, then at a given velocity V_W M and Re numbers, and with them and coefficients c_n^α and c_τ vary with altitude, coefficient c_n^α weakly, and coefficient c_τ substantially.

It must be noted that the coefficient of axial force also depends on rocket engine operation. When the engine is not operating axial force X_1 increases by the magnitude of the corresponding increase in wake drag.

Considering what has been said above, it is possible to state that aerodynamic forces depend upon the shape and the dimensions of the missile, the angle of attack, and the flight velocity and altitude.

The total aerodynamic moment acting on a missile, is usually broken down into the components for the axes Ox_1, Oy_1, Oz_1. These components M_{x1}, M_{y1}, and M_{z1} are respectively called bank, yaw and pitch moments.

The value and direction of total aerodynamic moment depend on a number of factors, among which the characteristics of missile motion

relative to the air medium have a significant value – the orientation of the missile relative to the velocity vectors of the center of mass and the angular velocity of the missile and the magnitudes of these velocities V_W and ω.

Let the center of mass of a nonrotating missile lie on its longitudinal axis at distance x_T from the tip of the nose section, and the center of pressure be located at a distance x_d from the tip of the missile. Then the value of total aerodynamic moment relative to the center of mass will be equal to

$$M = c_n qS(x_T - x_d) \quad (1.26)$$

or at small angles of attack

$$M = c_n^{\alpha} qS(x_T - x_d)\alpha. \quad (1.27)$$

This moment as well as total aerodynamic force, acts in the plane, passing through the longitudinal axis of the missile and the velocity vector.

The position of the center of pressure in the general case depends on the shape of the missile, M number and the angle of attack. Figure 1.11 shows a typical displacement of the center of pressure of a missile with variation in M number. It can usually be considered that the position of the center of pressure remains constant, if the angle of attack varies in a certain small vicinity of its zero value.

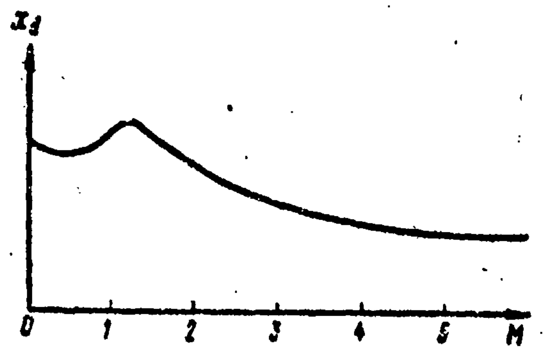


Fig. 1.11. The position of the center of pressure of a missile depending on M number.

When the missile is rotating, atmospheric drag is manifested, mainly, in the form of aerodynamic moment of resistance to rotation. This moment, called *damping moment,* is always oriented in the direction, opposite to the rotation and tends to cancel the angular velocity of rotation.

Damping moment depends on the value of angular velocity as well as on its orientation relative to the missile. Usually the damping moment vector is broken down with respect to the body axes into components, proportional to angular velocity relative to the corresponding axes:

$$\begin{aligned} M_{x_{x1}} &= m_{x1}^{\omega_{x1}} qS \frac{D^2}{V} \omega_{x1}; \\ M_{z_{y1}} &= m_{y1}^{\omega_{y1}} qS \frac{l^2}{V} \omega_{y1}; \\ M_{z_{z1}} &= m_{z1}^{\omega_{z1}} qS \frac{l^2}{V} \omega_{z1}, \end{aligned} \tag{1.28}$$

where D and l – the diameter and the length of the missile respectively.

Each of the dimensionless coefficients $m_{x1}^{\omega_{x1}}$, $m_{y1}^{\omega_{y1}}$, $m_{z1}^{\omega_{z1}}$ is always negative. The values of these coefficients in the flight range of angles of attack depend, mainly, on the geometric shape of the missile, its centering [position of its c.g.] x_T M number.

1.4. CHARACTERISTICS OF ROCKET ENGINES.

The basic characteristics of a rocket engine are its thrust P, specific thrust $P_{уд}$, propellant consumption per second $\dot{m}g_0$ and its propellant components ratio K.

As is known, the thrust force of a rocket engine P is connected with its mass consumption per second $\dot{m}$, the exhaust velocity of the products of combustion w, the gas pressure at the nozzle exit section p_a, the atmospheric pressure p and the area of the nozzle exit cross section S_a by dependence

$$P = \dot{m}w + (p_a - p) S_a. \tag{1.29}$$

As is evident, thrust force depends upon altitude. At sea level, where $p = p_0$, thrust force has its least value

$$P_0 = \dot{m}w + (p_a - p_0)S_a, \tag{1.30}$$

and in a vacuum – its greatest

$$P_\text{п} = \dot{m}w + p_a S_a = P_0 + p_0 S_a. \tag{1.31}$$

The increase in thrust force with a change in atmospheric pressure from p_0 to zero can attain 25% of thrust P_0 at sea level.

The dependence of thrust force on altitude can be written in the form

$$P = P_\text{п} - pS_a. \tag{1.32}$$

The ratio of engine thrust to mass consumption per second is usually called specific thrust:

$$P_\text{уд} = \frac{P}{\dot{m}g_0} = \frac{P_0 + p_0 S_a}{\dot{m}g_0} - \frac{pS_a}{\dot{m}g_0}. \tag{1.33}$$

Specific thrust is the thrust force created by an engine during the combustion of 1 kg of propellant in 1 s, and characterizes the efficiency of the engine system. Specific thrust in a vacuum is determined by the formula

$$P_\text{уд.п} = \frac{P_0 + p_0 S_a}{\dot{m}g_0} = \frac{w}{g_0} + \frac{p_0 S_a}{\dot{m}g_0}. \tag{1.34}$$

Using the concept of specific thrust in a vacuum, expression (1.32) can be rewritten in the form

$$P = \dot{m}g_0 P_\text{уд.п} - p_0 S_a \left(\frac{p}{p_0}\right). \tag{1.35}$$

The following formula for determining thrust is also finding application in design practice:

$$P = Ap_\text{к} - pS_a, \tag{1.36}$$

where $p_к$ – the combustion chamber pressure of the engine; A – the proportionality factor whose magnitude is determined on the basis of engine bench tests.

On the basis of expression (1.36) it is possible to write the appropriate formulas for determining specific thrust, for example:

$$P_{уд.п}=A\frac{p_к}{\dot{m}g_0}. \qquad (1.37)$$

The convenience of formulas (1.36) and (1.37) consists in the fact that thrust and specific thrust are connected in them by a linear dependence with the combustion chamber pressure whose magnitude is measured by bench tests.

The propellant components ratio $K = \dot{G}_{ок}/\dot{G}_г$ characterizes the propellant mixture composition. Here $\dot{G}_{ок}$ and $\dot{G}_г$ are the mass consumption rates respectively of the oxidizer and the fuel per unit of time.

Knowing the total propellant consumption in one second $\dot{G}_{топл}=\dot{m}g_0$ and the magnitude of parameter K, it is possible to determine the mass consumption per second of oxidizer and fuel by the formulas:

$$\left.\begin{aligned}\dot{G}_{ок}&=\frac{K}{K+1}\dot{G}_{топл};\\ \dot{G}_г&=\frac{1}{K+1}\dot{G}_{топл}.\end{aligned}\right\} \qquad (1.38)$$

Rocket engine thrust substantially varies in time under transient conditions (in starting and switching down the engines). The dependence of thrust on time (the transient characteristics of a rocket engine) is represented in Fig. 1.12. As is evident, the combustion chamber pressure, and thus, the thrust force also attain their optimum values not immediately after starting of the engine. A certain time passes from the moment of the introduction of the

command for starting the engine before the beginning of ignition. Thrust practically appears at the moment of ignition.

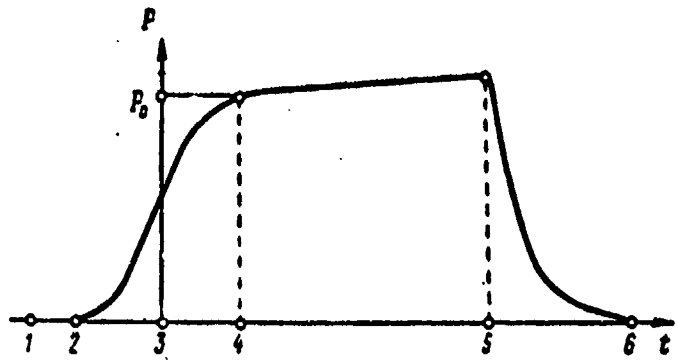


Fig. 1.12. The variation in the thrust force of a liquid-propellant rocket engine [ЖРД = LPRE] during the time of a rocket flight: 1 – command for starting; 2 – ignition; 3 – separation of the rocket from the launch pad; 4-5 – optimum thrust phase; 6 – command for engine shut-down; 5-6 – the aftereffect [thrust trailoff] phase.

Upon switching off of an engine the thrust does not also disappear instantaneously – a so-called, aftereffect phenomenon is observed. After the command for engine shut-down due to the afterburning of a specific quantity of propellant a certain thrust continues to be created whose impulse, called the aftereffect impulse, is expressed by the formula

$$I_{\text{пд}} = \int_{t_{\text{к}}}^{t_{P=0}} P(t)\,dt,$$

where $t_{\text{к}}$ – the time of the introduction of the command for engine shut-down; $t_{P=0}$ – the moment of time corresponding to zero thrust.

The aftereffect impulse is a random variable whose variance can constitute up to 15% of the mean value of this impulse [12]. This feature of the transient engine characteristics affects the conditions of stage separation and the separation of the nose section of missiles.

The variance in the value of the impulse aftereffect depends on the time variance corresponding to zero thrust, and the thrust variance which, in turn, depends on the pressure variance in the combustion chamber, the variance in parameter K and other factors.

1.5. THE MISSILE AS A BODY OF VARIABLE COMPOSITION.

The Mass of a Missile, the Position of the Center of Mass, Inertia Moments.

In connection with the continous burning of propellant the mass of a missile varies during flight and it can be found from the expression

$$m=m_0-\int_0^t \dot{m}dt, \tag{1.39}$$

where $\dot{m} = \left|\frac{dm}{dt}\right| = -\frac{dm}{dt}$ – the mass consumption per second; $t = 0$ – the moment of engine activation; m_0 – initial mass of the rocket.

Mass consumption per second $\dot{m}$ consists of the mass consumption per second passing through the combustion chambers of the main engines, through the exhaust pipes of the turbopump unit [THA = TPU] as well as through the combustion chambers of the controlling engines.

Usually during flight variations take place in the mass consumption rate per second, caused by a variation in the engine operating mode, and also by various random factors. The most significant variations in consumption per second occur in the transient modes of engine operation (activation, switching to smaller thrust, complete shut-down).

The position of the center of mass of a missile, determined by coordinate x_T (Fig. 1.13), and the moments of inertia of a missile vary in proportion to propellant burnup. These values can be taken into account dependent on the mass of a missile, i.e.,

$$x_T(m);\ J_{x1}(m);\ J_{y1}(m);\ J_{z1}(m).$$

Figure 1.14 for example contains graphs which illustrate the nature of the variation in mass and moment of inertia relative to the transverse axis for the first stage of a two-stage rocket.

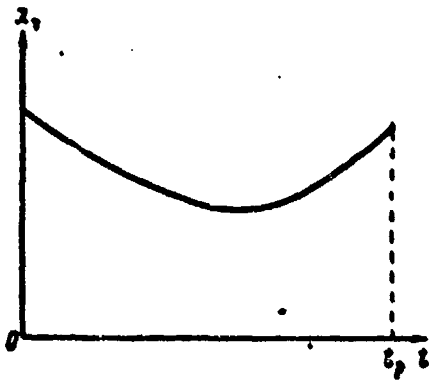


Fig. 1.13. The variation in the dimensionless coordinate of the center of mass of a single-stage rocket $\bar{x}_T = x_T/l$ during the time of flight: t_p – the moment of engine shut-down.

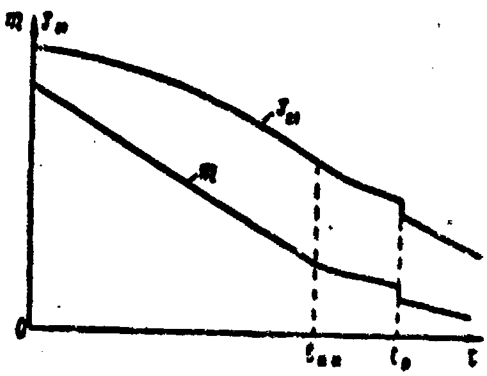


Fig. 1.14. The variation in mass and inertia moment of a two-stage rocket during the time of flight: $t_{p.\kappa.}$ – the moment of the preliminary command; t_p – the moment of stage separation.

The Principle of Composing Equations of Motion of a Missile as a Body of Variable Composition.

During the flight of a rocket with the engine operating emission of the products of combustion takes place and the composition of the rocket is continuously varying. In examining the motion of a rocket, it is convenient at each moment of time to include in its composition only those material particles which at that moment are located inside the specific volume occupied by the rocket itself. When the problem is composed in this way the rocket with the engine operating is a

system of variable composition, to which it is not possible to directly apply the theorems of the dynamics of rigid bodies. However, based on these classical theorems, it is possible to prove analogous theorems for a system of variable composition and to establish the principle for composing the equations of motion of a rocket.

Rejecting the proofs presented, for example, in book [6], let us examine the principle for composing the equations of motion of a rocket.

The equations of motion of a reaction flight vehicle at an arbitrary moment t can be written in the form of the equations of the motion of a solid body which is obtained as a result of the "solidification" of reaction vehicle at this instant, if we include the reactive forces among the forces, applied to this fictitious solid body.

Thus, the vector equation of the motion of the center of mass of the rocket can be written in the form

$$m\frac{d\overline{V}_a}{dt}=\sum\overline{F}_i+\sum\overline{P}_i. \qquad (1.40)$$

Here $m = m(t)$ – the mass of the rocket at moment of time t; $\frac{d\overline{V}_a}{dt}$ – the acceleration of the center of mass in an inertial coordinate system; $\sum\overline{F}_i$ – the sum of the external forces, applied to the rocket; $\sum\overline{P}_i$ – the sum of the reactive forces.

By the external forces, acting on a rocket, are meant such forces, as gravity $\overline{G}_T$, total aerodynamic force $\overline{R}$, the force of the interaction of the rocket with the launch pad or with a separated stage.

As is evident, the composing of equations of motion of a body of variable composition reduces to the determination of the reactive forces, which is a rather complex problem. The main one of these forces is the reactive force $\dot{m}w$, which cannot be directly measured. Thus it is customary to determine the thrust force of a rocket engine

by formula (1.29), in which is included the force caused by atmospheric pressure and by the gas pressure at the nozzle exit section $(p_\alpha - p)S_\alpha$. Although this force is external, it is combined with the strictly reactive power $\dot{m}w$, since during the testing of an engine on a stand the force acting on the stand mounts, determined by dependence (1.30), is measured. Accordingly the force $(p_\alpha - p)S_\alpha$ is eliminated from the number of external forces $\sum \bar{F}_i$.

Besides thrust force P, determined by formula (1.29), the following are included in the composition of the reactive forces:

1) the forces caused by the nonsteady-state of the motion of the propellant and the products of combustion relative to the missile body;

2) the Coriolis forces caused by the motion of propellant and the products of combustion in a rocket rotating relative to an inertial coordinate system;

3) the forces caused by the displacement of the center of mass of a rocket relative to its housing.

The enumerated forces are very small in comparison with thrust determined by the formula (1.29), and their direct measurement is not possible. Depending on the assumptions made, different authors obtain different theoretical expressions for them. In ballistics the indicated small reactive forces are usually disregarded and the thrust forces of rocket engines are determined by the expression (1.29).

The vector equation of the rotary motion of a rocket relative to the center of mass is composed in an analogous manner:

$$\frac{d\bar{K}}{dt} = \sum \bar{M}_{F_i} + \sum \bar{M}_{P_i}. \qquad (1.41)$$

Here $\bar{K}$ – the main moment (relative to the center of mass of the rocket) of the momenta of the particles of a "solidified" missile relative to the axes, passing through the center of mass of the rocket and moving translationally with velocity $\bar{V}_\alpha$ relative to the inertial system;

$\sum\overline{M}_{F_i}$ – the main moment (relative to the center of mass of the rocket) of all the external forces acting on a missile, with the exception of the forces of atmospheric pressure and gas pressure in the nozzle exit section; $\sum\overline{M}_{P_i}$ – the main moment (relative to the center of mass of the rocket) of the thrust force of a rocket engine, and also the forces caused by the motion of the propellant and the gases inside the rotating missile, by the nonsteady-state of this motion and by the displacement of the center of mass of the rocket relative to its h using.

Subsequently for the sake of simplifying the equations of rotary motion of a missile relative to its center of mass we will disregard the moments of forces caused by the nonsteady-state of the motion of the propellant and the gases inside the rocket and by the displacement of the center of mass of the rocket relative to its housing, since these moments are rather small.

It is necessary to note that significant (in value) moments caused by oscillations of the fluid in the tanks of the missile occur when a free surface exists. However with an appropriate selection of the parameters of a stabilization system the oscillations of the missile due to the mobility of the fluid in the tanks are small and their effect on the missile trajectory is insignificant. In this connection we will omit the study of moments caused by the interaction of the fluid with the missile body, especially because these questions are the subject of investigation by special sections of missile dynamics.[1]

1.6. PERTURBING FORCES AND MOMENTS.

In actual flight perturbing forces and moments caused by various perturbing factors always act on a missile.

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[1]See, for example, works [1], [17].

In composing a mathematical model of a missile flight and its investigation it is not possible to take all these perturbing factors into account. Depending on the actual conditions it is necessary to consider only those of them which substantially affect the solution of the given problem. Thus here we will limit ourselves to only a brief survey of the basic groups of perturbing factors, presenting a more detailed examination of them in the appropriate sections of this book.

Such perturbing factors as the deviations in the parameters of a missile and its engines (the weight of the missile, the thrust force of the main engines, the fuel consumption per second and others) from their optimum values, are caused, mainly, by production errors in the manufacture and the assembly of elements and subassemblies making up a missile, and by the variance in the propellant characteristics. These deviations in the parameters of a missile and engines from the optimum values and such manufacturing errors, as thrust eccentricity in the main engine, missile asymmetry, body misalignment, etc., cause the appearance of random perturbing forces (the forces of gravity, reactive and aerodynamic forces) and their moments.

The atmosphere is another source of perturbances. Deviations in the parameters of the atmosphere from standard values give rise to the appearance of perturbing aerodynamic forces and moments and to the deviation in thrust from the optimum value. Wind effects on a missile also cause the perturbances in aerodynamic forces and moments. Atmospheric perturbances are a random process and are accordingly described by random functions.

All these perturbing forces and moments are applied directly to the missile. Besides these, perturbing forces and moments arising as a result of various errors in control element deflection are always acting. The common sources of such perturbing effects are noises, errors in the operation of equipment and deviations in the parameters of the equipment from their optimum values, leading to various false signals in the control elements. As a result perturbing forces and moments appear which are, generally speaking, random variables.

Perturbing factors, acting directly on a missile and on the control processes, finally, lead to a reduction in maximum flight range and to deflection of the flight paths of the missile and nose sections.

1.7. CONTROLLING FORCES AND MOMENTS.

The control of the translational and rotary motion of a missile is carried out by varying the controlling forces and moments which are created with the rocket control elements upon commands from the control system. The selection of the type of control elements and their effectiveness is one of the most important questions in the dynamic designing of a missile. Its solution is directly connected with the selection of the structural layout of a missile.

Controlling Forces.

In the general case the following forces act on a missile during flight: gravity $\bar{G}$, total aerodynamic force $\bar{R}$ and engine thrust force $\bar{P}$. During the launching of a missile launch pad reaction forces can also act on it.

For varying the flight path of a missile it is necessary to vary the magnitude and the direction of the resultant forces indicated above. Since it is not possible to affect gravity, flight control is practically accomplished only by varying the magnitude and the direction of the resultant $\bar{N}$ of thrust forces of the engines and of the aerodynamic forces. The resultant $\bar{N}$ can be broken down into two components $\bar{N}_t$ and $\bar{N}_n$, directed respectively along velocity vector $\bar{V}$ and perpendicular to it (Fig. 1.15).

Tangential component $\bar{N}_t$, equal in value to,

$$N_t = P_t - X, \tag{1.42}$$

can serve to regulate flight velocity. The variation in tangential component $\bar{N}_t$ for ballistic rockets is attained by varying the thrust of the main engines (for instance, by regulating the propellant

consumption per second, if the engine is liquid-propellant) and by activating or shutting down various engines. That part of thrust force ΔP which can be used for regulating missile speed, let us call tangential controlling force.

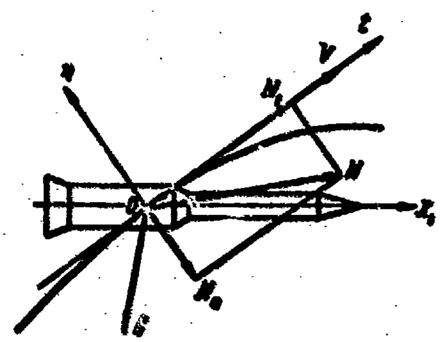

Fig. 1.15. Forces acting on a rocket in flight.

Normal force $\bar{N}_n$ is equal to the sum of the projections of total aerodynamic force and thrust forces in the plane, normal to the trajectory:

$$N_n = P_n + R_n = P_n + \bar{Y} + Z. \qquad (1.43)$$

Its component in the firing plane $\bar{N}_y$ we call normal controlling force.

By analogy with normal controlling force let us introduce the concept of lateral controlling force N_z, which is a projection of force N_n on a perpendicular to the plane of flight.

By creating the required (in magnitude and direction) tangential, normal and lateral controlling forces, it is possible to ensure an assigned flight trajectory of a rocket.

Various methods of creating normal and lateral controlling forces are possible. For ballistic rockets, in order to obtain a normal force of different magnitude, it is necessary to vary the angle of attack in the plane of flight α_y, turning the rocket around its center of mass. When a rocket has an angle of attack α_y, the normal force is equal to (Fig. 1.16)

$$N_y = P \sin\alpha_y + Y \approx (P + c_y^\alpha qS)\alpha_y. \qquad (1.44)$$

In order to obtain lateral controlling force, it is necessary to give the rocket angle of attack α_z in the plane, perpendicular to the firing plane.

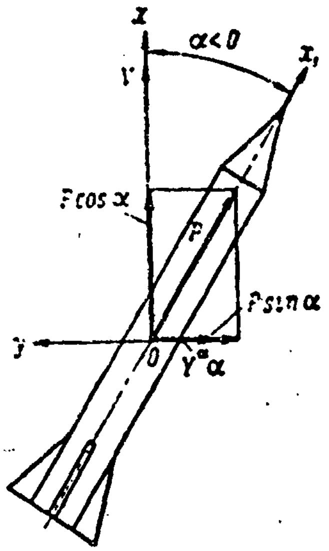

Fig. 1.16. Diagram of the onset of normal controlling force in a rocket.

Rocket Control Elements. Controlling Moments.

As was stated above, to obtain the required (in magnitude and direction) normal force it is necessary in a specific manner to regulate the orientation of a rocket relative to the velocity vector. This problem is solved by creating controlling moments which rotate the rocket around its axes Ox_1, Oy_1, and Oz_1. The corresponding motions are usually called roll, yaw and pitch motions. For producing controlling moments there are control elements on the rocket. The latter create comparatively small aerodynamic or reactive forces, whose moments relative to the center of mass of the rocket are sufficient for controlling the angular motions of the rocket.

For varying normal and lateral force rotation of the rocket around its Oy_1 and Oz_1 axes with the aid of yaw and pitch control elements is

employed. These same control elements are necessary for stabilizing the required orientation of a rocket in space. For stabilizing a rocket with respect to roll[1] still other elements controlling roll are necessary, creating controlling moment relative to the longitudinal axis Ox_1. And finally a control element is necessary for varying the thrust force of the main engines, if it is necessary to regulate rocket velocity.

At the present time the following basic types of elements are employed for controlling ballistic missiles:

1) air vanes;

2) jet vanes;

3) turning combustion chambers of the main engines (one or several);

4) turning nozzles of the main engines;

5) special adapters at the nozzle edge (spherical, cylindrical with an oblique edge and others);

6) slotted nozzles;

7) extensible flaps operating in the engine jet perpendicular to the flow;

8) the blowing of generator gas or the injection of a fluid into the supersonic part of the nozzle of the main engine;

9) multi-chamber main engine operating in a boosting-throttling mode;

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[1]Rotation relative to the longitudinal axis Ox_1 for varying the lateral controlling force of ballistic rockets is not employed. However for simplifying the control system it is necessary that rocket flight occur without rotation around this axis, i.e., without roll.

10) controlling engines (fixed and turning);

11) controlling nozzles (turning and fixed);

12) combined control element (for instance, air and jet vanes or air vanes with the main chambers operating in a boosting-throttling mode).

All the enumerated control elements can create controlling yaw and pitch moments, however not all of them are suitable for producing rolling moment. It is not possible to obtain rolling moment, if for pitch and yaw control, for example, one turning engine is used, or, if the forces creating pitching and yawing moments, are directed along the longitudinal axis of the rocket. In these cases for roll control it is necessary to use the special controlling engines whose thrust acts in the transverse plane.

In all other cases, when there are not less than two pairs of pitch and yaw control elements, creating transverse forces at a certain distance from the longitudinal axis, for producing rolling moment *differential control* of the control elements is employed. The latter can act symmetrically, creating pitching or yawing moment, or asymmetrically, creating rolling moment. With a combination of the indicated operations pitching (yawing) moment and rolling moment can also be simultaneously created.

The magnitude of the forces created by the control elements, depends on the displacement of these elements (most frequently angular) or on the propellant consumption per second, if misalignment of the thrusts of the main engines is used for control.

Let us examine the definition of the forces created by the control elements, and of controlling moments as an example of control of rocket motion with the aid of four controlling engines.

In many contemporary rockets control of rocket motion in the powered-flight phase is accomplished by four controlling engines.

The location of these engines and the directions of their deflection taken as positive, are shown in Fig. 1.17. We will consider its turning counterclockwise as positive deflection of a controlling engine, if it is looked at from the direction of the corresponding axis, i.e., in Fig. 1.17 deflections of engines II-IV downward, and engines I-III to the right will be positive.

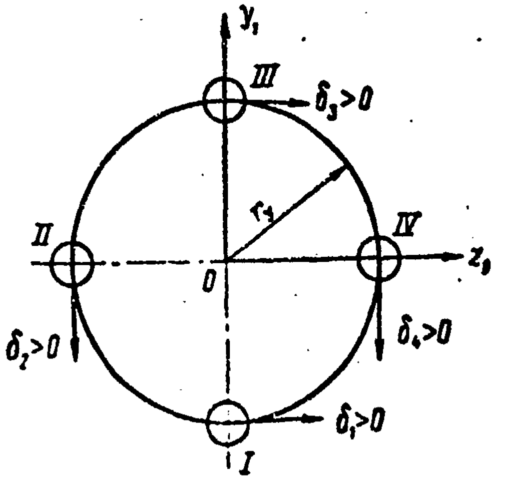

Fig. 1.17. Diagram of the onset of controlling moment upon deflection of the controlling engines.

Assuming the thrusts T of all four controlling engines equal, let us write the projections of their resultant on body coordinate axes in the form:

$$\left.\begin{aligned} T_{x1}&=T(\cos\delta_1+\cos\delta_2+\cos\delta_3+\cos\delta_4);\\ T_{y1}&=T(\sin\delta_2+\sin\delta_4);\\ T_{z1}&=-T(\sin\delta_1+\sin\delta_3),\end{aligned}\right\} \qquad (1.45)$$

where δ_1, δ_2, δ_3, δ_4 – respectively the angles of deflection of controlling chambers I, II, III, and IV.

Apparently, the controlling moments in this case will be equal to:

$$\left.\begin{aligned} M_{y\,x1}&=Tr_y(\sin\delta_1+\sin\delta_2-\sin\delta_3-\sin\delta_4);\\ M_{y\,y1}&=-Tl_y(\sin\delta_1+\sin\delta_3);\\ M_{y\,z1}&=-Tl_y(\sin\delta_2+\sin\delta_4),\end{aligned}\right\} \qquad (1.46)$$

where r_y – the distance from the longitudinal axis of the rocket to the longitudinal axes of the controlling chambers; $l_y = x_T - x_{дв}$ – distance from the center of mass of the rocket to the axes of rotation of the controlling engines.

In these expressions the thrust force of the controlling engine is determined depending on its parameters by formula (1.35).

Rocket Controllability.

Ensuring the controllability of a rocket is one of the major problems of dynamic design which reduces to the selection of the type and the effectiveness of the control elements. We will estimate control element effectiveness by the maximum controlling moment $M_{y\ max}$, created by deflecting the control elements, and by the corresponding angle of deflection of these elements δ_{max}.

The selection of the type and effectiveness of control elements is carried out taking into account the possibility of the design realization of the following conditions:

1) a sufficient amount of controlling moments (with certain reserves) for compensating for the perturbing forces and moments;

2) minimum energy losses during controlling.

The energy losses during controlling and, as a consequence of this, the reduction in firing range are due, mainly, to two causes, namely:

1) the installation of control elements gives rise to a reduction in the specific thrust of the engine system;

2) the installation of control elements causes an increase in the "dry" weights of rocket stages (because of the weights of the control elements themselves, their driving mechanisms and the energy sources for the latter).

Overestimation of control effectiveness can lead to unjustified design complications and large energy losses for the rocket. In connection with this the problem of correct determination of the necessary control element effectiveness acquires very vital importance. In solving this problem it is necessary to correctly estimate the perturbing forces and moments and to rationally select the dynamic rocket layout.

All the perturbing factors affecting the selection of the control elements, can be divided into the following groups:

1) wind;

2) technological errors in the manufacture and the assembly of a rocket;

3) rocket layout asymmetry.

At the present time voluminous statistical material has been accumulated characterizing the wind field of the earth.

Wind velocity $\overline{W}$ can be examined as a vectorial random variable with a nonzero mean value. The systematic component of wind velocity $\overline{W}_{\text{сист}}$ is oriented from west to east. The magnitude of a random component of wind velocity whose direction is equiprobable, is subject to the normal distribution principle.

Another group of perturbing factors is caused by technological errors in the manufacture and the assembly of a rocket; the basic ones are the following:

1) bias and misalignment of the axes of the nozzles of the engine system relative to the mounting base;

2) bias and misalignment of the mounting base relative to the correct position;

3) difference in chamber thrusts and a possible variance in engine thrusts;

4) elastic deformation of the engine system mount;

5) bias and misalignment of the joined missile sections;

6) errors in the installation of the stabilizers.

Technological manufacturing and assembly errors in a rocket can be considered subject to the normal distribution principle. In connection with this perturbing effects from each of the enumerated factors can also be considered distributed according to the normal distribution principle.

Asymmetric layout of a missile gives rise to systematic perturbing forces and moments:

1) perturbing moment due to weight asymmetry;

2) perturbing forces and moments due to asymmetric positioning of the exhaust nozzles of engine turbine-pump assemblies;

3) perturbing forces and moments due to elastic deformation of the engine system mount caused by its asymmetric loading.

To each group of perturbing forces and moments acting on a preceding stage of a rocket, there corresponds a group of initial perturbations for the following stage. Initial perturbations due to the stage separation process can be considered random and distributed according to normal distribution principle.

Thus, for evaluating effectiveness of control elements the complex of independent perturbing effects, applied to a missile during flight, which can be reduced to one systematic and to n random independent (between themselves) effects with the normal distribution principles.

During the various flight phases the role of the perturbing factors is not identical.

The basic perturbing factor for single-stage rockets and for the first stages of multi-stage rockets is the wind.

Perturbing forces and moments caused by manufacturing and assembly errors in a rocket and by layout asymmetry (both systematic and random), considerably less than wind forces and moments, do not play a substantial role when evaluating effectiveness of control elements. Thus for preliminary determination of the necessary angles of deflection of the control elements of single-stage and the first stages of multi-stage rockets it is sufficient to estimate the effect on a rocket of wind (taking into account its variation with height).

For second and subsequent stages of missiles with a separation height of more than 40 km the basic perturbing factors are manufacturing and assembly errors and asymmetry of missile layout. The effect of wind in these cases is unimportant.

In the initial flight phases after the separation of the stages it is necessary to consider the effect on the deflections of the control elements of the initial perturbations caused by stabilization errors in the previous stage and by the stage separation process.

For the preliminary selection of the type and the effectiveness of control elements it is possible to determine total perturbing moment

$$M_{\text{вz}} = \sum_{i=1}^{m} M_{\text{в}\, i\, \text{сист}} + \sqrt{\sum_{j=1}^{n} M^2_{\text{в}\, j\, \text{сл}}} \qquad (1.47)$$

and to start with a reserve of 10%, which is

$$M_{y\,max} = M_y^{\delta}\delta_{max} \geqslant 1{,}1 M_{\text{вz}}. \qquad (1.48)$$

Proceeding from this value of necessary controlling moment and taking the design characteristics of the missile into account, it is

possible to select the type of control elements. After this it is necessary to carry out evaluation of controllability (in other words, to estimate the sufficient amount of controlling moments), using the dynamic layout of rocket motion examined in Sect. 2.4.

The order of calculations for determining the deflections (loading) of the control elements in the first and subsequent stages of a missile is given below.

<u>The Order for Carrying Out Calculations for the First Stage</u>.

A. The deflections of the control organs are determined:

1) due to the effect of the systematic component of wind velocity $\delta_{W_{\text{сист}}}$;

2) due to the effect of the random component of wind velocity $\delta_{W_{\text{сл}}}$;

3) due to the effect of the perturbing factors caused by layout asymmetry $\delta_{\text{нс}}$;

4) due to the effect of perturbing forces and moments caused by technological errors in the manufacture and the assembly of a missile $\delta_{\text{п}}$.

B. The total deflection of the control elements is determined

$$\delta_{\Sigma}=\delta_{W_{\text{сист}}}+\delta_{\text{нс}}\pm\sqrt{\delta^2_{W_{\text{сл}}}+\delta^2_{\text{п}}}. \qquad (1.49)$$

C. The loading of the control elements is compared (taking into account the possible random deviations in the parameters of the missile, the guidance equipment and the atmosphere — by introducing a margin of safety 1.1) with the maximum possible deflection

$$1{,}1\ \delta_{\Sigma}\leqslant\delta_{\max}. \qquad (1.50)$$

The Order for Carrying Out Calculations for the Second and Subsequent Stages.

During determining control element loading in second and subsequent stages the combined effect on the motion of a missile of wind, technological errors, layout asymmetry, and also initial perturbations is considered.

The initial perturbations due to stabilization errors at the end of the flight of a previous stage cause the following deflections of the control elements:

– systematic component $\delta^0_{\text{сист}} = \delta^0_{W_{\text{сист}}} + \delta^0_{\text{ас}}$;

– random component $\delta^0_{W_{\text{сл}}}$, $\delta^0_{\text{п}}$.

Random perturbations due to the stage separation process lead to deflection of the control elements $\delta^0_{\text{р}}$.

The order for carrying out the calculations.

A. The component deflections of the control elements are determined:

1) $\delta_{\text{сист}}$ – due to the effect of the algebraic sum of the perturbing forces and moments resulting from the systematic component of wind velocity and the layout asymmetry during the initial perturbances $\delta^0_{\text{сист}}$;

2) $\delta_{W_{\text{сл}}}$ – due to the effect of the random component of wind velocity with zero initial perturbations;

3) $\delta_{\text{п}}$ – due to the effect of perturbing forces and moments caused by technological errors, with zero initial perturbations;

4) $\delta_{W_{\text{сл.н}}}$, $\delta_{\text{п}_{\text{н}}}$, $\delta_{\text{р}_{\text{н}}}$ – due to initial perturbations $\delta^0_{W_{\text{сл}}}$, $\delta^0_{\text{п}}$, $\delta^0_{\text{р}}$.

B. The total deflection of the control elements is determined

$$\delta_{\Sigma}=\delta_{\text{смст}}\pm\sqrt{\delta^{2}_{W_{\text{сл}}}+\delta^{2}_{\text{н}}+\delta^{2}_{W_{\text{сл.н}}}+\delta^{2}_{\alpha_{\text{н}}}+\delta^{2}_{p_{\text{ч}}}}. \qquad (1.51)$$

C. The loading of the control elements is composed (taking into account the possible random deviations in the parameters of the missile, the guidance equipment and the atmosphere) with the maximum possible deflection:

$$1{,}1\delta_{\Sigma}\leqslant\delta_{\text{max}} \qquad (1.52)$$

If this condition is not fulfilled, correction of earlier selected control effectiveness characterized by magnitude M_y^{δ} is carried out.

1.8. THE ROCKET FLIGHT CONTROL SYSTEM.

The Problems and the Makeup of a Control System.

A rocket flight control system controls its motion in the powered-flight phase, ensuring flight in a rather close vicinity of the required flight path, and separation of the stages and nose section of the rocket at the necessary moments of time. This very general formulation of the problems of a flight control system can be somewhat more concretely defined by separating the overall problem into the problems of guidance and stabilization.

The problem of rocket stabilization, and more precise, the controlling of its motion around the center of mass, reduces to controlling the orientation of the rocket axes in space and to maintaining the required orientation. This problem is solved by a group of devices located onboard the rocket, — by the *automatic angular stabilization equipment.*

Due to feedback the rocket and the automatic angular stabilization equipment form a single dynamic system, in which the rocket is one of

the links. Henceforth by *stabilization system* we will understand the closed automatic system consisting of the rocket and the automatic angular stabilization equipment.

Usually a rocket is stabilized (oriented) relative to all three (connected with it) coordinate axes. Accordingly the stabilization system consists of three channels: pitch, yaw, and roll.

The problem of missile guidance (controlling the motion of the center of mass) reduces to the control of the three components of the velocity of the center of mass (longitudinal, normal, and lateral), separation of the stages and the nose section in such a way that the parameters of the motion of the center of mass of the rocket at the moment of the separation of the nose section ensures the free flight of the latter along the required trajectory. The automatic control system, solving this problem, we customarily call the guidance system.

In general a guidance system consists of three channels for controlling of lateral, normal and longitudinal components of velocity and channels controlling the separation of the stages and the nose section. The first three channels operate with the use of feedback and form together with the rocket a closed three-channel dynamic system.

Control of the separation of the rocket parts is accomplished by an open system (without feedback): on the basis of the information about the motion of the center of the mass of the rocket in the powered-flight phase the moment of time is determined at which it is necessary to separate the appropriate part of the rocket and a series of single instructions is given for shutting down the engine system and for the separation of this part.

As a result of the presence of two different flight phases of a ballistic missile - powered-and unpowered-flight phases the main problem of guidance is controlling the separation of the nose section of the rocket in such a way that the nose section thereafter carrying out free flight, impacts in the vicinity of the target on the surface

of the earth with the required accuracy. The execution of this task is ensured by the channel controlling the separation of the nose section which is a group of devices which shape the signal for the shutting down of the engine system and for the separation of the nose section at that moment of time, when a certain function of the parameters of the motion of the rocket attains a value which ensures with the required accuracy the transit of the flight path of the nose section to the target whose position relative to the rocket launch site is known.

In order that at the moment of separation of the nose section the parameters of rocket motion are located in such a sufficiently small area which ensures the normal operation of the channel controlling the separation of the nose section, it is necessary to control the velocity component of the rocket.

The channel controlling lateral velocity maintains the flight of the rocket during the powered-flight phase in the indicated plane of firing. This is necessary so that the velocity vector of the rocket at the moment of separation of the nose section has the necessary direction with respect to azimuth. Since this channel strives to reduce lateral drift to zero - the deflection of the rocket from the plane of firing, then it also can be called the lateral drift stabilization channel or the channel of lateral stabilization.

The channel controlling normal velocity ensures the flight of the rocket in the plane of firing along the assigned flight path so that at the moment of separation of the nose section the velocity vector of the rocket has the necessary direction in the plane of firing.

The channel controlling velocity ensures the required principle of varying flight velocity. The control of velocity is accomplished for the purpose of reducing the parametric domain of the motion of the center of mass of the rocket in the plane of firing, at which separation of the nose section is possible for impacting on the

assigned target. The channel controlling velocity can be absent, if it is possible to ensure the required accuracy of firing without velocity control.

Each of the three closed channels controlling the components of rocket velocity usually carry out the following functions.

1. It obtains and processes information concerning the parameters of rocket motion, on the basis of which the guidance signals are worked out.

2. It transmits the guidance signals on board the rocket, if these signals are developed by off-board equipment.

3. It converts the guidance signals into lateral, normal and tangential controlling forces.

Since lateral and normal controlling forces are created by varying the angular position of the rocket relative to its velocity vector, i.e., by varying the angles of attack and sideslip, then for controlling these forces two channels of the angular stabilization system are used – the pitch and yaw channels. In this case the angular stabilization system simultaneously executes two functions:

1) it converts the guidance signals into lateral and normal controlling forces;

2) it stabilizes during the effect of perturbations the angular position of the rocket assigned by the guidance signals.

Thus, the two channels of the angular stabilization system are design elements of the [illegible]ing the channels of the guidance system. Thus, the channel stabilizing the angle of pitch is part of the channel controlling normal velocity, and the channel stabilizing the angle of yaw is included in the channel controlling lateral velocity. These two channels of the angular stabilization system are with respect to the guidance system a certain complex "object of control."

For the normal guidance system operation it is usually necessary that a rocket flight occur without roll. With rocket rolling the guidance signals will be executed inaccurately as a result of a reduction in normal controlling force N_y as compared with its required value N_y^* ($N_{y_*} = N_y^* \cos \eta$) and the appearance of lateral controlling force $N_\zeta = N_y \sin \eta$, which causes deviation of the center of mass of the rocket from the plane of firing. The problem of preventing rocket roll under the effect of external perturbations is solved by the roll stabilization channel. This channel is not included in the guidance system, but it facilitates its operation.

The principle of nose section separation control is based on the assumption a nose section can hit one and the same target point (at the end of the unpowered-flight phase), moving both along the optimum trajectory and along an infinite set of other possible trajectories. Because of this it is not at all mandatory for hitting a target, that the parameters of rocket motion at end of the unpowered-flight phase be equivalent to their optimum values. It is possible to separate a nose section at that instant, when the totality of parametic deviation of rocket motion from their optimum values will ensure the subsequent motion of the nose section along one of the trajectories, leading it to the target. This problem is also solved by the appropriate channel of the rocket control system.

Peculiarities of Rocket Control Systems With Controllable and Uncontrollable Thrust.

The principles of ballistic missile control system construction are determined by the rocket flight conditions and by the characteristics of its design, by the missions which are assigned to this system, and by the specifications, imposed on it, and also by the level of development of the corresponding fields of technology. In particular, a large role is played by the fact, of whether the magnitude of engine thrust is controlled or is not controlled. Rockets with engines which make it possible to vary the thrust level, are more refined objects of control. They can accomplish flight along a flight path, sufficiently close to the optimum, and thus, execute nose section separation with small parametric deviations at the end of the

powered-flight phase from the optimum values. This makes it possible to obtain a relatively high firing accuracy when using comparatively simple guidance systems and instruments.

For ensuring a rocket flight along a trajectory, close to optimum, it is possible to measure three components of the acceleration of the center of mass and to compare them (or integrals of them, i.e., three velocity components) with the programs of their variation which are stored as functions of time in a program unit (or in an onboard computer).

By controlling the normal forces by varying the orientation of the rocket and by the thrust level by boosting or throttling the engine, it is possible to accomplish a predetermined flight trajectory with an accuracy determined only by the characteristics of the control-system equipment. The narrow "tube" of perturbed trajectories obtained in this case makes it possible to construct a comparatively simple automatic device for controlling the separation of the nose section, that works in a standby mode and affecting neither the rocket motion process in the powered-flight phase, nor the operation of the control system. The algorithm of the operation of this automatic device can be comparatively simple, not requiring a complex computer.

It is rather simple to obtain a channel controlling normal velocity, maintaining the assigned program of its variation. A characteristic of this program is the gradual variation in normal speed during the course of tens of seconds (the flight duration of a rocket stage). The representation of a program in the form of a frequency spectrum shows that this spectrum occupies in practice the frequency band from zero to several tenths of a radian per second. For accurate reproduction of such a program high speed operation is not required from the channel controlling normal velocity and a stabilization mode is more characteristic of it than is a control mode. In connection with the noted fact for rockets with controllable thrust the channel controlling normal speed, ensuring an assigned program of its variation, can be called the *normal stabilization channel.*

Thus, for rockets with controllable thrust the optimum flight path in the powered-flight phase is assigned by programs controlling the projections of velocity for some three directions and by programs controlling the angles of pitch, yaw and roll, and a guidance system is intended for controlling a rocket in the powered-flight phase along a trajectory, as close as possible to the optimum. This problem is solved by three channels of the guidance system, which include the channels stabilizing the angles of pitch and yaw, and also a channel stabilizing angle of roll.

The examined approach to constructing a guidance system transfers a significant part of the problem of ensuring required firing accuracy to rocket and engine system designers. The guidance system in this case receives a simple instrument formulation due to the complicating of the rocket design.

The rocket, in which the thrust level of the engine system is not controlled, for example solid-propellant rockets, can noticeably deviate from the optimum trajectory. However the reducing to zero during the whole duration of the powered-flight phase of the parametric deviations in rocket motion from the optimum values is not an end in itself. The basic problem of a rocket control system is minimizing the deflection of the point of impact of the nose section from the target. An ideal guidance system should employ the parametric information about rocket motion in such a way as to ensure at the end of the flight the impact of the nose section on the target. The success in solving this problem by a real guidance system depends on how completely the algorithm for converting the information about rocket motion into the guidance signal controlling rocket orientation, considers all possible factors affecting impact accuracy.

Thus if an attempt is not made to ensure the smallest deviations in the parameters of rocket motion from the optimum values, which is more or less possible in rockets with controllable thrust, then control of the normal and lateral velocities and of nose section separation is accomplished in accordance with rather complete algorithms ensuring the required dispersion of the nose section,

despite the substantial deviations in the parameters of rocket motion from the optimum. In this case for shaping the guidance signals and for nose section separation a large number of computational operations is necessary which are performed by a digital computer: onboard – in an inertial system or ground-based – in a radio-controlled system. It is not difficult to see that for ensuring assigned nose section dispersion a less refined object of control (with uncontrollable thrust) requires the employment of a more refined guidance system using a digital computer.

Inertial Control Systems for Rockets With Controlled Thrust

On the basis of an analysis of the control systems of various ballistic missiles[1] it is possible to visualize a certain typical inertial guidance system for a rocket with controllable thrust. As it was already noted, such a rocket does not require complex control algorithms, and thus its guidance system does not contain complex computers.

The basis of a guidance system is a gyrostabilized platform (GSP) which preserves in flight in the powered-flight phase of the trajectory the directions of the axes of the initial launch coordinate system.

The Control Program

During the flight of a rocket in the powered-flight phase of the trajectory three projections of the apparent velocity of the center of mass of the rocket and the angles of turn of the body axis of the rocket relative to the axes of the inertial (initial launch) coordinate system, are measured, i.e., the angles of pitch ϕ, yaw ξ and roll η.

For establishing the components of the apparent velocity of the center of mass of a rocket as reference directions it is possible to

[1]See, for example, the books [12], [13], [26], [27], [29], [30].

use: the direction of body axis Ox_1, coinciding with the longitudinal axis of the rocket: the direction of axis Oy^*, coinciding with the reference (programmed) direction of body axis Oy_1; the direction of axis Oz_0, coinciding with the normal to the plane of firing Ox_0y_0, or the direction of body axis Oz_1.

The programs of lateral velocity w^*_{z0}, of the angles of yaw and roll are usually considered zero. The program of normal speed is also very simple to select as zero. Then the flight path will be assigned only by two programs: w^*_{x1} and ϕ^*.

The program of variation in pitch angle, assigned in the form of a dependence on time, is distinguished by the simplicity of its instrument execution. Other means of assigning a pitch angle program are also possible. For instance, a program can be assigned in the form of the dependence of pitch angle on the projection of the apparent velocity on the longitudinal axis: $\phi^* = \phi^*(w_{x1})$. The execution of the program in this case is accomplished by a program mechanism in accordance with the actual value of the apparent flight velocity.

Angular Stabilization System

A system of angular stabilization maintains the required orientation of a rocket determined by the zero values of the angles of yaw and roll and by the program value of pitch angle. This system has three channels: pitch, yaw and roll, constructed in an analogous manner. Certain differences in the operation of these channels are due to their interaction with the corresponding channels of the guidance system.

A characteristic of the channels for stabilizing yaw and roll is differential deflection of the control surfaces for creating rolling moment, which means using the same control drives and control elements for creating yaw and roll moments.

Since a rocket is dynamically axisymmetric, the channel for stabilizing the angle of yawing usually has the same structural

layout, as the channel for stabilizing pitch.

The block diagram of one of the channels of an angular stabilization system is shown in Fig. 1.18. Let us examine the equations of the links, included in this diagram.

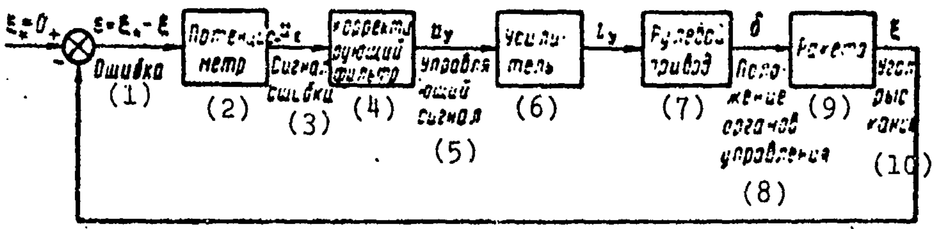

Fig. 1.18. Diagram of an angular stabilization channel.

Key: (1) Error. (2) Potentiometer. (3) Error signal. (4) Correcting filter. (5) Control signal. (6) Amplifier. (7) Control drive. (8) Position of the control elements. (9) Rocket. (10) Angle of yaw.

The gyrostabilized platform with an angular error sensor is a practically inertialess link, the unique characteristic of which is the transmission factor $K_{Д}$, i.e.

$$u_{\varepsilon}=K_{Д}\varepsilon, \qquad (1.53)$$

where $\varepsilon = \phi^{*} - \phi$; $\varepsilon = \xi$; $\varepsilon = \eta$ – for the pitch, yaw and roll channels respectively.

The amplifiers included in the amplifier-converter diagram, and also the converters of the signal type – modulators and demodulators, are characterized by a very small dynamic lag, which in analyzing a stabilization system can be disregarded. Then

$$i_{y}=K_{y}u_{y}. \qquad (1.54)$$

The hydraulic control drive taking the moment of load into account is characterized by the equation of the aperiodic link

$$T_{р.п}\dot{\delta}+\delta=K_{р.п}i_{y}. \qquad (1.55)$$

Other drive layouts are possible; other equations will correspond to them.

For correcting the dynamic properties of a system various types of RC-filters can be used. The equations single-link differentiating and integrating filters can be written in the general form as

$$T_\Phi \dot{u}_y + u_y = K_\Phi (\tau_\Phi \dot{u}_\text{ı} + u_\text{ı}). \tag{1.56}$$

Equations of two-link differentiating and integro-differentiating filters take the following form:

$$T_\Phi^2 \ddot{u}_y + 2\xi_\text{T} T_\Phi \dot{u}_y + u_y = K_\Phi (\tau_\Phi^2 \ddot{u}_\text{ı} + 2\xi_\Phi \tau_\Phi \dot{u}_\text{ı} + u_\text{ı}). \tag{1.57}$$

The given equations are employed in selecting the type of correction and the basic parameters of each of the channels of an angular stabilization system. However in solving various problems of the dynamic rocket design it is possible to enlist transitional processes of stabilization and to examine only the established modes of stabilization, using the following simplified equations;

control drive

$$\delta = K_{\text{p.п}} i_y; \tag{1.58}$$

single-link differentiating filter

$$u_y = K_\Phi (\tau_\Phi \dot{u}_\text{ı} + u_\text{ı}); \tag{1.59}$$

integrating filter

$$u_y = \frac{K_\Phi}{T_\Phi} \left(\tau_\Phi u_\text{ı} + \int_0^t u_\text{ı} dt \right); \tag{1.60}$$

two-link differentiating filter

$$u_y = K_\Phi (\tau_\Phi^2 \ddot{u}_\text{ı} + 2\xi_\Phi \tau_\Phi \dot{u}_\text{ı} + u_\text{ı}); \tag{1.61}$$

integro-differentiating filter

$$u_y = \frac{K_\Phi}{2\xi_\text{T} T_\Phi} \left(\tau_\Phi^2 \dot{u}_\text{ı} + 2\xi_\Phi \tau u_\text{ı} + \int_0^t u_\text{ı} dt \right). \tag{1.62}$$

Using equations (1.58)-(1.62), it is possible to compose for each of the channels of a stabilization system an approximation equation connecting the deflection of the control elements with the parameters of angular rocket motion.

For correcting the dynamic characteristics of channels stabilizing pitch and yaw it is usually necessary to use either a two-link differentiating, or an integro-differentiating filter. Then, for example, for a channel stabilizing angle of pitch we have the equations

$$\delta_\varphi = a_\varphi \Delta\varphi + a_{\dot\varphi} \Delta\dot\varphi + a_{\ddot\varphi} \Delta\ddot\varphi, \tag{1.63}$$

or

$$\delta_\varphi = a_\varphi \Delta\varphi + a_{\dot\varphi} \Delta\dot\varphi + a_{\int\varphi} \int \Delta\varphi dt. \tag{1.64}$$

The equations for the channel stabilizing the angle of yaw are written in an analogous manner.

For correcting the dynamic characteristics of a channel stabilizing angle of roll it is sufficient to employ a single-link differentiating filter. In this case we obtain

$$\delta_\eta = a_\eta \eta + a_{\dot\eta} \dot\eta. \tag{1.65}$$

The Guidance System.

The channel regulating apparent velocity (the PKC [AVR] system) is intended for maintaining the program value of the longitudinal component of apparent velocity by boosting or throttling the main engine system. Velocity control is accomplished for the purpose of reducing the variance in the motion parameters of the center of mass of the rocket; this necessary for simplifying the algorithm of engine system shut-down.[1]

[1]The AVR system can be absent, if it is possible to ensure the required firing accuracy without equipping the rocket with this system.

The regulation of the longitudinal component of apparent velocity can be accomplished in the following manner (Fig. 1.19). AVR is introduced into the system, and then program $w^*_{x1}(t)$ is reproduced onboard the rocket. This function (the storage and the reproduction of the program onboard the rocket) is carried out by some program unit.

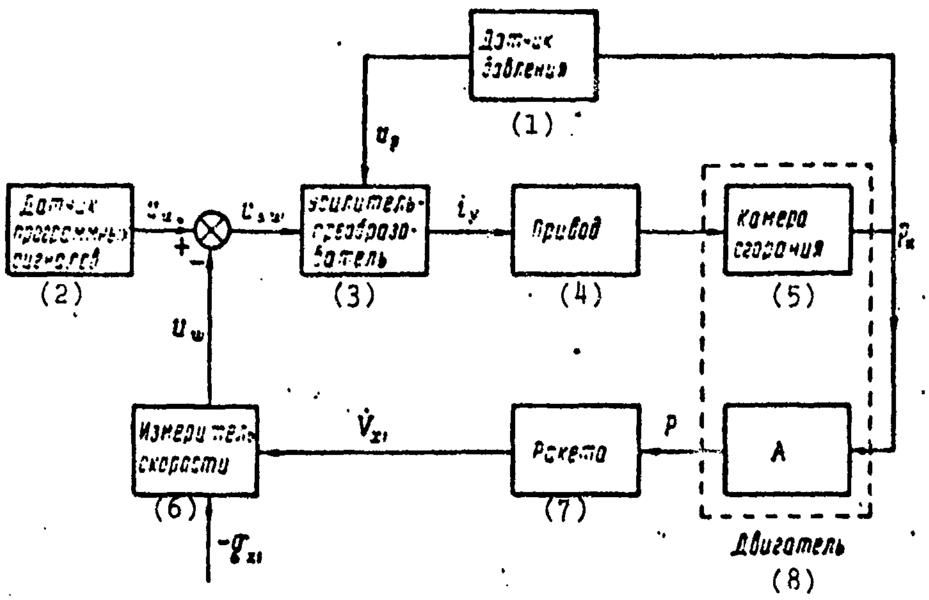

Fig. 1.19. Diagram of an apparent velocity regulating channel.

KEY: (1) Pressure sensor. (2) Program signal sensor. (3) Amplifier-converter. (4) Drive. (5) Combustion chamber. (6) Speed meter. (7) Rocket. (8) Engine.

A speed meter is a gyroscopic integrator mounted on the missile body so that its axis of sensitivity is parallel to the longitudinal axis of the rocket; it emits to the comparing device in the form of a corresponding signal the measured value of the apparent velocity w_{x1}. To the comparing device there also comes a signal of the programmed value of the apparent velocity w^*_{x1}. (The gyroscopic integrator and the comparing device can be combined in one instrument – the apparent velocity error sensor). A signal, proportional to the mismatch, is supplied to the amplifier-converter, where it is amplified and converted into a signal controlling the actuating element of the AVR

system. The actuating device can be for instance, the engine reduction screw drive. The turning of the reduction screw varies the flow rate per second of the propellant components into the turbopump unit. As a result of this the per-second flow rate of propellant going into the combustion chambers leads to a change in pressure in the combustion chambers and thus, in the thrust of the engine system.

The thrust increases, if the actual velocity w_{x1} is less than the programmed velocity w^*_{x1} and vice versa. As a result an automatic closed AVR system is obtained, in which the rocket is included as the object of control.

For improving the dynamic properties of this system correction methods, well known in the theory of automatic control, can be employed: the introduction of derivatives from the error signal Δw_{x1} into the control signal, the utilization of internal feedback, etc. One of the possible means is the introduction of internal feedback with respect to pressure in the combustion chambers. In this case the error signal of the apparent velocity $u_{\Delta w}$ is added with the feedback signal with respect to pressure in the combustion chambers (see Fig. 1.19).

The lateral stabilization channel. The channel controlling the lateral component of velocity maintains the flight of the rocket, during the powered-flight phase of the trajectory, in the assigned plane of firing. This is necessary in order to as simply as possible ensure the required direction with respect to the azimuth of the velocity vector of the rocket at the moment of engine shut-down.

The channel controlling lateral velocity ensures during the powered-flight phase of the rocket with the necessary accuracy the zero values of the lateral component of velocity and of lateral drift, i.e., deviations of the center of mass of the rocket from the plane of firing. In accordance with this problem the examined guidance system channel is usually called the *channel* (or system) *of lateral stabilization* (БС = [LS]).

In an inertial system of lateral stabilization the measuring instrument (accelerometer or gyrointegrator) is mounted on the rocket in such a way that its axis of sensitivity is directed either along body axis Oz_1 (in the case of immobile mounting of the measuring element relative to the rocket body), or perpendicular to the plane of firing, i.e., along the initial launch axis Oz_0 (with the mounting of the measuring element on the GSP). The inertial system of lateral stabilization in this manner ensures during the course of the powered-flight phase of the rocket with sufficient accuracy the zero values of the lateral component of apparent velocity w_z and the integral in time from this velocity $s_z = \int_0^t w_z(\tau)\,d\tau$, i.e., the apparent path in the lateral direction.

A signal, proportional to the apparent path in the lateral direction s_z, is obtained by twofold integration of the accelerometer signal or by single integration of the gyrointegrator signal. Integration can be accomplished with the aid of integrators of various types, for example an electrolytic integrator or an integrating RC-circuit.

The problem of selecting the type of correction and the basic parameters of the lateral stabilization circuit is solved proceeding from the conditions of ensuring the assigned guidance accuracy characterized by the magnitude of lateral drift, and the required dynamic properties of the circuit, characterized by the quality of the transitional process. In an inertial lateral stabilization system for correcting the dynamic characteristics of the system a signal, proportional to the lateral component of apparent velocity is used. In this case the deflection of the control elements is described by the approximate equation

$$\delta_z = a_z z + a_{\dot{z}} \dot{z}, \tag{1.66}$$

in which it is assumed that: $\dot{z} \approx w_z$; $z \approx s_z$.

<u>The normal stabilization channel</u> of a rocket guidance system, as well as the lateral stabilization channel, consists of two circuits:

th normal stabilization circuit and the pitch angle stabilization circuit. It operates in an analogous manner to the inertial lateral stabilization channel. The difference consists only in the fact that the programmed values of the angle of pitch are not zero. The angle of pitch $\phi^*(t)$, corresponding to flight along an optimum trajectory, is assigned by the program mechanism and is put into practice by the angle of pitch stabilization circuit.

A programmed variation in pitch angle gives rise to a corresponding variation in the normal component of velocity. In connection with this in the simplest guidance systems the control of the normal velocity of motion of the center of mass was replaced by control of the angular position of the rocket with respect to pitch. However the monitoring of the angle of pitch in certain cases is insufficient for ensuring small deviation in the parameters of the motion of the center of mass of the rocket in the plane of firing. Control of the normal component of velocity in addition to control of the angle of pitch makes it possible to reduce the indicated deviation.

In the simplest case control of normal velocity reduces to the stabilization of the zero values of the normal component of apparent velocity w_{y1} (along body axis Oy_1 or in the programmed direction) and the integral for time due to this velocity

$$s_y = \int_0^t w_y(\tau)\, d\tau.$$

The information, necessary for normal stabilization, comes from the inertial measuring element — the accelerometer of gyrointegrator.

If the measuring element of the normal stabilization system is mounted on a OSP, then with the aid of the angle of pitch program mechanism it is turned during flight so that its axis of sensitivity is always perpendicular to the programmed direction of the longitudinal axis of the rocket.

The equation of the deflection of the control elements of a rocket for a steady-state process of normal stabilization takes the form

$$\delta_y = a_y s_y + a_{\dot{y}} w_y. \tag{1.67}$$

The automatic equipment controlling nose section separation. The inertial guidance system examined here, which includes lateral and normal stabilization channels and an apparent velocity regulating channel, ensures rather small dispersion of all the coordinates and of the components of velocity of the center of mass of the rocket and thus makes it possible to simplify the automatic equipment controlling nose section separation. This automatic equipment contains missile apparent velocity meters mounted on GSP, — accelerometers or gyro-integrators oriented in an appropriate manner. The information from the meters concerning the projections of the apparent velocity and the apparent path goes to a computer for the shaping of a signal for engine system shut-down and for nose section separation.

The control algorithm, the corresponding systematic and instrumental errors and their effect on nose section dispersion are examined in Chapter V.

Missile Inertial Control Systems with Uncontrollable Thrust

Let us visualize a certain typical inertial control system with an onboard digital computer for a rocket with uncontrollable thrust, for which let us take into account the diagrams of the control system of various rockets. Such a control system, naturally, consists of guidance and stabilization systems.

The stabilization system receives the information, necessary for shaping the control signal going to the control drive, from the guidance system and from the meters measuring the parameters of rocket motion: the angles of pitch, yaw and roll, the angles of attack, the angular velocities, the accelerations of the center of mass. Thus,

for instance, besides the usual stabilization systems with angles of pitch, yaw and roll meters, stabilization systems with angular velocity and linear acceleration sensors can be used.

The stabilization system is digital, i.e., the controlling signals are shaped by digital filters, which makes it possible to substantially improve the dynamic properties of the system.

The *guidance system* equipment consists of an inertial measuring system (navigation system), which determines the velocities and the coordinates of the rocket, and an onboard computer shaping the guidance signals and the engine shut-down signal.

The inertial measuring system is set up designwise as a single unit, which includes:

- a three-degree-of-freedom gyrostabilized platform with the analog stabilization circuit elements;

- meters for measuring rocket motion parameters (angle, acceleration or center of mass velocity sensors).

The output signals of the inertial measuring system in digital form go to the onboard computer which determines the velocity components and the coordinates of the rocket, it solves guidance equations and emits signals in digital form to the stabilization system, for stage separation, engine shut-down, etc.

Three accelerometers mounted on the GSP, simultaneously perform the functions of normal and lateral velocity control system meters and automatic range control equipment meters, and also the functions of stabilization system meters in the case of the corresponding construction of the latter. Actually the automatic range control equipment as an independent unit is eliminated, specific meters for normal and lateral velocities are absent and as a result the number of meters is reduced to three.

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With such guidance system construction the totality of its equipment is a rather general-purpose device which makes it possible to use it in various rockets without substantial alterations.

1.9. BASIC CHARACTERISTICS OF A ROCKET AS A DYNAMIC SYSTEM

Before going on to the composition of a mathematical model of controlled rocket flight, let us note the specific characteristics of a rocket as a dynamic system.

A rocket together with its control system forms a dynamic closed system, the processes in which (rocket motion, the elastic oscillations of the rocket and the oscillations of the liquid propellant in the tanks, the conversion of electric signals, the deflections of the control elements and others) are described by a complex system of differential equations.

It is possible to examine the following components of rocket motion:

1) the motion of the center of mass;

2) the motion around the center of mass;

3) the elastic oscillations of the housing (flexural oscillations in two planes, longitudinal and torsional oscillations);

4) the oscillations of the liquid propellant in the tanks relative to the missile body with the presence of free propellant surfaces.

It is possible to consider a rocket as absolutely solid only as a first approximation. In the general case the flexural vibrations of the missile body can interact with the oscillations in the control system and with the oscillations of the liquid propellant. When the frequencies of the oscillations are rather close to each other, when investigating rocket motion it is necessary to consider the interrelation of the corresponding oscillatory processes.

The motion of the center of mass of a rocket is unsteady and the parameters of the rocket and the parameters of its motion very substantially vary during flight. The variations in the motion parameters are connected with the great propellant consumption per second and the displacement of the rocket with variable speed in the atmosphere, the density of which sharply drops with height. As a result of the large fuel consumption such characteristics of the rocket, as mass, the inertial moments and the position of the center of mass (see Fig. 1.13-1.14) vary. The variation in flight altitude (Fig. 1.20) and atmospheric density in conjunction with the sharp variation in the velocity of motion of the rocket (Fig. 1.21) give rise to a very specific character of variation in the magnitude of dynamic head (Fig. 1.22).

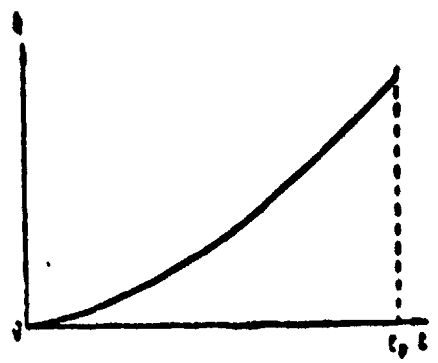

Fig. 1.20. Variation in the flight altitude of a single-stage rocket.

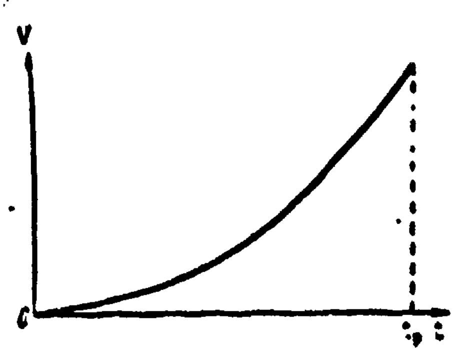

Fig. 1.21. Variation in the flight velocity of a single-stage rocket.

With variation in M number the aerodynamic characteristics of the rocket vary, in particular, c_τ – the coefficient of tangential force, c_α^2 – derivative from the coefficient of normal force with respect to

angle of attack, $\bar{x}_d$ – the relative coordinate of the center of pressure (Fig. 1.23). In connection with the fact that in the last seconds of the powered-flight phase, as a rule, the engine system goes over to a low thrust mode; in this phase of motion there is an abrupt change in thrust level and axial overload (Fig. 1.24 and 1.25).

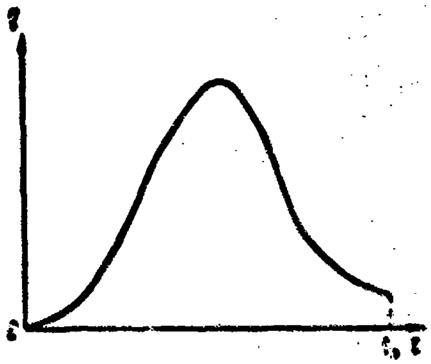

Fig. 1.22. Variation in dynamic head during the flight of a single-stage rocket.

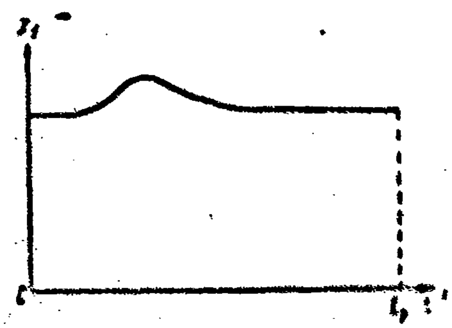

Fig. 1.23. Variation in the position of the center of pressure of a single-stage rocket during flight.

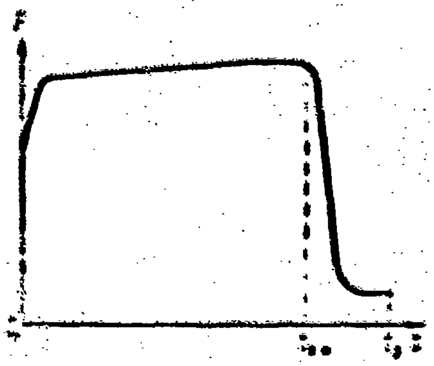

Fig. 1.24. Variation in the engine thrust of a single-stage rocket during flight.

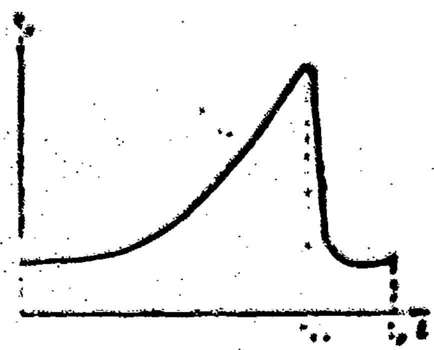

Fig. 1.25. Variation in axial overload of a single-stage rocket during flight.

For multistage rockets, besides the indicated variations in continuous character, there are also intermittent variations in the rocket parameters and motion parameters connected with the separation

of a used-up stage and the beginning of operation of the engine system of the following stage (see Fig. 1.13 and 1.14). Figure 1.26 for example presents a graph illustrating the nature of the variation in overload for a two-stage missile.

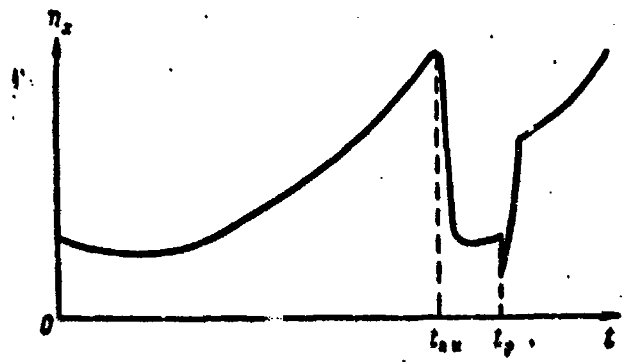

Fig. 1.26. Variation in the axial overload for a two-stage missile during flight.

Intermittent parametric variations can take place both at moments of stage separation and at moments of the separation of the structural elements of rocket jettisonable after its exit from the dense layers of the atmosphere (the nose section fairing, elements of the tail section and others). As a result of the variations in the rocket parameters and the parameters of its motion during flight the dynamic properties of the rocket as an object of control (control element effectiveness, the reaction of the rocket to the deflection of the control elements and others) are substantially changed.

The system of differential equations, rather completely describing the flight of a guided rocket, has a very high order and is a nonlinear stochastic simultaneous system of equations. Let us briefly examine the noted characteristics of these equations.

The motion of a rocket as an absolute solid body, consisting of the motion of the center of mass and the motion around the center of mass, is characterized by six degrees of freedom and is appropriately described by a system of differential equations of the 12th order.

The elastic properties of a rocket design and the presence of free liquid-propellant surfaces considerably increase the number of degrees of freedom of such a mechanical system, as a rocket. The numerous elements of the control system affecting the motion of a rocket increase even more the number of degrees of freedom of the dynamic system formed by the rocket and by its guidance equipment.

Generally speaking, all processes in nature are described by nonlinear equations, and linear equations give only a model of a process, more or less corresponding to reality. So the equations of motion of a rocket are in general nonlinear. Among the many nonlinear dependences in the equations of motion of a rocket it is possible to indicate, for example, the nonlinear dependences of aerodynamic forces and moments on the parameters of motion, the limitation in the deflections of the control elements, the characteristics with saturation and with zones of insensitivity of the control system elements, and others.

The stochastic nature of the differential equations is conditioned by the action of numerous random perturbations on the process of the rocket's flight.

CHAPTER II

GENERAL EQUATIONS OF MOTION

In deriving the equations of motion of a ballistic missile it is assumed that the rocket is an absolutely rigid body, i.e., the elasticity of the rocket and the presence of liquid-propellant in the tanks are not considered.

2.1. VECTORIAL EQUATIONS OF ROCKET MOTION

The motion of a rocket can be considered as the sum of translatory motion, determined by the motion of the rocket, and the rotation of the rocket about this point as fixed.

The motion of the center of mass of a rocket is determined by the equation

$$m\frac{d\bar{V}_a}{dt}=\bar{F}+\bar{P}, \qquad (2.1)$$

where $m = m(t)$ - the mass of the rocket; $\bar{V}_a$ - the vector of the absolute velocity of the center of mass of the rocket, i.e., velocity relative to an inertial coordinate system; $\bar{F}$ - the main vector of all the external forces, applied to the rocket; $\bar{F}$ - the main vector of the reactive forces.

Absolute acceleration $j_a = dV_a/dt$ can be represented in the form

$$\bar{j}_a = \bar{j} + \bar{j}_e + \bar{j}_c, \quad (2.2)$$

where $\bar{j}$ - relative acceleration; $\bar{j}_e$ - translatory acceleration; $\bar{j}_c$ - coriolis acceleration.

Consequently, the equation of motion of the center of mass of a rocket relative to a certain mobile coordinate system will take the form

$$m\bar{j} = \bar{F} + \bar{P} + (-m\bar{j}_e) + (-m\bar{j}_c), \quad (2.3)$$

where $(-m\bar{j}_e)$ and $(-m\bar{j}_c)$ - respectively the translatory and Coriolis inertial forces.

Let, for example, the motion of a rocket be examined in a coordinate system rotating together with the earth with angular velocity $\bar{\omega}_3$. The origin O of this coordinate system is located at the center of the earth; axes Ox and Oy lie in the equatorial plane; axis Oz coincides with the axis of rotation of the earth. The relative acceleration $\bar{j}$ will then be the acceleration of the center of mass of the rocket relative to the earth. Since

$$\bar{j}_e = \bar{j}_0 + \frac{d\bar{\omega}_3}{dt} \times \bar{r} + \bar{\omega}_3 \times (\bar{\omega}_3 \times \bar{r}), \quad (2.4)$$

and $j_0 = 0$ and $d\bar{\omega}_3/dt = 0$, then the translatory acceleration is

$$\bar{j}_e = \bar{\omega}_3 \times (\bar{\omega}_3 \times \bar{r}). \quad (2.5)$$

The Coriolis acceleration arising due to the rotation of the earth when relative velocity $\bar{V}$ exists, is determined by the dependence

$$\bar{j}_c = 2(\bar{\omega}_3 \times \bar{V}). \quad (2.6)$$

Formulas (2.5) and (2.6) preserve their form for any coordinate system connected with the earth.

The equation of motion of the center of mass of a rocket in a coordinate system rotating together with the earth, if it is assumed

that $\overline{j} = d\overline{V}/dt$, can be rewritten in the following form:

$$m\frac{d\overline{V}}{dt}=\overline{F}+\overline{P}-m\overline{j}_e-m\overline{j}_c \tag{2.7}$$

Let us now examine an arbitrary mobile coordinate system with its origin at the center of mass of the rocket. Let $\overline{\Omega}$ - the angular velocity of rotation of the axes of this system relative to the terrestrial axes. Then

$$\frac{d\overline{V}}{dt}=\frac{d'\overline{V}}{dt}+\overline{\Omega}\times\overline{V}, \tag{2.8}$$

where $\frac{d'\overline{V}}{dt}$ - the local derivative of vector $\overline{V}$ with respect to time, characterizing the rate of variation of the vector in the mobile coordinate system being examined.

Thus, the vectorial equation of motion of the center of mass of the rocket can be written in the form

$$m\left(\frac{d'\overline{V}}{dt}+\overline{\Omega}\times\overline{V}\right)=\overline{F}+\overline{P}-m\overline{j}_e-m\overline{j}_c. \tag{2.9}$$

The rotation of the rocket relative to its center of mass is determined by equation (1.41) written in the form

$$\frac{d\overline{K}}{dt}=\sum\overline{M}, \tag{2.10}$$

where $\overline{K}$ - the main moment of momentum of the rocket, or its angular momentum; $\Sigma\overline{M}$ - the main moment of all the external forces relative to the center of mass of the rocket (including the reactive forces).

In determining the main moment of momentum usually the rotation of the earth is disregarded, examining the terrestrial axes as inertial axes.

According to the theorem of local derivative

$$\frac{d\overline{K}}{dt}=\frac{d'\overline{K}}{dt}+\overline{\Omega}\times\overline{K}, \tag{2.11}$$

where $\frac{d'\overline{K}}{dt}$ - is the local derivative of vector $\overline{K}$.

Then

$$\frac{d'\overline{K}}{dt}+\overline{\Omega}\times\overline{K}=\Sigma\overline{M}. \tag{2.12}$$

Projecting equations (2.9) and (2.12) on various coordinate axes, it is possible to obtain various forms of scalar equations of the motion of a rocket. These equations can also be used for composing the equations of motion of the nose section.

The position of the center of mass of an object - a rocket or a nose section - in vectorial form is determined by radius-vector $\overline{r}$, drawn from the origin of the coordinate system being examined to the center of mass of the object.

The kinematic equation of the motion of the center of mass of an object in vectorial form has the aspect

$$\frac{d\overline{r}}{dt}=\overline{V}, \tag{2.13}$$

where $\overline{V}$ - the velocity vector of the object relative to the coordinate system in question.

The orientation of the object in space relative to the selected coordinate system is determined by the three Eulerian angles: κ, λ, μ. The kinematic equation of the rotary motion of the object connects angular velocities of $\overline{\dot{\kappa}}$, $\overline{\dot{\lambda}}$, $\overline{\dot{\mu}}$ with angular velocity of the object $\overline{\omega}$:

$$\overline{\dot{\kappa}}+\overline{\dot{\lambda}}+\overline{\dot{\mu}}=\overline{\omega}. \tag{2.14}$$

Projecting equations (2.13) and (2.14) for the selected coordinate axes, it is possible to obtain scalar kinematic equations.

In solving ballistic and dynamic problems of a rocket and its nose section various coordinate systems can be used. In many instances the successful selection of the coordinate system significantly simplifies the research. For studying flight Cartesian rectangular right handed coordinate systems are commonly used and spherical coordinate systems corresponding to them.

2.2 EQUATIONS OF MOTION OF A ROCKET IN PROJECTIONS OF COORDINATES ON TERRESTRIAL AXES.

The investigation of rocket flight can be considerably simplified by the successful selection of a coordinate system. It is practically always more advisable to obtain equations of the rotary motion of an object by projecting the corresponding vectorial equation on axes connected with the object. However the selection of a coordinate system for composing scalar equations of the motion of the center of mass of an object in many respects depends on the particular problem. Thus, for instance, in investigating the controlled motion of a rocket in the powered-flight phase of the trajectory it is advantageous to examine the motion relative to terrestrial axes.

Coordinate Systems

Terrestrial Coordinate System

The axes of this system $Ox_3y_3z_3$ (Fig. 2.1) are rigidly connected with the earth and they participate in its diurnal rotation. For short they are called *terrestrial axes*.

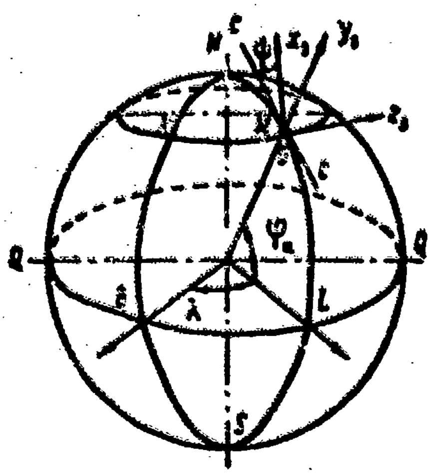

Fig. 2.1. Terrestrial system of coordinate axis: M - launch point; NABS - prime (Greenwich) meridian; NMLS - local meridian; Φ_u - geocentric latitude of point M; λ - longitude of point M; cc - tangent to the local meridian at point M; ψ - launch azimuth.

The origin O of the coordinates is located at the launch point; axis Oy_3 is directed along the radius-vector drawn from the center of the general terrestrial ellipsoid through the launch point; axis Ox_3 forms with the plane of the local meridian angle Ψ, called the Launch Azimuth; axis Oz_3 is directed so that the coordinate system is right-handed.

Launch Coordinate System

The launch coordinate system $Ox_cy_cz_c$ (Fig. 2.2) is also connected with the earth and rotates together with it. The origin of the coordinates is located at the launch point; axis Oy_c is directed upward along the plumb line, i.e., it is opposite to the direction of the force of gravity; axis Ox_c forms with the plane of the local meridian the launch azimuth angle Ψ; axis Oz_c corresponds to a right-handed coordinate system.

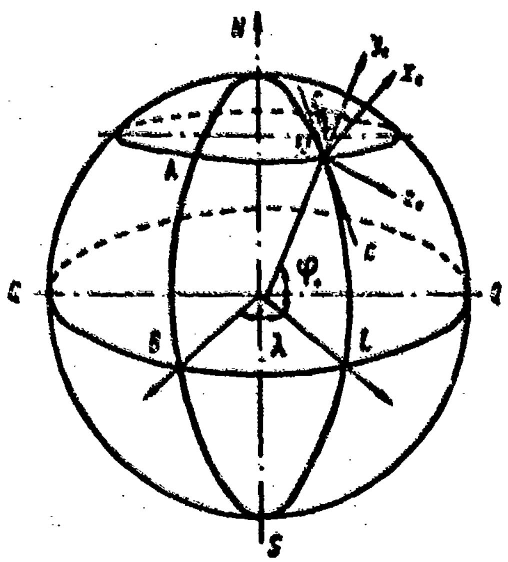

Fig. 2.2. Launch coordinate axis system: M - Launch point; NABS - prime (Greenwich) meridian; NMLS - local meridian; ϕ_A - astronomical latitude of point M; λ - longitude of point M; cc - tangent to local meridian at point M; Ψ - launch azimuth.

At launch the body axes of the rocket are oriented along the axes of the launch system (Fig. 2.3). The logitudinal axis of the rocket Ox_1 coincides with axis Oy_c; the transverse axis Oy_1 is oriented in the direction, opposite to axis Ox_c; axis Oz_1 is directed along axis Oz_c.

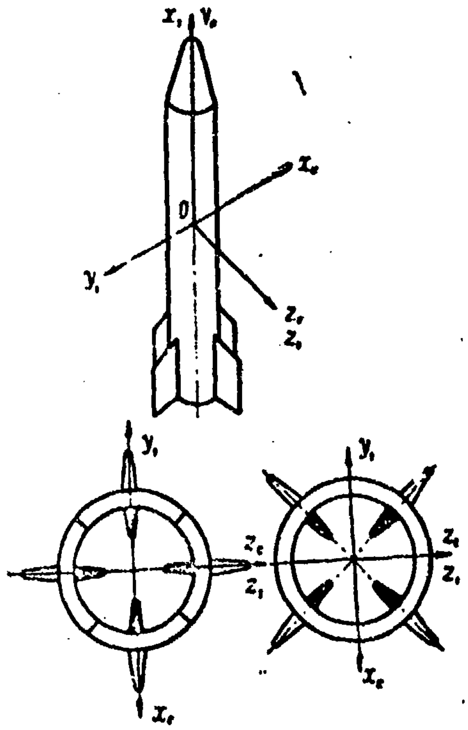

Fig. 2.3. The orientation of the body axes at the launch of a missile.

Initial Launch System of Coordinate Axes

The axes of an initial launch coordinate system $Ox_0y_0z_0$ at the moment of launch coincide with the axes of the launch system. Subsequently the axes of the initial launch system do not vary their initial direction relative to inertial space, and the axes of the launch system, rigidly connected with the earth, turn during time t by angle $\omega_3 t$ around the axis of rotation of the earth. An initial launch coordinate system is an inertial coordinate system.

Direction Cosine Matrices

The Cosines of Angles Included Between the Axes of Body and Initial Launch Coordinate Systems

The orientation of a rocket relative to an initial launch coordinate system is determined by the three angles included between body $Ox_1y_1z_1$ and initial launch $Ox_0y_0z_0$ coordinate systems (Fig. 2.4):

by the angle of yaw ξ - between the projection of the longitudinal axis of the rocket Ox_1 to plane Ox_0z_0 and axis Ox_0;

by the angle of pitch ϕ - between the longitudinal axis of the rocket Ox_1 and plane Ox_0z_0;

by the angle of roll η - between transverse axis Oy_1 and the plane, passing through axes Ox_1 and Oy_0.

The cosines of angles included between axes of body and initial launch coordinate systems are given in Table 2.1.

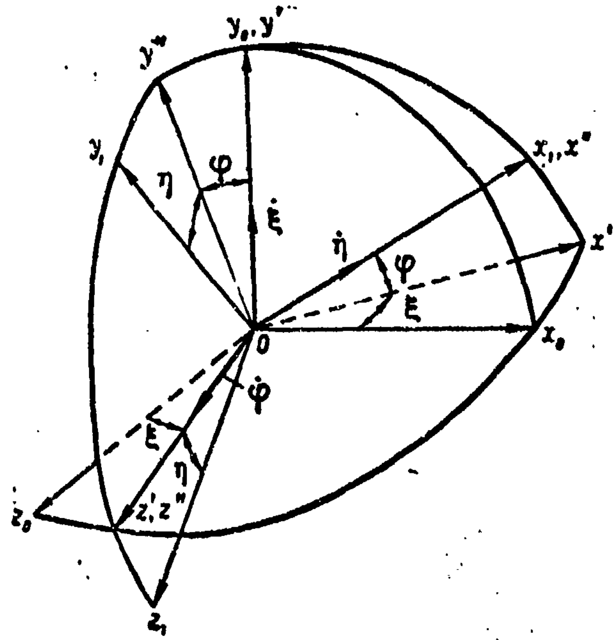

Fig. 2.4. The orientation of the body axes relative to the initial launch axes.

Table 2.1

axes	Ox_0	Oy_0	Oz_0
Ox_1	$\cos\xi\cos\varphi$	$\sin\varphi$	$-\sin\xi\cos\varphi$
Oy_1	$-\cos\xi\sin\varphi\cos\eta+$ $+\sin\xi\sin\eta$	$\cos\varphi\cos\eta$	$\cos\xi\sin\eta+$ $+\sin\xi\sin\varphi\cos\eta$
Oz_1	$\cos\xi\sin\varphi\sin\eta+$ $+\sin\xi\cos\eta$	$-\cos\varphi\sin\eta$	$\cos\xi\cos\eta-$ $-\sin\xi\sin\varphi\sin\eta$

Let us find the cosines of the angles included between the axes of initial launch and body coordinate systems. For this through the center of mass of the rocket - the origin of the body coordinate system, let us draw axes $Ox_0y_0z_0$ parallel to the axes of the initial launch coordinate system. Let us turn this system by angle ξ around axis Oy_0 so that plane $Ox'y'$ passes through axis Ox_1. Let us designate the obtained system by $Ox'y'z'$.

It is convenient to write the formulas for coordinate transformation in matrix form. Designating the matrix-column with the elements x', y', z' through $[\bar{x}']$, i.e.,

$$[\bar{x}']=\begin{bmatrix}x'\\y'\\z'\end{bmatrix},$$

and with the elements x_0, y_0, z_0, through $[\bar{x}']$, we obtain the following formula for transforming from system $Ox_0y_0z_0$ to system $Ox'y'z'$:

$$[\bar{x}']=\Gamma_\xi[\bar{x}_0], \tag{2.15}$$

where matrix Γ_ξ of the transformation from coordinate system $Ox_0y_0z_0$ to system $Ox'y'z'$ takes the form

$$\Gamma_{\xi}=\left\|\begin{matrix}\cos\xi & 0 & -\sin\xi\\ 0 & 1 & 0\\ \sin\xi & 0 & \cos\xi\end{matrix}\right\|. \tag{2.16}$$

Let us turn system $Ox'y'z'$ around axis Oz' by angle ϕ so that axis Oy' coincides with axis Ox_1; let us designate the obtained system by $Ox''y''z''$. The corresponding formula for the transformation of the coordinates upon turning the axes by angle ϕ will take the form

$$[\bar{x}''] = \Gamma_{\varphi}[\bar{x}'], \tag{2.17}$$

where

$$\Gamma_{\varphi}=\left\|\begin{matrix}\cos\varphi & \sin\varphi & 0\\ -\sin\varphi & \cos\varphi & 0\\ 0 & 0 & 1\end{matrix}\right\|. \tag{2.18}$$

By turning axis Ox'' around by angle η we bring axes Oy'' and Oz'' into line with axes Oy_1 and Oz_1. The transformation of system $Ox''y''z''$ to system $Ox_1y_1z_1$ will be accomplished by formula

$$[\bar{x}_1] = \Gamma_{\eta}[\bar{x}''], \tag{2.19}$$

where

$$\Gamma_{\eta}=\left\|\begin{matrix}1 & 0 & 0\\ 0 & \cos\eta & \sin\eta\\ 0 & -\sin\eta & \cos\eta\end{matrix}\right\|. \tag{2.20}$$

Substituting (2.15) and (2.17) into dependence (2.19), we obtain the matrix equation of the transformation from initial launch axes to body axes:

$$[\bar{x}_1] = \Gamma_{\eta}\Gamma_{\varphi}\Gamma_{\xi}[\bar{x}_0] = \Gamma[\bar{x}_0], \tag{2.21}$$

in which matrix Γ is a table of direction cosines, i.e., of the cosines of the angles included between the axes of initial launch and body

coordinate systems (see Table 2.1):

$$\Gamma=$$

$$=\left\|\begin{matrix} \cos\xi\cos\varphi & \sin\varphi & -\sin\xi\cos\varphi \\ \sin\xi\sin\eta-\cos\xi\sin\varphi\cos\eta & \cos\varphi\cos\eta & \cos\xi\sin\eta+\sin\xi\sin\varphi\cos\eta \\ \sin\xi\cos\eta+\cos\xi\sin\varphi\sin\eta & -\cos\varphi\sin\eta & \cos\xi\cos\eta-\sin\xi\sin\varphi\sin\eta \end{matrix}\right\|. \quad (2.22)$$

<u>Cosines of Angles Included Between Axes of Initial Launch and Launch Coordinate Systems</u>

At the moment of rocket firing the initial launch coordinate system $Ox_0y_0z_0$ and the launch coordinate system $Ox_cy_cz_c$ coincide. During the flight of a rocket the launch coordinate system will turn together with the earth relative to its initial position by angle $\omega_3 t$, where t - rocket flight time (Fig. 2.5).

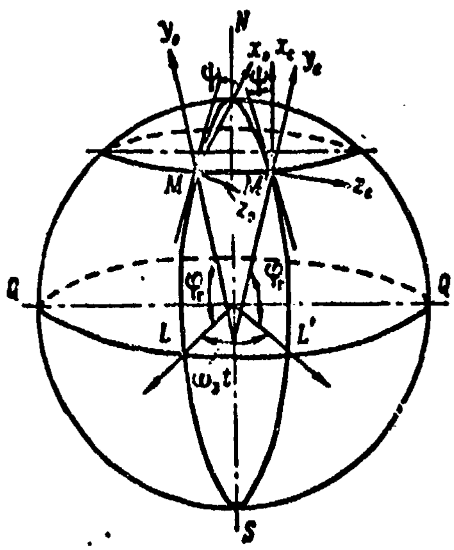

Fig. 2.5. Transformation from an initial launch system to a launch system of coordinate axes.

In order to accomplish the transformation from a rotating launch coordinate system to an initial launch system, let us draw five sequential turns of an auxiliary coordinate system which coincides with the launch system. The first turn of this system let us draw around axis Oy_c by angle Ψ so that the azimuth of axis Ox' becomes equal to zero. Let us turn the obtained system Ox'y'z' around axis Oz' by angle ϕ_r so that axis Ox" becomes parallel to the axis of rotation of the earth,

and axis Oy" - parallel to the equatorial plane. Now we can carry out turning by angle $\omega_3 t$. For this let us turn the new system Ox"y"z" around axis Ox" by angle $\omega_3 t$ so that plane Ox"'y"' posses through axis Oy_0. Let us further turn system Ox‴y‴z‴ around axis Oz‴ by angle $-\phi_r$ so that axis Oy^{IV} coincides with axis Oy_0. By turning by azimuth angle ψ around axis Oy^{IV} let us line up system $Ox^{IV}y^{IV}z^{IV}$ with the initial launch system $Ox_0y_0z_0$.

The transformations of the coordinate systems carried out are described by matrix equation

$$[\bar{x}_0]=\Delta\,[\bar{x}_c], \tag{2.23}$$

in which the transformation matrix Δ takes the form

$$\Delta=\begin{Vmatrix}\delta_{11} & \delta_{12} & \delta_{13}\\ \delta_{21} & \delta_{22} & \delta_{23}\\ \delta_{31} & \delta_{32} & \delta_{33}\end{Vmatrix}, \tag{2.24}$$

and the coefficients of the matrix

$$\delta_{11}=\cos^2\psi\cos^2\varphi_r(1-\cos\omega_3 t)+\cos\omega_3 t;$$
$$\delta_{21}=\cos\psi\sin\varphi_r\cos\varphi_r(1-\cos\omega_3 t)-\sin\psi\cos\varphi_r\sin\omega_3 t;$$
$$\delta_{31}=-\sin\psi\cos\psi\cos^2\varphi_r(1-\cos\omega_3 t)-\sin\varphi_r\sin\omega_3 t;$$
$$\delta_{12}=\cos\psi\sin\varphi_r\cos\varphi_r(1-\cos\omega_3 t)+\sin\psi\cos\varphi_r\sin\omega_3 t;$$
$$\delta_{22}=\sin^2\varphi_r(1-\cos\omega_3 t)+\cos\omega_3 t;$$
$$\delta_{32}=-\sin\psi\sin\varphi_r\cos\varphi_r(1-\cos\omega_3 t)+\cos\psi\cos\varphi_r\sin\omega_3 t;$$
$$\delta_{31}=-\sin\psi\cos\psi\cos^2\varphi_r(1-\cos\omega_3 t)+\sin\varphi_r\sin\omega_3 t;$$
$$\delta_{32}=-\sin\psi\sin\varphi_r\cos\varphi_r(1-\cos\omega_3 t)-\cos\psi\cos\varphi_r\sin\omega_3 t;$$
$$\delta_{33}=\sin^2\psi\cos^2\varphi_r(1-\cos\omega_3 t)+\cos\omega_3 t.$$

<u>The Cosines of the Angles Included Between the Axes of the Launch and the Terrestrial Coordinates System</u>

The transformation from a launch $Ox_cy_cz_c$ to a terrestrial $Ox_3y_3z_3$ coordinate system can be accomplished in the following manner (Fig. 2.6). Let us turn the axes system which coincides first with the

launch system $Ox_cy_cz_c$, around axis Oy_c by angle ψ so that the azimuth of axis Ox' becomes equal to zero. Let us further turn system Ox'y'z' around axis Oz' by angle $\gamma = \phi_r - \phi_u$ so that axis Oy" coincides with axis Oy_3. By turning around axis Oy_3 by angle ψ in the opposite direction we line up the intermediate system Ox"y"z" with the terrestrial system $Ox_3y_3z_3$.

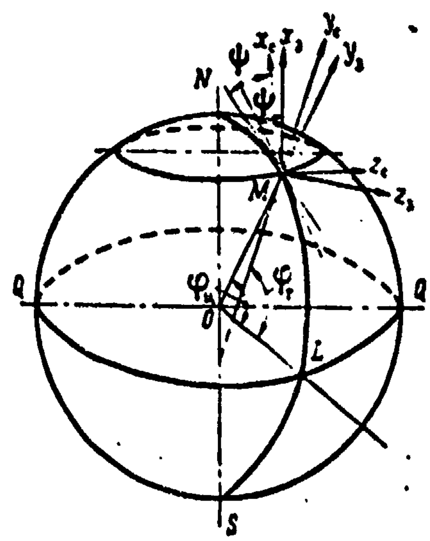

Fig. 2.6. Orientation of terrestrial axes relative to launch coordinate axes.

The matrix equation of the transformation takes the form

$$[\bar{x}_c] = \mathbf{E}[\bar{x}_3], \tag{2.25}$$

where

$$\mathbf{E} = \begin{Vmatrix} \sin^2\psi + \cos^2\psi\cos\gamma & -\cos\psi\sin\gamma & \sin\psi\cos\psi(1-\cos\gamma) \\ \cos\psi\sin\gamma & \cos\gamma & -\sin\psi\sin\gamma \\ \sin\psi\cos\psi(1-\cos\gamma) & \sin\psi\sin\gamma & \cos^2\psi + \sin^2\psi\cos\gamma \end{Vmatrix}. \tag{2.26}$$

<u>The Cosines of Angles Included Between the Axes of Initial Launch and Terrestrial Coordinates Systems</u>

Matrix B of the transformation from a terrestrial coordinate system to an initial launch coordinate system can be found by multiplying

the transformation matrixes above obtained from a launch to an initial launch coordinate system (Δ) and from a terrestrial to launch coordinate system (E). As a result we will obtain

$$[\bar{x}_0]=\Delta[\bar{x}_c]=\Delta\mathrm{E}[\bar{x}_3]=\mathrm{B}[\bar{x}_3], \qquad (2.27)$$

where

$$\mathrm{B}=\Delta\mathrm{E}=\begin{Vmatrix}\beta_{11} & \beta_{12} & \beta_{13}\\ \beta_{21} & \beta_{22} & \beta_{23}\\ \beta_{31} & \beta_{32} & \beta_{33}\end{Vmatrix}, \qquad (2.28)$$

and the matrix coefficients:

$$\beta_{11}=\cos^2\psi\cos\varphi_r\cos\varphi_u(1-\cos\omega_3 t)+(\cos^2\psi\cos\gamma+\sin^2\psi)\cos\omega_3 t+$$
$$+\sin\psi\cos\psi(\sin\varphi_r-\sin\varphi_u)\sin\omega_3 t;$$
$$\beta_{21}=\cos\psi\sin\varphi_r\cos\varphi_u(1-\cos\omega_3 t)+\cos\psi\sin\gamma\cos\omega_3 t-$$
$$-\sin\psi\cos\varphi_r\sin\omega_3 t;$$
$$\beta_{31}=-\cos\psi\sin\psi\cos\varphi_r(1-\cos\omega_3 t)\cos\varphi_u+\cos\psi\sin\psi(1-\cos\gamma)\times$$
$$\times\cos\omega_3 t-(\cos^2\psi\sin\varphi_u+\sin^2\psi\sin\varphi_r)\sin\omega_3 t;$$
$$\beta_{12}=\cos\psi\cos\varphi_r\sin\varphi_u(1-\cos\omega_3 t)-\cos\psi\sin\gamma\cos\omega_3 t+$$
$$+\sin\psi\cos\varphi_u\sin\omega_3 t;$$
$$\beta_{22}=\sin\varphi_r\sin\varphi_u(1-\cos\omega_3 t)+\cos\gamma\cos\omega_3 t;$$
$$\beta_{32}=-\sin\psi\cos\varphi_r\sin\varphi_u(1-\cos\omega_3 t)+\sin\psi\sin\gamma\cos\omega_3 t+$$
$$+\cos\psi\cos\varphi_u\sin\omega_3 t;$$
$$\beta_{13}=-\sin\psi\cos\psi\cos\varphi_r\cos\varphi_u(1-\cos\omega_3 t)+$$
$$+\sin\psi\cos\psi(1-\cos\gamma)\cos\omega_3 t+(\sin^2\psi\sin\varphi_u+\cos^2\psi\sin\varphi_r)\sin\omega_3 t;$$
$$\beta_{23}=-\sin\psi\sin\varphi_r\cos\varphi_u(1-\cos\omega_3 t)-\sin\psi\sin\gamma\cos\omega_3 t-$$
$$-\cos\psi\cos\varphi_r\sin\omega_3 t;$$
$$\beta_{33}=\sin^2\psi\cos\varphi_r\cos\varphi_u(1-\cos\omega_3 t)+(\sin^2\psi\cos\gamma+\cos^2\psi)\cos\omega_3 t+$$
$$+\sin\psi\cos\psi(\sin\varphi_u-\sin\varphi_r)\sin\omega_3 t.$$

The Cosines of the Angles Included Between the Axes of Body and Terrestrial Coordinate Systems

The matrix of transformation from a terrestrial to a body coordinate system can be found by multiplying matrixes Γ and B. As a result we will obtain

$$[\bar{x}_1]=\Gamma[\bar{x}_0]=\Gamma B[\bar{x}_3]=A[\bar{x}_3], \tag{2.29}$$

where

$$A=\Gamma B=\begin{Vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{Vmatrix}, \tag{2.30}$$

and the matrix coefficients:

$$a_{11}=\beta_{11}\cos\varphi\cos\xi+\beta_{21}\sin\varphi-\beta_{31}\sin\xi\cos\varphi;$$

$$a_{21}=\beta_{11}(-\cos\xi\sin\varphi\cos\eta+\sin\xi\sin\eta)+\beta_{21}\cos\varphi\cos\eta+$$
$$+\beta_{31}(\sin\varphi\cos\eta\sin\xi+\cos\xi\sin\eta);$$

$$a_{31}=\beta_{11}(\cos\xi\sin\varphi\sin\eta+\sin\xi\cos\eta)-\beta_{21}\cos\varphi\sin\eta+$$
$$+\beta_{31}(-\sin\xi\sin\varphi\sin\eta+\cos\xi\cos\eta);$$

$$a_{12}=\beta_{12}\cos\varphi\cos\xi+\beta_{22}\sin\varphi-\beta_{32}\sin\xi\cos\varphi;$$

$$a_{22}=\beta_{12}(-\cos\xi\sin\varphi\cos\eta+\sin\xi\sin\eta)+\beta_{22}\cos\varphi\cos\eta+$$
$$+\beta_{32}(\sin\varphi\cos\eta\sin\xi+\cos\xi\sin\eta);$$

$$a_{32}=\beta_{12}(\cos\xi\sin\varphi\sin\eta+\sin\xi\cos\eta)-\beta_{22}\cos\varphi\sin\eta+$$
$$+\beta_{32}(-\sin\xi\sin\varphi\sin\eta+\cos\xi\cos\eta);$$

$$a_{13}=\beta_{13}\cos\varphi\cos\xi+\beta_{23}\sin\varphi-\beta_{33}\sin\xi\cos\varphi;$$

$$a_{23}=\beta_{13}(-\cos\xi\sin\varphi\cos\eta+\sin\xi\sin\eta)+\beta_{23}\cos\varphi\cos\eta+$$
$$+\beta_{33}(\sin\varphi\cos\eta\sin\xi+\cos\xi\sin\eta);$$

$$a_{33}=\beta_{13}(\cos\xi\sin\varphi\sin\eta+\sin\xi\cos\eta)-\beta_{23}\cos\varphi\sin\eta+$$
$$+\beta_{33}(-\sin\xi\sin\varphi\sin\eta+\cos\xi\cos\eta).$$

Kinematic Equations

For investigating rocket flight it is necessary to have kinematic equations describing the variation in the angular coordinates of the rocket ϕ, ξ and η depending on the projections of the angular velocity vector of the rocket on the body axes ω_{x1}, ω_{y1}, ω_{z1}.

In order to obtain the indicated equations, let us examine Fig. 2.4 given earlier, from which it follows that the angular velocity vector $\bar{\dot{\xi}}$ is directed along axis Oy_0, vector $\bar{\dot{\phi}}$ - along axis Oz', and vector $\bar{\dot{\eta}}$ - along axis Ox_1.

The angular velocity vector of a rocket $\bar{\omega}$ can be represented as the sum

$$\bar{\omega}=\bar{\dot{\xi}}+\bar{\dot{\varphi}}+\bar{\dot{\eta}}. \tag{2.31}$$

The cosines of the angles included between vectors $\bar{\dot{\xi}}$, $\bar{\dot{\phi}}$, and $\bar{\dot{\eta}}$, and the body axes are given in Table 2.2. Using this table, we find that the projections of the angular velocity of the rocket on the body axes are equal to:

$$\left.\begin{aligned}
\omega_{x1}&=\frac{d\eta}{dt}+\frac{d\xi}{dt}\sin\varphi;\\
\omega_{y1}&=\frac{d\xi}{dt}\cos\varphi\cos\eta+\frac{d\varphi}{dt}\sin\eta;\\
\omega_{z1}&=\frac{d\varphi}{dt}\cos\eta-\frac{d\xi}{dt}\cos\varphi\sin\eta.
\end{aligned}\right\} \tag{2.32}$$

The kinematic equations of the motion of the center of mass of the object we obtain, by projecting expression (2.13) on terrestrial axes:

$$\frac{dx_3}{dt}=V_{x3};\quad \frac{dy_3}{dt}=V_{y3};\quad \frac{dz_3}{dt}=V_{z3}. \tag{2.33}$$

Table 2.2

axes	$\overline{\dot{\xi}}$	$\overline{\dot{\varphi}}$	$\overline{\dot{\eta}}$
Ox_1	$\sin\varphi$	0	1
Oy_1	$\cos\varphi\cos\eta$	$\sin\eta$	0
Oz_1	$-\cos\varphi\sin\eta$	$\cos\eta$	0

Resolving Forces and Moments with Respect to Coordinate Axes

Let us find the components of the forces and moments acting on a rocket with respect to coordinates axes, taking into account that the equations of motion of the center of mass of the rocket are projected on terrestrial coordinates axes, and the equations of rotation - on body axes.

For determining the components of thrust forces and aerodynamic forces it is necessary to know the flight altitude, and for determining the components of attractive forces - the geocentric latitude depending on the coordinates of the rocket in terrestrial axes x_3, y_3, z_3. Let us give the appropriate formulas.

First let us break down into components with respect to terrestrial axes the radius-vector $\bar{r}$ of the center of mass of the rocket relative to the center of the earth. Let $\bar{x}_3^0$, $\bar{y}_3^0$, and $\bar{z}_3^0$ be the unit vectors of the terrestrial axes (Fig. 2.7). Then

$$\bar{r}=x_3\vec{x}_3^0+(R_0+y_3)\vec{y}_3^0+z_3\vec{z}_3^0, \tag{2.34}$$

where R_0 - distance from the center of the earth to the launch point, determined by the formula

$$R_0=a\frac{\sqrt{1-e^2}}{\sqrt{1-e^2\cos^2\varphi_{c0}}}; \tag{2.35}$$

e - the eccentricity of the meridional cross section of the general terrestrial ellipsoid; ϕ_{u0} - the geocentric latitude of the launch point which can be determined, knowing the geodetic latitude, using the formula

$$\varphi_{u0}=\varphi_{r0}-\gamma\approx\varphi_{r0}-\alpha\sin 2\varphi_{r0}. \qquad (2.36)$$

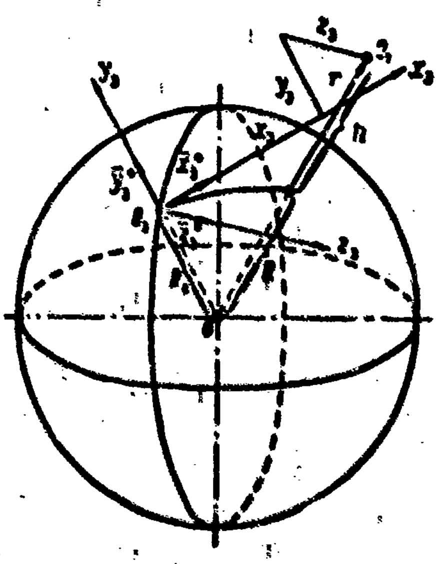

Fig. 2.7. Coordinates of the center of mass of a rocket in terrestrial axes and the flight altitude.

The flight altitude is determined by the dependence

$$h=r-R, \qquad (2.37)$$

where r - the distance from the center of the earth to the rocket, equal to

$$r=\sqrt{x^2+(R_0+y)^2+z^2}; \qquad (2.38)$$

R - distance along radius-vector $\vec{r}$ from the center of the earth to its surface, equal to

$$R=a\frac{\sqrt{1-e^2}}{\sqrt{1-e^2\cos^2\varphi_u}}\approx a(1-\alpha\sin^2\varphi_u). \qquad (2.39)$$

The geocentric latitude of the point in space, at which the rocket is located, is determined by the assigned coordinates of this point in

the terrestrial coordinate system by the formula

$$\sin\varphi_{ш} = \frac{x_3}{r}\cos\psi\cos\varphi_{ш0} + \frac{R_0 + y_3}{r}\sin\varphi_{ш0} - \frac{z_3}{r}\sin\psi\cos\varphi_{ш0}. \qquad (2.40)$$

Upon resolving the attractive force of the earth with respect to terrestrial axes we will examine two components of the acceleration due to terrestrial attraction: $\bar{g}_{Tr}$ - directed toward the center of the earth and $\bar{g}_{T\omega}$ - directed parallel to the axis of rotation of the earth. The expressions for their values were given above [see formulas (1.13) and (1.14)].

Having combined centrifugal acceleration $\bar{j}_{ц} = -\bar{j}_e$ with the acceleration due to terrestrial attraction $\bar{g}_T$, we obtain the acceleration due to gravity

$$\bar{g} = \bar{g}_T + \bar{j}_{ц}. \qquad (2.41)$$

Taking into account that centrifugal acceleration

$$\bar{j}_e = -\bar{\omega}_3 \times (\bar{\omega}_3 \times \bar{r}) = \bar{y}^0 \omega_3^2 r \cos\varphi_{ш}, \qquad (2.42)$$

where $\bar{y}^0$ - the unit vector of geocentric axes Oy (Fig. 2.8), let us resolve centrifugal acceleration, and also the acceleration due to terrestrial attraction, into two components - along radius $\bar{r}$ and along the axis of rotation of the earth (see Fig. 2.8):

$$\left.\begin{aligned} \bar{j}_{цr} &= \omega_3^2 r; \\ \bar{j}_{ц\omega} &= \omega_3^2 r \sin\varphi_{ш}. \end{aligned}\right\} \qquad (2.43)$$

Then the components of acceleration due to gravity will be equal to:

$$\left.\begin{aligned} g_r &= g_{Tr} - \omega_3^2 r, \\ g_\omega &= g_{T\omega} + \omega_3^2 r \sin\varphi_{ш}. \end{aligned}\right\} \qquad (2.44)$$

The cosines of the angles included between these components of acceleration due to gravity and the terrestrial axes are given in Table 2.3.

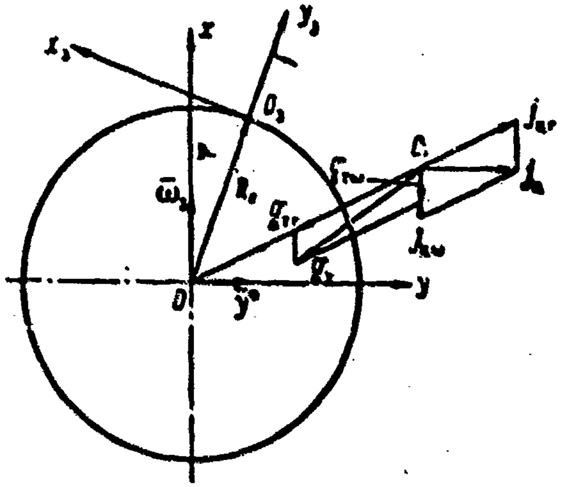

Fig. 2.8 The resolution of the acceleration due to gravity and centrifugal acceleration into components with respect to the direction toward the center of the earth and with respect to the axis of rotation of the earth.

Table 2.3

axes	Ox_3	Oy_3	Oz_3
$\bar{g}_r$	$-\frac{x_3}{r}$	$-\frac{R_0+y_3}{r}$	$-\frac{z_3}{r}$
$\bar{g}_\omega$	$-\frac{\omega_{3x_3}}{\omega_3}$	$-\frac{\omega_{3y_3}}{\omega_3}$	$-\frac{\omega_{3z_3}}{\omega_3}$

The projections of gravity on the terrestrial axes are determined by the formulas:

$$\left.\begin{aligned} G_{x3} &= -mg_r\frac{x_3}{r} - mg_\omega\frac{\omega_{3x_3}}{\omega_3};\\ G_{y3} &= -mg_r\frac{R_0+y_3}{r} - mg_\omega\frac{\omega_{3y_3}}{\omega_3};\\ G_{z3} &= -mg_r\frac{z_3}{r} - mg_\omega\frac{\omega_{3z_3}}{\omega_3}. \end{aligned}\right| \quad (2.45)$$

In order to determine the components of Coriolis acceleration, let us first resolve the angular velocity vector of the earth $\bar{\omega}_3$ into components along the terrestrial axes. First let us resolve it into

two components in the meridian plane (Fig. 2.9): on the vertical

$$\omega_{3y_3} = \omega_3 \sin \varphi_{u0}$$

and horizontal

$$\omega_{3.r} = \omega_3 \cos \varphi_{u0}.$$

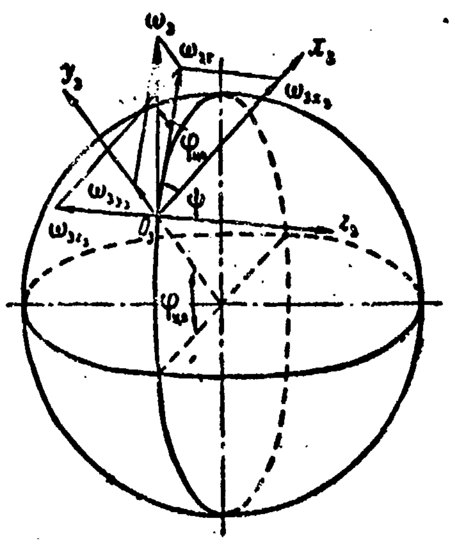

Fig. 2.9 Resolution of the angular velocity of the rotation of the earth into its components along terrestrial coordinate axes.

The horizontal component in its turn can be resolved along axes Ox_3 and Oz_3 into components:

$$\omega_{3x_3} = \omega_3 \cos \varphi_{u0} \cos \psi;$$
$$\omega_{3z_3} = -\omega_3 \cos \varphi_{u0} \sin \psi.$$

Thus, vector $\bar{\omega}_3$ can be represented in the form

$$\vec{\omega}_3 = \omega_3 (\cos \varphi_{u0} \cos \psi \vec{x}_3^{\circ} + \sin \varphi_{u0} \vec{y}_3^{\circ} - \cos \varphi_{u0} \sin \psi \vec{z}_3^{\circ}). \qquad (2.46)$$

Using expressions (2.6), (2.34) and (2.46), let us find the resolution of Coriolis acceleration

$$\bar{J}_c = 2(\bar{\omega}_3 \times \bar{V}) = 2 \begin{vmatrix} \bar{x}_3^{\circ} & \bar{y}_3^{\circ} & \bar{z}_3^{\circ} \\ \omega_{3x_3} & \omega_{3y_3} & \omega_{3z_3} \\ V_{x3} & V_{y3} & V_{z3} \end{vmatrix} \qquad (2.47)$$

along the terrestrial axes. Then we will obtain:

$$\left.\begin{aligned} -j_{cx_3} &= b_{12}V_{y_3} + b_{13}V_{z_3}; \\ -j_{cy_3} &= b_{21}V_{x_3} + b_{23}V_{z_3}; \\ -j_{cz_3} &= b_{31}V_{x_3} + b_{32}V_{y_3}, \end{aligned}\right\} \quad (2.48)$$

where

$$\left.\begin{aligned} b_{12} &= -b_{21} = -2\omega_3 \cos\varphi_{u0} \sin\psi; \\ b_{13} &= -b_{31} = -2\omega_3 \sin\varphi_{u0}; \\ b_{23} &= b_{32} = 2\omega_3 \cos\varphi_{u0} \cos\psi. \end{aligned}\right\} \quad (2.49)$$

For determining the projections on the terrestrial axes of force $\bar{N}$, i.e., the resultant of thrust force, aerodynamic forces and the forces created by the control elements, let us first resolve these forces along the body axes, and then, using the matrix of the cosines of the angles included between the axes of body and terrestrial coordinate systems, let us find the desired projections of the forces. Thus, if the components of the forces in question are represent along the body axes in the form of the sums:

$$\left.\begin{aligned} N_{x1} &= P_{x1} + T_{x1} + X_1; \\ N_{y1} &= T_{y1} + Y_1; \\ N_{z1} &= T_{z1} + Z_1, \end{aligned}\right\} \quad (2.50)$$

then the sums of the projections on the terrestrial axes of thrust force, aerodynamic forces and the forces created by the control elements, are determined by the formulas:

$$\left.\begin{aligned} N_{x3} &= N_{x1}a_{11} + N_{y1}a_{21} + N_{z1}a_{31}; \\ N_{y3} &= N_{x1}a_{12} + N_{y1}a_{22} + N_{z1}a_{32}; \\ N_{z3} &= N_{x1}a_{13} + N_{y1}a_{23} + N_{z1}a_{33}. \end{aligned}\right\} \quad (2.51)$$

It remains to find the expressions for the normal and transverse components of aerodynamic force Y_1 and Z_1.

Resultant of these forces $(\bar{Y}_1 + \bar{Z}_1)$ is perpendicular to the longitudinal axis of the rocket and lies in the plane of the angle of attack, passing through the velocity vector and this axis. Let us introduce the unit vectors: $\bar{x}_1^\circ$, directed along longitudinal axis Ox_1, and $\bar{v}^\circ$, directed along the velocity vector, and let us write in vectorial form the direction of force $(\bar{Y}_1 + \bar{Z}_1)$. This force is perpendicular to vectors $\bar{x}_1^\circ$ and $\bar{v}^\circ \times \bar{x}_1^\circ$ and thus, coincides in direction with vector $(\bar{v}^\circ \times \bar{x}_1^\circ) \times \bar{x}_1^\circ$ (Fig. 2.10).

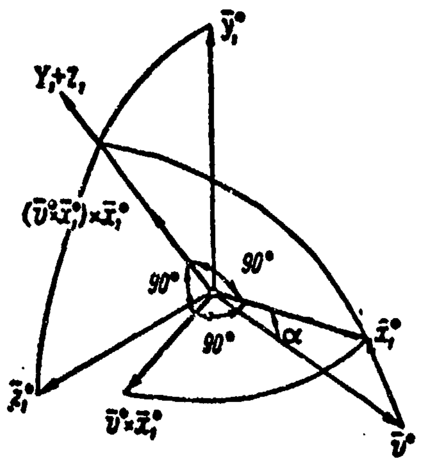

Fig. 2.10. For determining the direction of aerodynamic transverse force $\bar{N}_1 = \bar{Y}_1 + \bar{Z}_1$.

The modulus of the vectorial product $\bar{v}^\circ \times \bar{x}_1^\circ$ is equal to $\sin\alpha \approx \alpha$. The modulus of vector $(\bar{v}^\circ \times \bar{x}_1^\circ) \times \bar{x}_1^\circ$ is also equal to α because the vectors $\bar{v}^\circ \times \bar{x}_1^\circ$ and $\bar{x}_1^\circ$ are mutually perpendicular. Having noted this, let us represent the transverse force in the following manner:

$$\bar{Y}_1+\bar{Z}_1=c_n^\alpha qS[(\bar{v}^\circ\times\bar{x}_1^\circ)\times\bar{x}_1^\circ]. \tag{2.52}$$

In order to determine the components of vector $(\bar{Y}_1 + \bar{Z}_1)$ on body axes, let us make use of the equality $\bar{x}_1^\circ = \bar{y}_1^\circ \times \bar{z}_1^\circ$ and convert the vectorial product $\bar{v}^\circ \times \bar{x}_1^\circ$ to the form

$$\bar{v}^\circ\times\bar{x}_1^\circ=\bar{v}^\circ\times(\bar{y}_1^\circ\times\bar{z}_1^\circ)=\bar{y}_1^\circ(\bar{v}^\circ\bar{z}_1^\circ)-\bar{z}_1^\circ(\bar{v}^\circ\bar{y}_1^\circ). \tag{2.53}$$

Taking the obtained expression into account let us convert the double vectorial product

$$(\bar{v}^{\circ}\times\bar{x}_1^{\circ})\times\bar{x}_1^{\circ}=[\bar{y}_1^{\circ}(\bar{v}^{\circ}\bar{z}_1^{\circ})-\bar{z}_1^{\circ}(\bar{v}^{\circ}\bar{y}_1^{\circ})]\times\bar{x}_1^{\circ}=$$
$$=(\bar{v}^{\circ}\bar{z}_1^{\circ})\,\bar{y}_1^{\circ}\times\bar{x}_1^{\circ}-(\bar{v}^{\circ}\bar{y}_1^{\circ})\,\bar{z}_1^{\circ}\times\bar{x}_1^{\circ}=-(\bar{v}^{\circ}\bar{z}_1^{\circ})\,\bar{z}_1^{\circ}-(\bar{v}^{\circ}\bar{y}_1^{\circ})\,\bar{y}_1^{\circ}. \quad (2.54)$$

Then expression (2.52) will take the form

$$\bar{Y}_1+\bar{Z}_1=-c_n^{\alpha}qS(\bar{v}^{\circ}\bar{z}_1^{\circ})\,\bar{z}_1^{\circ}-c_n^{\alpha}qS(\bar{v}^{\circ}\bar{y}_1^{\circ})\,\bar{y}_1^{\circ}. \quad (2.55)$$

Thus, the forces $\bar{Y}_1$ and $\bar{Z}_1$ are determined by the expressions:

$$\left.\begin{aligned}\bar{Y}_1&=-c_n^{\alpha}qS(\bar{v}^{\circ}\bar{y}_1^{\circ})\,\bar{y}_1^{\circ};\\ \bar{Z}_1&=-c_n^{\alpha}qS(\bar{v}^{\circ}\bar{z}_1^{\circ})\,\bar{z}_1^{\circ}.\end{aligned}\right\} \quad (2.56)$$

Scalar products $(\bar{v}^{\circ}\bar{y}_1^{\circ})$ and $(\bar{v}^{\circ}\bar{z}_1^{\circ})$ can be considered as the values of angles of attack α_y and α_z in the planes Ox_1y_1 and Ox_1z_1.

As can be seen from Fig. 2.11, if angles α_y and α_z are small, then

$$\left.\begin{aligned}(\bar{v}^{\circ}\bar{y}_1^{\circ})&=\cos(\bar{v}^{\circ},\widehat{\ \bar{y}_1^{\circ}})\approx\cos(90^{\circ}+\alpha_y)=-\sin\alpha_y\approx-\alpha_y;\\ (\bar{v}^{\circ}\bar{z}_1^{\circ})&=\cos(\bar{v}^{\circ},\widehat{\ \bar{z}_1^{\circ}})\approx\sin\alpha_z\approx\alpha_z.\end{aligned}\right\} \quad (2.57)$$

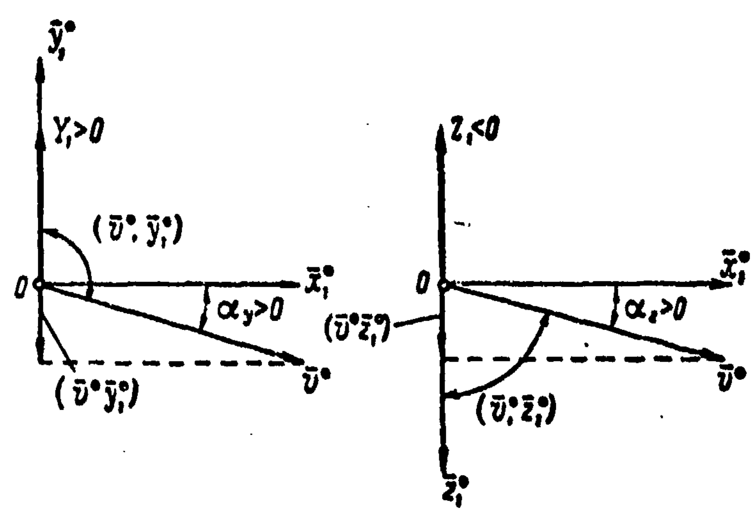

Fig. 2.11. For determining angles of attack α_y and α_z.

Taking formula (2.56) into account, we obtain the following expressions for the aerodynamic forces:

$$\left.\begin{aligned} Y_1 &= c_n^* qS\alpha_y; \\ Z_1 &= -c_n^* qS\alpha_z. \end{aligned}\right\} \qquad (2.58)$$

In order to find the expressions of angles α_y and α_z, let us determine the direction in space of the velocity vector $\bar{V}$ of the center of mass of the rocket. For this let us examine coordinate system $Ox_3y_3z_3$ whose origin coincides with the center of mass of the rocket, and the axes are directed parallel to the axes of the terrestrial system.

The direction of velocity vector $\bar{V}$ relative to the terrestrial coordinate axes let us determine by the following two angles (Fig. 2.12):

1) By angle σ between the projection of the velocity vector $\bar{V}$ on plane Ox_3z_3 and axis Ox_3 and

2) by angle τ between velocity vector $\bar{V}$ and plane Ox_3z_3.

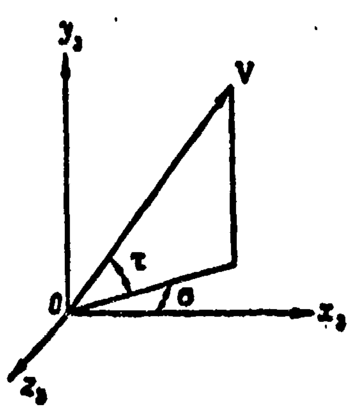

Fig. 2.12. Orientation of the velocity vector of a rocket relative to the terrestrial coordinate axes.

The projections of the velocity vector on the terrestrial axes are equal to:

$$\left.\begin{aligned} V_{x_3} &= V\cos\tau\cos\sigma; \\ V_{y_3} &= V\sin\tau; \\ V_{z_3} &= -V\cos\tau\sin\sigma, \end{aligned}\right\} \tag{2.59}$$

The cosines of the angles included between the velocity vector $\bar{V}$ and the terrestrial axes are given in Table 2.4.

Table 2.4

axes	Ox_3	Oy_3	Oz_3
V	$\cos\tau\cos\sigma$	$\sin\tau$	$-\cos\tau\sin\sigma$

Knowing the cosines of the angles included between the body and the terrestrial axes α_{ij}, and also between the velocity vector and the terrestrial axes (see Table 2.4), let us determine angle of attack α_y:

$$\alpha_y = -(\vec{v}^{\circ}\vec{y}_1^{\circ}) = -\cos(\widehat{\vec{v}^{\circ}, \vec{x}_3})\cos(\widehat{\vec{y}_1, \vec{x}_3}) - -\cos(\widehat{\vec{v}^{\circ}, \vec{y}_3})\cos(\widehat{\vec{y}_1, \vec{y}_3}) - \cos(\widehat{\vec{v}^{\circ}, \vec{z}_3})\cos(\widehat{\vec{y}_1, \vec{z}_3}),$$

or

$$\alpha_y = -\alpha_{21}\cos\tau\cos\sigma - \alpha_{22}\sin\tau + \alpha_{23}\cos\tau\sin\sigma. \tag{2.60}$$

In an analogous manner let us find angle α_z:

$$\alpha_z = \alpha_{31}\cos\tau\cos\sigma + \alpha_{32}\sin\tau - \alpha_{33}\cos\tau\sin\sigma. \tag{2.61}$$

In these expressions angles τ and σ are determined by the formulas:

$$\left.\begin{aligned} \tau &= \arcsin\frac{V_{y_3}}{V} = \arcsin\frac{V_{y_3}}{\sqrt{V_{x_3}^2 + V_{y_3}^2 + V_{z_3}^2}}; \\ \sigma &= \operatorname{arctg}\left(-\frac{V_{z_3}}{V_{x_3}}\right), \end{aligned}\right\} \tag{2.62}$$

following from the relationships of (2.59).

Let us now determine the projections of the moments of force on the body axes. The moments of normal and transverse aerodynamic forces we will obtain, using expression (2.58):

$$\left.\begin{aligned} M_{y1} &= -Z_1(x_т - x_d) = c_n^* qS(x_т - x_d)\alpha_z; \\ M_{z1} &= Y_1(x_т - x_d) = c_n^* qS(x_т - x_d)\alpha_y. \end{aligned}\right\} \tag{2.63}$$

Since the axial aerodynamic force and the thrust force of the main engine are directed along the longitudinal axis, passing through the center of mass of the rocket, the moments of these forces are equal to zero. The aerodynamic damping moments and the controlling moments are determined respectively by formulas (1.28) and (1.46).

System of Equations of Motion

Let us project equation (2.9) on terrestrial coordinate axes. Let us first represent the forces $\bar{F}+\bar{P}-m\bar{j}_e$, acting on the rocket, in the form

$$-m\bar{j}_e+\bar{F}+\bar{P}=\bar{N}+\bar{G}. \tag{2.64}$$

Here $\bar{F}=\bar{R}+\bar{G}_т$ - resultant of the total aerodynamic force and the force of attraction; $\bar{P}$ - (THA = TPU), and the forces of the controlling engines; $\bar{G}$ - gravity; $\bar{N} = \bar{R} + \bar{P}$ - the resultant of the total aerodynamic forces and the thrust forces.

The equations of motion of the center of mass of a rocket in projections on terrestrial axes will take the form:

$$\left.\begin{aligned} m\frac{dV_{x_3}}{dt} &= N_{x_3}+G_{x_3}-mj_{ex_3}; \\ m\frac{dV_{y_3}}{dt} &= N_{y_3}+G_{y_3}-mj_{ey_3}; \\ m\frac{dV_{z_3}}{dt} &= N_{z_3}+G_{z_3}-mj_{ez_3}. \end{aligned}\right\} \tag{2.65}$$

To these equations it is still necessary to add the three kinematic equations of motion of the center of mass of a rocket of (2.33).

Let us project the equations of the rotation of the rocket around the center of mass (2.12) on the rocket body coordinate axis rotating relative to the terrestrial axes with angular velocity $\bar{\omega}$. Let ω_{x1}, ω_{y1}, ω_{z1} - projections of the angular velocity of the rocket $\bar{\omega}$ on its body axes. The projections of the vector of angular momentum K on these axes are respectively equal to $J_{x1}\omega_{x1}, J_{y1}\omega_{y1}, J_{z1}\omega_{z1}$. Then, projecting expression (2.12) on the rocket body axes, we obtain the so-called "dynamic Euler equations":

$$\left.\begin{aligned} J_{x1}\frac{d\omega_{x1}}{dt}+(J_{z1}-J_{y1})\,\omega_{y1}\omega_{z1}&=\sum M_{x1};\\ J_{y1}\frac{d\omega_{y1}}{dt}+(J_{x1}-J_{z1})\,\omega_{z1}\omega_{x1}&=\sum M_{y1};\\ J_{z1}\frac{d\omega_{z1}}{dt}+(J_{y1}-J_{x1})\,\omega_{x1}\omega_{y1}&=\sum M_{z1}.\end{aligned}\right\} \qquad (2.67)$$

The relationships between the projections of the angular velocity of a rocket on the body coordinate axes ω_{x1}, ω_{y1}, ω_{z1} and angles ϕ, ξ, η, which determine the orientation of the rocket relative to the initial launch axes, are determined by the kinematic equations of (2.32).

The system of equations (2.32), (2.33), (2.65), (2.66) can be used for describing the motion of an unguided rocket, but for a guided rocket it is still not closed. The fact is that an unguided rocket as a solid body has six degrees of freedom. With respect to this its motion is described by the system of 12-differential equations of the first order (2.32, (2.33), (2.65), (2.66) which is closed because the forces P, X_1, Y_1, Z_1, acting on the rocket, and their moments M_{x1}, M_{y1}, M_{z1} relative to the body axes are uniquely determined by the parameters of the rocket motion and the number of unknown functions

$$x_3,\ y_3,\ z_3,\ V_{x3},\ V_{y3},\ V_{z3},\ \varphi,\ \xi,\ \eta,\ \omega_{x1},\ \omega_{y1},\ \omega_{z1} \qquad (2.67)$$

is equal to the number of differential equations. In this case, if random perturbations are absent, the flight path is completely determined by the initial conditions - by the values of the kinematic parameters of motion at the initial moment of time:

$$x_3(t_0),\ y_3(t_0),\ z_3(t_0),\ \ldots,\ \omega_{z1}(t_0). \qquad (2.68)$$

Guided rocket, if we disregard its elasticity and examine it as a mechanical system, already possesses in general 12 degrees of freedom: the six degrees of freedom for the motion of the center of mass and rotation around the center of mass and the six degrees of freedom of the corresponding control elements. In the particular case examined above, in Section 1.7, when normal controlling forces are created by the rotation of the rocket around two axes, the rocket has four control elements: the elements controlling the rotary motions of pitch, yaw and roll and engine thrust. The system of 12 differential equations (2.32), (2.33), (2.65), (2.66) in this case is not closed because the projections of the forces and moments, going into the right sides of the equation, depend on the displacements of the elements controlling the motions of pitch δ_ϕ, yaw δ_η, roll δ_ξ and engine thrust δ_P.

If we apart from the initial conditions assign variation with time of values $\delta_\phi(t)$, $\delta_\eta(t)$, $\delta_\xi(t)$, $\delta_P(t)$, then the missile trajectory will be determined by this. In actual flight the displacements of the control elements are accomplished by the control system depending on the flight mission being carried out. So that the problem of determining flight path can be carried out, it is necessary to add the equations describing the processes in the control system and connecting the displacements of the control elements with the parameters of rocket motion to the system of equations of rocket motion (2.32), (2.33), (2.65), (2.66). These equations can take a completely different specific form depending on the operating principle and the control system layout.

In the most general form the equations of the control system can be written in the following manner:

$$\left.\begin{aligned} F_1[\delta_\varphi(t), x_3(t), y_3(t), z_3(t), \varphi(t), \xi(t), \eta(t)]=0; \\ F_2[\delta_\xi(t), x_3(t), y_3(t), z_3(t), \varphi(t), \xi(t), \eta(t)]=0; \\ F_3[\delta_\xi(t), x_3(t), y_3(t), z_3(t), \varphi(t), \xi(t), \eta(t)]=0; \\ F_4[\delta_P(t), x_3(t), y_3(t), z_3(t), \varphi(t), \xi(t), \eta(t)]=0. \end{aligned}\right\} \quad (2.69)$$

where F_1, F_2, F_3, and F_4 - functionals of the functions enclosed in the square brackets.

Sixteen equations (2.32), (2.33), (2.65), (2.66), (2.69) now make up the closed system determining the 16 unknown functions:

$$x_3,\ y_3,\ z_3,\ V_{x3},\ V_{y3},\ V_{z3},\ \omega_{x1},$$
$$\omega_y,\ \omega_{z1},\ \varphi,\ \xi,\ \eta,\ \delta_\varphi,\ \delta_\xi,\ \delta_\xi,\ \delta_p.$$

In this case the trajectory of guided flight (the solution of the system) is determined by assigning the initial conditions and the actual connections (2.69) imposed on the rocket motion by the control system.

2.3 THE EQUATION OF MOTION IN PROJECTIONS ON SEMI-WIND COORDINATE AXES

The obtained above general equations of motion in projection on terrestrial axes can be used in principle for solving any technical problems. However it is always advantageous to introduce into the equations under investigation these or other simplifications whose essence is intimately connected with the content of the actual problem. Because of this it is frequently convenient in investigating the dynamics of a rocket or a nose section to use the equations of motion in projections on semi-wind axes.

Coordinate Systems

Geocentric Coordinate System

This coordinate system with its origin at the center of the earth and with its axes connected with the earth, was already used above in studying the earth's gravitational field. The reference planes in the coordinate system in question are the equatorial and the prime meridian planes.

The position of the center of mass of a rocket in this case can be determined either by three Cartesian coordinates x, y, z, or which

is more convenient, by three spherical coordinates λ, ϕ_u, r. Longitude λ and geocentric latitude ϕ_u are reckoned, as was shown above, in Fig. 1.3. Coordinate r is the distance from the center of the earth to the center of mass of the rocket.

Wind and Semi-Wind Coordinate Systems

In some problems of dynamics the equations of motion of the center of mass of a rocket are conveniently written as projections on coordinate axes connected with the velocity vector $\bar{V}$ of the rocket. The origin O of the coordinates of such a system is located at the center of mass of the rocket; axis Ox is directed along the velocity vector $\bar{V}$, i.e., tangentially to the trajectory in the direction of flight; axes Oy and Oz lie in the plane, normal to the flight path. In this case in flight dynamics axis Oy is selected both in the plane of symmetry of the object Ox_1y_1 and in the vertical plane. The first coordinate system we will call wind, the second - semi-wind.

Local Geographical Coordinate System

The origin of this coordinate system $Ox_гy_гz_г$ (Fig. 2.13) coincides with the center of mass of the object; axis $Ox_г$ is drawn parallel to the tangent toward the meridian of the site northwards; axis $Oy_г$ is directed along radius-vector $\bar{r}$; axis $Oz_г$ is parallel to the equatorial plane.

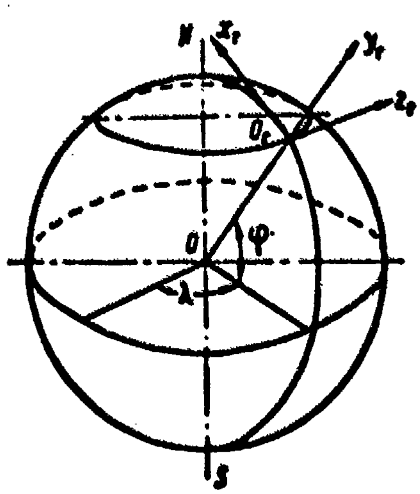

Fig. 2.13. Local geographical coordinate system.

Seim-Body Coordinate System

The origin of the semi-body coordinate system $Ox'y'z'$ (Fig. 2.14) coincides with the center of mass of the object; axis Ox' is directed along the longitudinal axis of the object in the direction of the nose; axis Oy' is perpendicular to the plane, passing through the velocity vector and the longitudinal axis of the object; axis Oz' completes the system to the right.

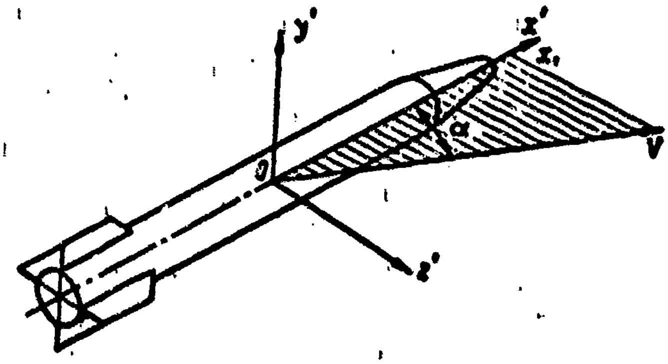

Fig. 2.14. Semi-body system of coordinate axes.

Direction Cosines Matrices

The Cosines of Angles Included Between the Axes of Body and Semi-Wind Coordinate Systems

In order to determine the cosines of the angles included between these coordinate axes, let us examine the sequential turns of the semi-wind coordinate system $Oxyz$ until its coincidence first with the semi-body $Ox'y'z'$, and then with the body $Ox_1y_1z_1$ coordinate systems (Fig. 2.15).

The transformation from a semi-wind coordinate system $Oxyz$ to a semi-body $Ox'y'z'$ can be accomplished by two sequential turns: first by angle μ around axis Ox, and then by angle of attack α around axis Oy'' in accordance with Fig. 2.15.

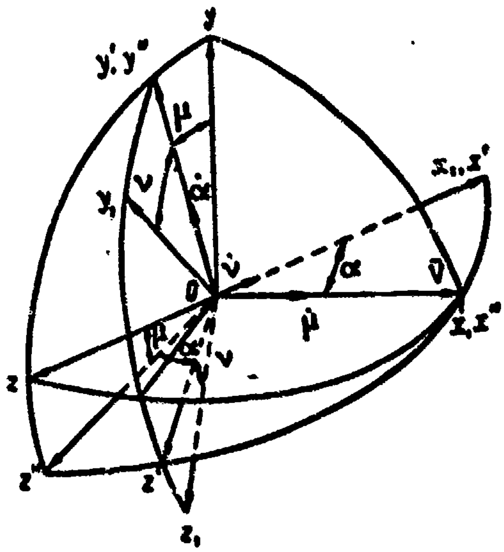

Fig. 2.15. Transformation from a semi-wind coordinate system to a body coordinate system.

The matrix of direction cosines included in the equation of the transformation of coordinates

$$[\overline{x'}] = \mathbf{Z}[\overline{x}], \tag{2.70}$$

has the form

$$\mathbf{Z} = \begin{Vmatrix} \cos\alpha & \sin\alpha\sin\mu & -\sin\alpha\cos\mu \\ 0 & \cos\mu & \sin\mu \\ \sin\alpha & -\cos\alpha\sin\mu & \cos\alpha\cos\mu \end{Vmatrix}. \tag{2.71}$$

In order to change from a semi-wind coordinate system to body, it is necessary to carry out one additional turn - to turn the semi-body coordinate axes Ox'y'z' by angle ν around axis Ox'.

As a result we obtain the following coordinate transformation equation:

$$[\overline{x}_1] = \mathbf{H}[\overline{x}], \tag{2.72}$$

where the matrix of the direction cosines H takes the following form:

$$H = \begin{Vmatrix} \cos\alpha & \sin\alpha\sin\mu & -\sin\alpha\cos\mu \\ \sin\alpha\sin\nu & \cos\mu\cos\nu - \\ & -\cos\alpha\sin\mu\sin\nu & \begin{matrix}\sin\mu\cos\nu + \\ +\cos\alpha\cos\mu\sin\nu\end{matrix} \\ \sin\alpha\cos\nu & \begin{matrix}-\cos\mu\sin\nu - \\ -\cos\alpha\sin\mu\cos\nu\end{matrix} & \begin{matrix}-\sin\mu\sin\nu + \\ +\cos\alpha\cos\mu\cos\nu\end{matrix} \end{Vmatrix}. \tag{2.73}$$

Cosines of The Angles Included Between the Axes of Semi-Wind and Local Geographical Coordinate Systems

Let us carry out sequential turns of the local geographical coordinate system $Ox_r y_r z_r$ by angles Ψ and Θ until the coincidence of the direction of its axes with the semi-wind coordinate system Oxyz (Fig. 2.16). As a result we obtain:

$$[\bar{x}] = A[\bar{x}_r]; \tag{2.74}$$

$$A = \begin{Vmatrix} \cos\Psi\cos\Theta & \sin\Theta & -\sin\Psi\cos\Theta \\ -\cos\Psi\sin\Theta & \cos\Theta & \sin\Psi\sin\Theta \\ \sin\Psi & 0 & \cos\Psi \end{Vmatrix}. \tag{2.75}$$

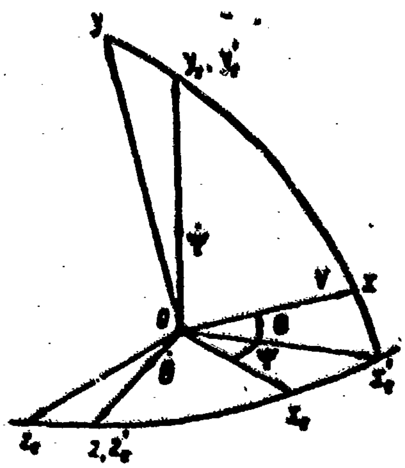

Fig. 2.16. Transformation from a local geographical coordinate system to a semi-wind coordinate system.

The equations for transforming from a local geographical coordinate system to a semi-wind system, and from the latter to body and

semi-body systems take the form

$$[\bar{x}'] = \mathbf{Z}[\bar{x}] = \mathbf{Z}\Lambda[\bar{x}_r]; \quad (2.76)$$

$$[\bar{x}_1] = \mathbf{H}[\bar{x}] = \mathbf{H}\Lambda[\bar{x}_r]. \quad (2.77)$$

Equations of Motion

For deriving the scalar equations of the motion of the center of mass of an object (a rocket or nose section) let us project vectorial equation (2.9) on the axes of semi-wind coordinate system Oxyz. In this case we will determine the position of the center of mass in a geocentric spherical coordinate system by geocentric latitude $\phi_ц$, by longitude λ and by radius-vector $\bar{r}$, drawn from the center of the earth to the center of mass of the object.

Let us determine the projections of relative, translatory and Coriolis accelerations on the axes of a semi-wind coordinate system. The semi-wind coordinate system rotates relative to the earth with angular velocity Ω which we will represent in the form of the sum of the angular velocities of the rotation of the semi-wind axes relative to geographical axes and of geographical axes relative to certain terrestrial axes. As the axes connected with the earth, it is convenient to take geographical axes $O_0x_{r0}y_{r0}z_{r0}$ at a certain initial moment of flight, for example at the moment of the firing of a rocket or at the moment of separation of the nose section from the rocket body (Fig. 2.17). As a result we will have (see also Fig. 2.16)

$$\bar{\Omega} = \bar{\dot{\Psi}} + \bar{\dot{\Theta}} + \bar{\dot{\lambda}} + \bar{\dot{\varphi}}_ц. \quad (2.78)$$

Let us find the projections of vector $\bar{\Omega}$ on semi-wind axes. First let us express angular velocities $\dot{\lambda}$ and $\dot{\phi}_ц$ by projections on the axes of a geographical coordinate system (see Fig. 2.17)

$$\left.\begin{aligned} \bar{\dot{\lambda}} &= \dot{\lambda}\left(\bar{x}_r^{\circ}\cos\varphi_ц + \bar{y}_r^{\circ}\sin\varphi_ц\right); \\ \bar{\dot{\varphi}} &= -\bar{z}_r^{\circ}\dot{\varphi}_ц, \end{aligned}\right\} \quad (2.79)$$

where $\bar{x}^{\circ}_{\Gamma}$, $\bar{y}^{\circ}_{\Gamma}$, $\bar{z}^{\circ}_{\Gamma}$, - the unit vectors of the geographical axes.

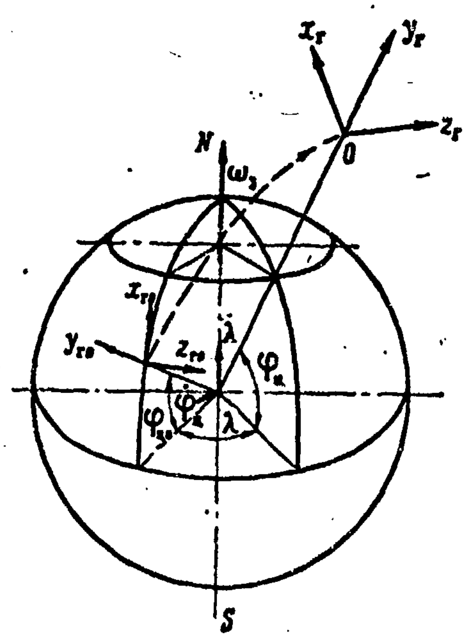

Fig. 2.17. Variation in the orientation of local geographical axes during the flight of a rocket.

In order to express $\dot{\bar{\lambda}}$ and $\dot{\bar{\varphi}}_{\text{ц}}$ by projections on semi-wind axes, we use formula (2.74). Then we will obtain:

$$\begin{aligned} \dot{\bar{\lambda}} &= \dot{\lambda}\left[\bar{x}^{\circ}(\cos\varphi_{\text{ц}}\sin\Psi\cos\Theta+\sin\varphi_{\text{ц}}\sin\Theta)+\right. \\ &\left.+\bar{y}^{\circ}(-\cos\varphi_{\text{ц}}\cos\Psi\sin\Theta+\sin\varphi_{\text{ц}}\cos\Theta)+\bar{z}^{\circ}(\cos\varphi_{\text{ц}}\sin\Psi)\right]; \\ \dot{\bar{\varphi}}_{\text{ц}} &= -\dot{\varphi}_{\text{ц}}\left[\bar{x}^{\circ}(-\sin\Psi\cos\Theta)+\bar{y}^{\circ}(\sin\Psi\sin\Theta)+\bar{z}^{\circ}\cos\Psi\right]. \end{aligned} \quad (2.80)$$

Vectors $\dot{\bar{\Psi}}$ and $\dot{\bar{\Theta}}$ we express by projections on semi-wind axes, using Fig. 2.16:

$$\left.\begin{aligned} \dot{\bar{\Psi}} &= \dot{\Psi}(\bar{x}^{\circ}\sin\Theta+\bar{y}^{\circ}\cos\Theta); \\ \dot{\bar{\Theta}} &= \dot{\Theta}\bar{z}^{\circ}. \end{aligned}\right\} \quad (2.81)$$

Thus, the projections of vector $\bar{\Omega}$ on semi-wind axes are determined by the following expressions:

$$\left.\begin{aligned}
\Omega_x &= \dot{\lambda}(\cos\varphi_{\text{ц}}\cos\Psi\cos\Theta+\sin\varphi_{\text{ц}}\sin\Theta)+\\
&+\dot{\varphi}_{\text{ц}}\sin\Psi\cos\Theta+\dot{\Psi}\sin\Theta;\\
\Omega_y &= \dot{\lambda}(-\cos\varphi_{\text{ц}}\cos\Psi\sin\Theta+\sin\varphi_{\text{ц}}\cos\Theta)-\\
&-\dot{\varphi}_{\text{ц}}\sin\Psi\sin\Theta+\dot{\Psi}\cos\Theta;\\
\Omega_z &= \dot{\lambda}\cos\varphi_{\text{ц}}\sin\Psi-\dot{\varphi}_{\text{ц}}\cos\Psi+\dot{\Theta}.
\end{aligned}\right\} \tag{2.82}$$

Now it is possible to find the projections of relative acceleration. Taking into account that

$$\bar{j}=\frac{d\bar{V}}{dt}=\frac{d'\bar{V}}{dt}+\bar{\Omega}\times\bar{V}=\frac{d'\bar{V}}{dt}+\begin{vmatrix}\bar{x}^\circ & \bar{y}^\circ & \bar{z}^\circ\\ \Omega_x & \Omega_y & \Omega_z\\ V_x & V_y & V_z\end{vmatrix}; \tag{2.83}$$

$$\frac{d'V}{dt}=\dot{V}\bar{x}^\circ;\quad V_x=V;\quad V_y=V_z=0,$$

we will obtain:

$$\left.\begin{aligned}
\bar{j}_x &= \dot{V};\\
\bar{j}_y &= V\Omega_z=V(\dot{\lambda}\cos\varphi_{\text{ц}}\sin\Psi-\dot{\varphi}_{\text{ц}}\cos\Psi+\dot{\Theta});\\
\bar{j}_z &= -V\Omega_y=-V[\dot{\lambda}(-\cos\varphi_{\text{ц}}\cos\Psi\sin\Theta+\\
&+\sin\varphi_{\text{ц}}\cos\Theta)-\dot{\varphi}_{\text{ц}}\sin\Psi\sin\Theta+\dot{\Psi}\cos\Theta].
\end{aligned}\right\} \tag{2.84}$$

Let us express angular velocities $\dot{\lambda}$ and $\dot{\phi}_{\text{ц}}$ by velocity V. For this let us find the projections of velocity V on geographical axes x_{Γ} and z_{Γ} (see Fig. 2.16):

$$\left.\begin{aligned}
V_{x_\Gamma} &= V\cos\Theta\cos\Psi;\\
V_{z_\Gamma} &= -V\cos\Theta\sin\Psi.
\end{aligned}\right\} \tag{2.85}$$

Having resolved the meridianal component of velocity $V_{x\Gamma}$ into sphere radius r, and the latitudinal component of $V_{z\Gamma}$ into small circle

$r \cos \phi_{ц}$, we obtain:

$$\left.\begin{aligned} \dot{\varphi}_{ц} &= \frac{V}{r} \cos \Psi \cos \Theta; \\ \dot{\lambda} &= -\frac{V}{r} \frac{\sin \Psi \cos \Theta}{\cos \varphi_{ц}}. \end{aligned}\right\} \qquad (2.86)$$

Substituting dependence (2.86) in the formulas of (2.84), we obtain the final expressions for the projections of the acceleration of the object relative to the earth on semi-wind coordinate axes:

$$\begin{aligned} \bar{j}_x &= \dot{V}; \\ \bar{j}_y &= V\dot{\Theta} - \frac{V^2}{r} \cos \Theta; \\ \bar{j}_z &= -V\dot{\Psi} \cos \Theta + \frac{V^2}{r} \operatorname{tg} \varphi_{ц} \sin \Psi \cos^2 \Theta. \end{aligned} \qquad (2.87)$$

Let us now determine the projections of Coriolis acceleration on semi-wind axes. As is known,

$$\bar{j}_c = 2\bar{\omega}_3 \times \bar{V} = 2 \begin{vmatrix} \bar{x} & \bar{y} & \bar{z} \\ \omega_{3x} & \omega_{3y} & \omega_{3z} \\ V & 0 & 0 \end{vmatrix}. \qquad (2.88)$$

Let us find the projections of the angular velocity of the earth $\bar{\omega}_3$ on semi-wind axes, using the expressions of (2.80) because the direction of vector $\bar{\omega}_3$ coincides with the direction of $\bar{\lambda}$ (see Fig. 2.17). Then we will obtain

$$\begin{aligned} \bar{\omega}_3 = \omega_3 [\bar{x}^\circ (\cos \varphi_{ц} \cos \Psi \cos \Theta + \sin \varphi_{ц} \sin \Theta) + \\ + \bar{y}^\circ (-\cos \varphi_{ц} \cos \Psi \sin \Theta + \sin \varphi_{ц} \cos \Theta) + \bar{z}^\circ (\cos \varphi_{ц} \sin \Psi)]. \end{aligned} \qquad (2.89)$$

The projections of Coriolis acceleration on semi-wind axes are determined by the expressions:

$$\left.\begin{aligned} j_{cx} &= 0; \\ j_{cy} &= 2V\omega_{3z} = 2V\omega_3 \cos \varphi_{ц} \sin \Psi; \\ j_{cz} &= -2V\omega_{3y} = 2V\omega_3 (\cos \varphi_{ц} \cos \Psi \sin \Theta - \sin \varphi_{ц} \cos \Theta). \end{aligned}\right\} \qquad (2.90)$$

Let us determine the projections of acceleration due to gravity on semi-wind axes. The direction cosines of vectors $\bar{g}_r$ and $\bar{g}_\omega$ will be the same as for vectors $\bar{\Psi}$ and $\bar{\omega}_3$ respectively [see formulas (2.81) and (2.89)], but with opposite signs. Then we will obtain:

$$\begin{gathered} g_x = -g_r \sin\Theta - g_\omega(\cos\varphi_{\text{a}}\cos\Psi^{*}\cos\Theta + \sin\varphi_{\text{a}}\sin\Theta); \\ g_y = -g_r\cos\Theta - g_\omega(-\cos\varphi_{\text{a}}\cos\Psi\sin\Theta + \sin\varphi_{\text{a}}\cos\Theta); \\ g_z = -g_\omega\cos\varphi_{\text{a}}\sin\Psi. \end{gathered} \tag{2.91}$$

Let us now find the projections of total aerodynamic force $\bar{R}$ on semi-wind axes. Let us resolve this force into its components along semi-body axes:

$$\bar{R} = -X_1(\bar{x}')^{*} - Z'\bar{z}'^{*}. \tag{2.92}$$

Now using matrix (2.71), we obtain:

$$\left.\begin{aligned} R_x &= -X_1\cos\alpha - Z'\sin\alpha; \\ R_y &= (-X_1\sin\alpha + Z'\cos\alpha)\sin\mu; \\ R_z &= (X_1\sin\alpha - Z'\cos\alpha)\cos\mu. \end{aligned}\right\} \tag{2.93}$$

Drag X and lift Y are connected with axial force X_1 and lateral force Z' by the following relationships:

$$\left.\begin{aligned} -X &= -X_1\cos\alpha - Z'\sin\alpha; \\ Y &= -X_1\sin\alpha + Z'\cos\alpha, \end{aligned}\right\} \tag{2.94}$$

which are obtained with aid of Table 1.1, if one considers that $Z' = Y_1$ when $\mu = 0$. Thus, we have another variant of the projections of total aerodynamic force on semi-wind axes:

$$\left.\begin{aligned} R_x &= -X; \\ R_y &= Y\sin\mu; \\ R_z &= -Y\cos\mu. \end{aligned}\right\} \tag{2.95}$$

The projections of the resultant $\bar{R}$ of thrust force, aerodynamic forces and the forces created by the control elements, on semi-wind axes take the form:

$$\left.\begin{aligned}N_x&=(P_{x1}+T_{x1})\eta_{11}+T_{y1}\eta_{21}+T_{z1}\eta_{31}-X;\\N_y&=(P_{x1}+T_{x1})\eta_{12}+T_{y1}\eta_{22}+T_{z1}\eta_{32}+Y\sin\mu;\\N_z&=(P_{x1}+T_{x1})\eta_{13}+T_{y1}\eta_{23}+T_{z1}\eta_{33}-Y\cos\mu,\end{aligned}\right\} \quad (2.96)$$

where η_{ij} - the matrix elements (2.73).

Thus, the projections of all the terms of the vectorial equation of motion of the center of mass of an object (2.9) on semi-wind axes are determined by the formulas (2.87), (2.90), (2.91) and (2.96).

The dynamic equations of the motion of the center of mass of an object in projections on semi-wind axes take the form:

$$\left.\begin{aligned}\dot V&=\frac{N_x}{m}-g_r\sin\Theta-g_\omega(\cos\varphi_u\cos\Psi\cos\Theta+\sin\varphi_u\sin\Theta);\\\dot\Theta&=\frac{N_y}{mV}-\frac{g_r}{V}\cos\Theta-\frac{g_\omega}{V}(-\cos\varphi_u\cos\Psi\sin\Theta+\\&\quad+\sin\varphi_u\cos\Theta)+\frac{V}{r}\cos\Theta-2\omega_3\cos\varphi_u\sin\Psi;\\\dot\Psi&=-\frac{N_z}{mV\cos\Theta}+\frac{g_\omega}{V}\frac{\cos\varphi_u\sin\Psi}{\cos\Theta}+\frac{V}{r}\operatorname{tg}\varphi_u\sin\Psi\cos\Theta+\\&\quad+2\omega_3(\cos\varphi_u\cos\Psi\operatorname{tg}\Theta-\sin\varphi_u).\end{aligned}\right| \quad (2.97)$$

It is necessary to supplement these equations with the kinematic equations of the motion of the center of mass of the object in a geocentric spherical coordinate system:

$$\left.\begin{aligned}\dot\varphi_u&=\frac{V}{r}\cos\Psi\cos\Theta;\\\dot\lambda&=-\frac{V}{r}\frac{\sin\Psi\cos\Theta}{\cos\varphi_u};\\\dot r&=V\sin\Theta\end{aligned}\right| \quad (2.98)$$

and with the formula for determining flight altitude h, on which aerodynamic forces depend:

$$h=r-a\sqrt{\frac{1-e^2}{1-e^2\cos\varphi_u}}. \quad (2.99)$$

Let us write the equations of motion around the center of mass in projections on body axes [see formula (2.67)]:

$$\left.\begin{aligned}
J_{x1}\frac{d\omega_{x1}}{dt}+(J_{z1}-J_{y1})\omega_{y1}\omega_{z1}&=\sum M_{x1};\\
J_{y1}\frac{d\omega_{y1}}{dt}+(J_{x1}-J_{z1})\omega_{z1}\omega_{x1}&=\sum M_{y1};\\
J_{z1}\frac{d\omega_{z1}}{dt}+(J_{y1}-J_{x1})\omega_{x1}\omega_{y1}&=\sum M_{z1}.
\end{aligned}\right\}\qquad(2.100)$$

For the projections of aerodynamic moment instead of the expressions of (2.63) it is now necessary to take the expressions:

$$\left.\begin{aligned}
M_{y1}&=c_n^{\alpha}qS\alpha(x_{\tau}-x_d)\cos\nu;\\
M_{z1}&=-c_n^{\alpha}qS\alpha(x_{\tau}-x_d)\sin\nu.
\end{aligned}\right\}\qquad(2.101)$$

Let us compose the kinematic relationships connecting the time derivatives of angles μ, α, ν with the projections of the angular velocity of body ω_{x1}, ω_{y1}, ω_{z1}.

The angular velocity of the body axes $\bar{\omega}$ is made up of the angular velocity $\bar{\Omega}$ of the semi-wind axes relative to the terrestrial axes and the angular velocity $\bar{\omega}'$ of the body axes relative to the semi-wind axes:

$$\bar{\omega}=\bar{\Omega}+\bar{\omega}'.\qquad(2.102)$$

Angular velocity $\bar{\omega}'$ can be represented in the form of the sum

$$\bar{\omega}'=\bar{\dot{\mu}}+\bar{\dot{\alpha}}+\bar{\dot{\nu}}.\qquad(2.103)$$

By projecting vector $\bar{\omega}'$ on body axes, we obtain (see Fig. 2.17):

$$\left.\begin{aligned}
\omega'_{x_1}&=\dot{\mu}\cos\alpha+\dot{\nu};\\
\omega'_{y_1}&=\dot{\alpha}\cos\nu+\dot{\mu}\sin\alpha\sin\nu;\\
\omega'_{z_1}&=-\dot{\alpha}\sin\nu+\dot{\mu}\sin\alpha\cos\nu.
\end{aligned}\right\}\qquad(2.104)$$

For determining the projections of angular velocity $\bar{\Omega}$ on body axes let us use its projections (2.82) on semi-wind axes and the table of the cosines of the angles included between semi-wind and body axes,

written in the form of the matrix of (2.73)

$$\mathbf{H}=\begin{Vmatrix}\eta_{11} & \eta_{12} & \eta_{13}\\ \eta_{21} & \eta_{22} & \eta_{23}\\ \eta_{31} & \eta_{32} & \eta_{33}\end{Vmatrix}. \tag{2.105}$$

Then we will obtain

$$\begin{aligned}\Omega_{x1}&=\eta_{11}\Omega_x+\eta_{12}\Omega_y+\eta_{13}\Omega_z;\\ \Omega_{y1}&=\eta_{21}\Omega_x+\eta_{22}\Omega_y+\eta_{23}\Omega_z;\\ \Omega_{z1}&=\eta_{31}\Omega_x+\eta_{32}\Omega_y+\eta_{33}\Omega_z.\end{aligned} \tag{2.106}$$

Taking expressions (2.102), (2.104) and (2.106) into account, we find:

$$\begin{aligned}\omega_{x1}&=\eta_{11}\Omega_x+\eta_{12}\Omega_y+\eta_{13}\Omega_z+\dot{\nu}+\dot{\mu}\cos\alpha;\\ \omega_{y1}&=\eta_{21}\Omega_x+\eta_{22}\Omega_y+\eta_{23}\Omega_z+\dot{\alpha}\cos\nu+\dot{\mu}\sin\alpha\sin\nu;\\ \omega_{z1}&=\eta_{31}\Omega_x+\eta_{32}\Omega_y+\eta_{33}\Omega_z-\dot{\alpha}\sin\nu+\dot{\mu}\sin\alpha\cos\nu.\end{aligned} \tag{2.107}$$

Substituting the values of the cosines η_{ij} from matrix (2.73) and solving the equations of (2.107) relative to μ, α, ν, we obtain:

$$\begin{aligned}\dot{\alpha}&=\omega_{y1}\cos\nu-\omega_{z1}\sin\nu-\Omega_y\cos\mu-\Omega_z\sin\mu;\\ \dot{\mu}&=\omega_{y1}\frac{\sin\nu}{\sin\alpha}+\omega_{z1}\frac{\cos\nu}{\sin\alpha}-\Omega_x+\Omega_y\operatorname{ctg}\alpha\sin\mu-\\ &\quad-\Omega_z\operatorname{ctg}\alpha\cos\mu;\\ \dot{\nu}&=\omega_{x1}-\omega_{y1}\operatorname{ctg}\alpha\sin\nu-\omega_{z1}\operatorname{ctg}\alpha\cos\nu-\Omega_y\frac{\sin\mu}{\sin\alpha}+\Omega_z\frac{\cos\mu}{\sin\alpha}.\end{aligned} \tag{2.108}$$

In these equations the values of Ω_x, Ω_y, and Ω_z are determined by the formulas of (2.82)

Equations of (2.97), (2.98), (2.100) and (2.108) make up a system of equations of the motion of an unguided object. For describing the

controlled flight of a rocket they should be supplemented by control system equations.

2.4 BASIC SIMPLIFICATIONS OF EQUATIONS OF MOTION

Depending on the problem being solved the general equations of motion obtained above can be more or less substantially simplified. Since the selection of one or another type of simplifications is inseparably connected with the actual conditions of the problem, we will examine the simplification of the equations of motion in appropriate sections of this book. Let us limit ourselves here only to certain general remarks and to one example of the simplifications of the equations used in ballistics.

Taking Trajectory Phase into Account

In the first place in composing equations of motion it is necessary to consider, which phase of the trajectory is being examined.

During the powered-flight phase the motion of a rocket should be examined taking control into account. Since we are interested in rocket flight relative to the earth, and the control system of a rocket is usually inertial, it is necessary to examine the motion of its center of mass in terrestrial coordinate axes, and the orientation of the rocket - in inertial axes, i.e., in initial launch axes. This fact makes it possible to use more or less simplified equations based on the general equations examined in Section 2.2.

In examining motion in the unpowered-flight phase beyond the limits of the atmosphere the investigation of trajectory is facilitated by the absence of thrust force, aerodynamic forces and forces created by the control elements, and also of the moments of all these forces. However due to the great range, altitude and flight speed it is necessary to consider the variation in acceleration due to gravity and the effect of the rotation of the earth.

During the phase of the descent of the nose section into the

the atmosphere a large role is played by the aerodynamic forces and moments. Since flight in this phase is unguided, there is no need to rely on an inertial coordinate system and for the investigation it is possible to use various simplifications of the equations of motion in projections on semi-wind axes.

"Quasi-Steady-State" Motion

The motion of a rocket or of a nose section, as well as the motion of any body, can be represented in the form of the motion of the center of mass of the object and of its rotation around the center of mass. The presence of control during the powered-flight phase makes it necessary to examine the motion of the center of mass of the rocket together with the motion of the rocket around the center of mass. During the descent of a nose section in the atmosphere it is also necessary to examine the oscillations around the center of mass together with the motion of the center of mass.

Standard for rockets is an investigation of the motion of the center of mass with simplified equations of the control system and of the rotation of the rocket.

The control system equations depend substantially on its structure and the composition of its elements. Thus their actual simplifications cannot be examined in this book. The most substantial simplifications consist of replacing control system equations with equations of ideal controlling connection.

Let us examine the simplification of the equations of rocket rotation.

The left sides of the Euler equation of (2.67) with controlled rocket flight, if we eliminate such non-steady-state flight modes, as launch, stage separation and nose section separation, are close to zero, thus in investigating the motion of the center of mass the left sides of the Euler equation are usually disregarded and these equations are written in the form of steady-state equations of the moments of force acting on the rocket, relative to the rocket body axes. Thus

the transitional processes in rotary motion are disregarded and the rocket is examined with a control system ideal in the sense, that upon deflection of the control elements the angle of attack instantaneously assumes the "steady-state" (balanced) value [21] corresponding to the equilibrium equation of the moments.

Simplification of Direction Cosine Matrices

The following common simplification in equations of rocket motion consists in simplifying the expressions of the cosines of the angles included between the coordinate axes. Thus, for instance, in calculating the optimum trajectory of a ballistic missile it is possible to set the angles of roll η and yaw ξ equal to zero. Such a possibility is brought about by the fact that a system controlling flight, by getting rid of perturbations, tends to reduce these angles to zero, as a result of which their actual values are small. Let us thus assume $\cos\eta\approx 1$, $\sin\eta\approx\eta$, $\cos\xi\approx 1$, $\sin\xi\approx\xi$.

We will also disregard the products of angles ξ and η. Then we will obtain the approximate matrix Γ of the cosines of the angles included between the initial launch and body axes in the following form:

$$\Gamma\approx\begin{Vmatrix}\cos\varphi & \sin\varphi & -\xi\cos\varphi\\ -\sin\varphi & \cos\varphi & \eta+\xi\sin\varphi\\ \xi+\eta\sin\varphi & -\eta\cos\varphi & 1\end{Vmatrix}.$$

In an analogous manner we will obtain the approximate matrix of the cosines of the angles included between the body and terrestrial axes.

Resolving General Motion into Longitudinal and Lateral Motion

A substantial simplification of a system of equations of rocket motion is attained, when it is possible to break this system down into two independent groups of equations describing the motion of a rocket in two mutually perpendicular planes [21]. The main possibility for such a breakdown is due to the dynamic symmetry of a rocket relative to its longitudinal axis Ox_1.

Let us represent the general motion of a rocket as made up of longitudinal motion, in which parameters $V_{x3}, V_{y3}, x_3, y_3, \omega_{x1}, \varphi$, vary, and lateral motion, in which parameters $V_{z3}, z_3, \omega_{x1}, \omega_{y1}, \xi, \eta$ vary. In the general case there is interaction between these two motions. Let us explain the conditions, under which each of the two motions in question, can occur independently of the other.

Lateral parameters of motion $V_{x3}, V_{y3}, x_3, y_3, \omega_{x1}, \varphi$; will not be included in the equations describing the variation in the longitudinal parameters of motion $V_{z3}, z_3, \omega_{x1}, \omega_{y1}, \xi, \eta$, if the channels stabilizing the angles of yaw and roll and lateral drift are operating ideally. In practice it is possible to exclude the parameters of lateral motion from the equations of rocket motion in plane Ox_3y_3, when the lateral parameters $V_{z3}, z_3, \omega_{x1}, \omega_{y1}, \xi, \eta$ are rather small. Then we obtain the following system of equations of rocket motion in the Ox_3y_3 plane:

$$\left.\begin{aligned}
&m\frac{dV_{x3}}{dt}=N_{x3}+G_{x3}-mj_{ex_3};\\
&m\frac{dV_{y3}}{dt}=N_{y3}+G_{y3}-mj_{ey_3};\\
&\frac{dx_3}{dt}=V_{x3};\qquad \frac{dy_3}{dt}=V_{y3};\\
&J_{z1}\frac{d\omega_{z1}}{dt}=\sum M_{z1};\qquad \frac{d\varphi}{dt}=\omega_{z1};\\
&F_1[\delta_\varphi(t),\ x_3(t),\ y_3(t),\ \varphi(t)]=0;\\
&F_2[\delta_P(t),\ x_3(t),\ y_3(t),\ \varphi(t)]=0.
\end{aligned}\right| \tag{2.109}$$

The equations of lateral motion in the general case take the form:

$$\left.\begin{aligned}
&m\frac{dV_{z3}}{dt}=N_{z3}+G_{z3}-mj_{ez3};\\
&\frac{dz_3}{dt}=V_{z3};\\
&J_{y1}\frac{d\omega_{y1}}{dt}=\sum M_{y1}-(J_{x1}-J_{z1})\omega_{x1}\omega_{z1};
\end{aligned}\right|$$

$$
\left.\begin{aligned}
&J_{x1}\frac{d\omega_{x1}}{dt}=\sum M_{x1}-(J_{z1}-J_{y1})\omega_{y1}\omega_{z1};\\
&\omega_{x1}=\frac{d\eta}{dt}+\frac{d\xi}{dt}\sin\varphi;\\
&\omega_{y1}=\frac{d\xi}{dt}\cos\varphi\cos\eta+\frac{d\varphi}{dt}\sin\eta;\\
&F_3[\delta_\xi(t),\ z(t),\ \xi(t),\ \eta(t)]=0;\\
&F_4[\delta_\eta(t),\ z(t),\ \xi(t),\ \eta(t)]=0.
\end{aligned}\right| \qquad (2.110)
$$

It is not possible to exclude all the parameters of longitudinal motion from these equations. Thus for independent investigation of lateral motion it is first necessary to determine all the necessary longitudinal parameters, for example, by solving the equations of (2.109).

Linearizing Equations

The method of linearizing equations is very widespread in all the technical sciences (see, for example, book [21]). In rocket ballistics and dynamics this method is most frequently used in investigating the dispersion of nose sections, for evaluating the controllability of rockets and many other problems.

Simplifying Equations for Evaluating Rocket Controllability

For determining the maximum deflection of the control elements, necessary to compensate for perturbing forces and moments, let us compose simplified equations of rocket motion, examining it as an absolutely rigid solid body. For this purpose let us resolve the general motion of a rocket into longitudinal and lateral motions and let us examine only the lateral motion, since the maximum perturbations are caused by cross wind.

Let us simplify the system of equations of lateral motion (2.110), having made the following assumptions.

1. The projections of gravity and Coriolis force on the Oz_3 axis are negiligibly small.

2. We will assume angles ϕ, ξ, η, δ_ϕ, δ_ξ, δ_η, α, β to be small; we will consider the cosines of these angles to be equal to unity, the sines of the angles - equal to the angles; we will disregard the products of these angles.

3. Let us linearize the forces and moments, representing them in the form:

$$X_1 \approx X;$$
$$Y_1 \approx (X+Y^\alpha)\alpha;$$
$$Z_1 \approx (-X+Z^\beta)\beta = -(X+Y^\alpha)\beta;$$
$$M_{y1} \approx M_{y1}^\beta\beta + M_{y1}^\delta\delta_\xi$$
$$M_{z1} \approx M_{z1}^\beta\beta + M_{z1}^\delta\delta_\eta$$
$$P = P_{x1} \approx P + 4T;$$
$$T_{y1} \approx 2T\delta_\xi;$$
$$T_{z1} \approx -2T\delta_\eta.$$

4. We represent angle β in the form

$$\beta = \xi - \sigma \approx \xi + \frac{\dot z}{V}.$$

Then, taking the perturbing forces and moments Z_B, M_{By_1}, M_{Bz_1}, into account on the right sides of the equations (2.110), we obtain:

$$\left.\begin{aligned}
\ddot z &= a_{zz}\dot z + a_{z\xi}\xi + a_{z\eta}\eta + a_{z\delta}\delta_\xi + Z_B;\\
\ddot \xi &= a_{\xi z}\dot z + a_{\xi\xi}\xi + a_{\xi\delta}\delta_\xi + M_{By_1};\\
\ddot \eta &= a_{\eta z}\dot z + a_{\eta\xi}\xi + a_{\eta\delta}\delta_\eta + \tilde M_{Bz_1}.
\end{aligned}\right\} \quad (2.111)$$

Here

$$\left.\begin{aligned}
&a_{zz}=-\frac{X+Y^{\alpha}}{mV};\quad a_{z\xi}=-\frac{P+Y^{\alpha}}{m};\\
&a_{z\eta}=\frac{Y}{m};\quad a_{z\delta}=-\frac{2T}{m};\\
&a_{\xi z}=\frac{Y^{\alpha}(x_{\text{т}}-x_d)}{J_{y1}V};\quad a_{\xi\xi}=\frac{Y^{\alpha}(x_{\text{т}}-x_d)}{J_{y1}};\\
&a_{\xi\delta}=\frac{2T(x_{\text{т}}-x_{\text{зп}})}{J_{y1}};\quad a_{\eta z}=\frac{M_x^{\beta}}{J_{x1}V};\\
&a_{\eta\xi}=\frac{M_x^{\beta}}{J_{x1}};\quad a_{\eta\delta}=\frac{4Tr_y}{J_{x1}};\\
&\tilde{Z}_{\text{в}}=\frac{Z_{\text{в}}}{m};\quad \tilde{M}_{\text{в}y_1}=\frac{M_{\text{в}y1}}{J_{y1}};\quad \tilde{M}_{\text{в}x_1}=\frac{M_{\text{в}x_1}}{J_{x1}}.
\end{aligned}\right\} \tag{2.112}$$

Let us write as control system equations the lineraized equations of the channels stabilizing yaw, roll and lateral drift, composed in accordance with the procedure examined in Section 1.8:

$$\begin{aligned}
&\delta_{\xi}=a_{\xi}\xi+a_{\dot{\xi}}\dot{\xi}+a_{\int\xi}\int\xi dt+a_z z+a_{\dot{z}}\dot{z};\\
&\delta_{\eta}=a_{\eta}\eta+a_{\dot{\eta}}\dot{\eta}.
\end{aligned} \tag{2.113}$$

The total deflection of the control elements is equal to:

$$\begin{aligned}
&\delta_1=\delta_{\xi}+\delta_{\eta};\\
&\delta_3=\delta_{\xi}-\delta_{\eta}.
\end{aligned} \tag{2.114}$$

As a result of solving the obtained system of equations of rocket motion with a given set of perturbations the deflections of the control elements $\delta_1(t)$ and $\delta_3(t)$ necessary for compensating for these perturbations are determined, and they are compared with the maximum possible deflection $\delta_{max}>0$. If the values of the angles of deflection of the control elements are less than the maximum possible angle (with a certain margin $\delta_{\text{зап}}>0$), then in this case the control elements are effective:

$$\left.\begin{aligned}
&|\delta_1|+\delta_{\text{зап}}\leqslant\delta_{max};\\
&|\delta_3|+\delta_{\text{зап}}\leqslant\delta_{max}.
\end{aligned}\right\} \tag{2.115}$$

CHAPTER III

TRANSITIONAL TRAJECTORY PHASES

Of all the questions of rocket ballistics and dynamics it is possible to segregate, as independent, the questions of the dynamics of the transitional trajectory phases - launch, stage separation and nose section separation. These questions are connected by the similarity of their dynamic processes and by identity of the formulation of their problems.

Sharp variations in the thrust of the main engines, propellant consumption per second, and also the operating modes of the control system are characteristic for transitional trajectory phases. Furthermore, the perturbations acting on a rocket in these phases are specific. From the point of view of mechanics, the investigation of the transitional trajectory phase is the solving of problems concerning the relative motion of two or several bodies, especially, a rocket relative to the launch pad, of the two separating parts of the rocket relative to each other, etc. In all the problems it is necessary to determine the forces acting on the rocket, and the parameters of relative motion taking into account the actual design features of the rocket and the operation of its systems.

The standard methods of launch, stage separation and the separation of other objects are examined below; the equations of motion of rockets and separating parts are given; it is pointed out, which questions of dynamics are solved in rocket designing. The exposition employs a

two-stage rocket as an example executed according to "tandem" layout.

3.1. ROCKET LAUNCH

Free Launch from an Open Ground-Based Launch Pad

Free rocket launch is carried from a launch pad located on the surface of the earth. The rocket stands freely on the pad and when the engine thrust attains a value, greater than the launch weight of the rocket, the latter lifts off from the pad.

The basic problem of dynamic design with such a launch setup are determining the perturbing forces and moments acting on the rocket, and investigating the perturbed motion of a rocket in the initial trajectory phase for the purpose of selecting the stabilization system parameters and evaluating the controllability of the rocket.

During free launch from an open ground-based launch pad the perturbing forces and moments acting on a rocket are caused by:

- cross wind;

- errors in the manufacture and the assembly of the rocket and the engine system;

- the time differential in starting and the thrust differential of the engines of the engine system (or of the combustion chambers of one engine).

The perturbing forces and moments can be determined by the formulas given in [22].

Let us write the equations of motion of the center of mass of a rocket as projections on the axes of the coordinate system, the origin which when $t = 0$ coincides with the center of mass of the rocket; axis Ox is directed vertically upward; axis Oy - is opposite to the direction to the target; axis Oz - so that the coordinate system is right-handed (Fig. 3.1)

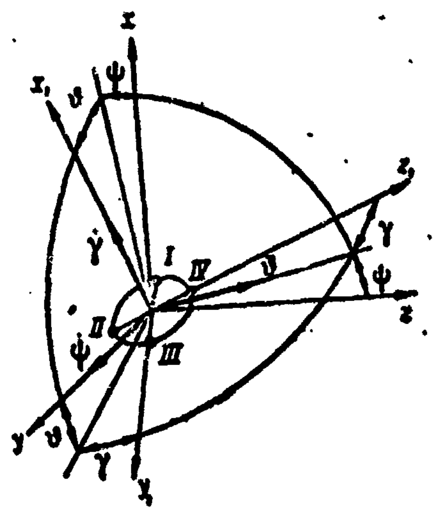

Fig. 3.1. Coordinate system for calculating rocket launch from an open ground-based launch pad.

We will determine the orientation of the rocket relative to this coordinate system by the angles of pitch ϑ, yaw ψ and roll γ which are formed similar to angles ϕ, η, and ξ, determining the orientation of a rocket relative to an initial launch coordinate system (see Fig. 2.4). In this case the kinematic equations connecting the projections of the angular velocity of the rocket $\omega_{x1}, \omega_{y1}, \omega_{z1}$ with the angular velocities $\dot{\gamma}, \dot{\psi}, \dot{\vartheta}$, will be analogous to the kinematic equations of (2.32). From these equations it follows that with rather small angles γ, ψ, ϑ and angular velocities $\dot{\psi}$ and $\dot{\vartheta}$ the approximate equalities occur:

$$\omega_{x1} \approx \dot{\gamma}; \ \omega_{y1} \approx \dot{\psi}; \ \omega_{z1} \approx \dot{\vartheta}.$$

Let us simplify and more accurately refine the general system of equations of (2.65)-(2.66) with respect to the conditions of the problem in question. In this case let us make an assumption about the fact, that angles γ, ψ, ϑ and angular velocities $\dot{\psi}$ and $\dot{\vartheta}$ and also the angles of deflection of the controlling engines $\delta_\gamma, \delta_\phi$ and δ_ϑ are small. It is evident that in the case in question it is possible to disregard the Coriolis forces.

In determining the projections of the forces on the selected axes let us take into account that:

- in unperturbed (vertical) flight drag is the only one of the aerodynamic forces acting on the rocket;

- the Coriolis forces are negligibly small;

- the forces and moments created by the controlling engines, are determined by the formulas of (1.45) and (1.46);

- the operation of the control system is described by the equations of (1.63), (1.65), (1.66), (1.67).

Taking into account what has been said the equations of motion of the rocket take the form:

$$\left.\begin{aligned}
&m\ddot{x}=P_\Sigma-G-X;\\
&m\ddot{y}=P_\Sigma\vartheta+2T\delta_\vartheta+Y_\text{в};\\
&m\ddot{z}=-P_\Sigma\psi-2T\delta_\psi+Z_\text{в};\\
&J_{x1}\ddot{\gamma}=4Tr_y\delta_\gamma+M_{\text{в}x_1},\\
&J_{y1}\ddot{\psi}=2T(x_\text{т}-x_{\text{цм}})\delta_\psi+M_{\text{в}y_1};\\
&J_{z1}\ddot{\vartheta}=2T(x_\text{т}-x_{\text{цм}})\delta_\vartheta+M_{\text{в}z_1};\\
&\delta_\gamma=a_\gamma\gamma+a_{\dot\gamma}\dot{\gamma};\\
&\delta_\psi=a_\psi\psi+a_{\dot\psi}\dot{\psi}+a_{\ddot\psi}\ddot{\psi}+a_z z+a_{\dot z}\dot{z};\\
&\delta_\vartheta=a_\vartheta\vartheta+a_{\dot\vartheta}\dot{\vartheta}+a_{\ddot\vartheta}\ddot{\vartheta}+a_y y+a_{\dot y}\dot{y};\\
&\delta_1=\delta_\psi+\delta_\gamma;\\
&\delta_2=\delta_\vartheta+\delta_\gamma;\\
&\delta_3=\delta_\psi-\delta_\gamma;\\
&\delta_4=\delta_\vartheta-\delta_\gamma.
\end{aligned}\right\} \tag{3.1}$$

Here $m = m_0 - \int_0^t \dot{m}(t)dt$ - the mass of the rocket; $P_\Sigma = P+4T$ - the total thrust of the engine system, and P - the thrust of the main engine; T - the thrust of the controlling engine; $Y_\text{в}$, $Z_\text{в}$ - the projections of the vector of the perturbing forces acting on the rocket; $M_{\text{в}x_1}$, $M_{\text{в}y_1}$, $M_{\text{в}z_1}$ - the projections of the vector of perturbing moment acting on the rocket; a_γ, $a_{\dot\gamma}$, $a_\psi, \ldots, a_y$, $a_{\dot y}$ the transmission coefficients of the control system of the rocket.

The system of equations of (3.1), as a rule, can be solved only by numerical methods on analog or digital computers. In this case, since the initial conditions and the perturbations acting on the rocket are in general random, the method of statistical testing is usually used for the solution. Sometimes for reducing machine time the solution of the indicated system of equations is found for the worst combination of perturbing forces and moments acting in a certain plane. Furthermore, since the position of this plane for investigating launch dynamics in a vertical phase is neutral, in most cases the problem is solved for the plane of pitch.

As a result of the solution the following launch problems are distinguished: necessary control element effectiveness, the rational values of the transmission coefficients of the control system, the statistical characteristics of the phase coordinates of the rocket, the design parameters of the launch pad and others.

Free Launch from a Silo

The free launch of a rocket from a silo complex is accomplished in the same way as a launch from an open ground-based launch pad, only the launch pad is located inside a silo complex. Free launch from a silo imposes particular requirements on a rocket, the silo complex and the control system as part of ensuring shock-free egress from the silo - it requires very high accuracy in stabilizing the motion of the rocket in the silo trajectory phase and the specific relationship of the diameters of the silo and the rocket. The basic questions of dynamic rocket design in launching from a silo are: evaluating the necessary effectiveness of the control elements, selecting the parameters of the control system and determining the overall dimensions of the silo complex. For this it is necessary to know the parameters of the perturbed motion of a rocket during its egress from a silo.

The dynamic procedure of a free launch of a rocket from a silo is identical to the dynamic procedure of a free launch from an open ground-based launch pad. However during the motion in the silo additional perturbations act on the rocket caused by the gas-dynamic forces due to the gas streams coming from the nozzles of the engines.

Furthermore, as the rocket makes its egress from the silo the wind begins to act on it. The formulas for calculating the wind perturbations acting on the rocket, are the same as before (see appendix on application). However the coefficients of normal aerodynamic force c_n and the coordinate of the center of pressure x_d in this case depend not only on the angle of attack, but also on the length of the part of the rocket which has exited from the silo. For calculating the parameters of perturbed rocket motion during free launch from a silo the same system of equations can be used, as for a launch from an open ground-based launch pad.

Launching from a Silo on Guides

The launching of a rocket from a silo on guides ensures the shock-free egress of the rocket from the silo and does not impose such rigid specifications on the silo complex and the rocket control system, as during free launch from a silo.

In launching from a silo on guides along with the evaluation of the effectiveness of the rocket control elements and the selection of the control system parameters it is also necessary to determine the reactions acting in the support girdles of the rocket during its motion along the guides. Furthermore, it is necessary to select a scheme of control system activation (lift contact or egress contact response), ensuring minimum loads (reactions of the guides) and initial perturbations.

Below are examined the equations of rocket motion for one of the possible variants of the design execution of the guides and support girdles of a rocket: the rocket moves in the silo along two vertical guides, with which it is connected by two elastic support girdles (upper and lower) which are two diametrically positioned lugs. The perturbations acting on the rocket, are the same as in free launching. In composing the expressions for the reactions of the support girdles it is necessary to consider the basic design features of the rocket and of guides; the clearances between the lugs and the guides, the preliminary compression of the spring of the lugs and the restriction of their motion, the elasticity of the rocket body under the lug, etc.

In a first approximation in investigating the launching of a rocket along guides the flexural vibrations of the housing, the vibrations of the guides, and also the possible displacements of the silo launch jacket are disregarded. For composing the appropriate equations of rocket motion let us use a right-handed coordinate system whose axes are oriented in the following manner: Ox is directed upward along the vertical, Oz lies in the plane of the guides; Oy is perpendicular to this plane (Fig. 3.2). The equations of the motion of the rocket along the guides are derived in an analogous manner to the equations of (3.1)

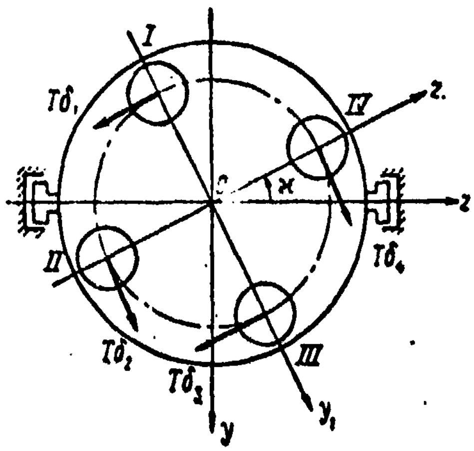

Fig. 3.2. Diagram for composing the equations of motion of a rocket during launch from a silo along guides: κ - angle included between the plane of the guides and plane Ox_1z_1.

The distinctive feature of the problem in question as compared with the previous one is calculating the elastic reaction forces arising as a result of the interaction of the elastic lugs on the rocket with the guides (Fig. 3.3). These reactions are functions of the rocket coordinates y, z and in many respects they are determined by the elastic properties of the material, from which the lugs are made.

Let us introduce the follwoing designations (see Fig 3.3):

$R_1^0(z)$, $R_2^0(z)$ - the radial reactions on the upper and lower support gridles respectively;

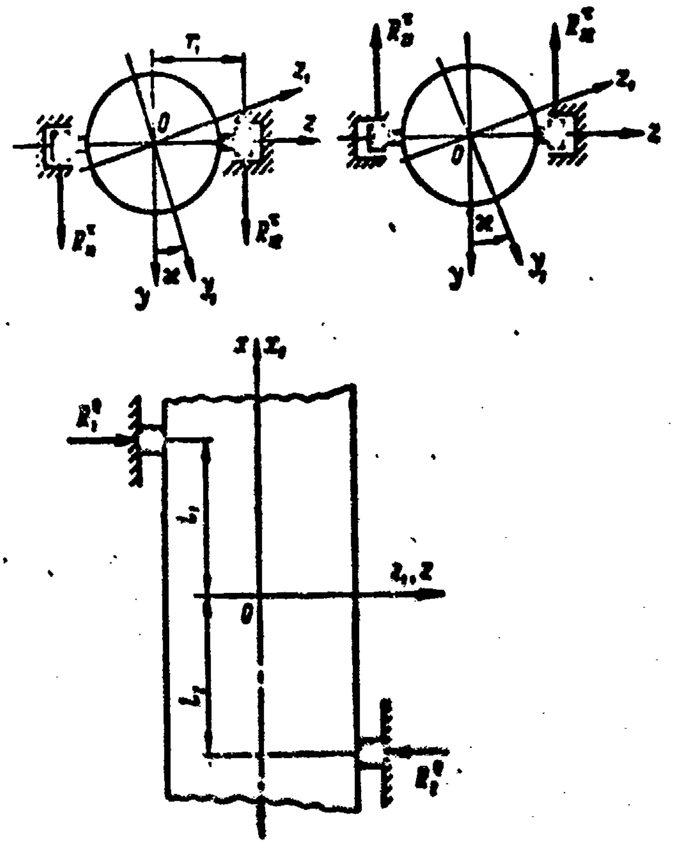

Fig. 3.3 Reacting forces of the supports acting on a rocket during launch from a silo along guides.

$R^{\tau}_{11}(y), R^{\tau}_{12}(y)$ - the tangential reactions on the upper support girdle;

$R^{\tau}_{21}(y), R^{\tau}_{22}(y)$ - the tangential reactions on the lower support girdle;

l_1, l_2 - the distance to thr gravity center of the object from the upper and lower support girdles respectively;

$\varkappa$ - the angle included between the plane of the guides and the yaw plane.

The equations of motion of the rocket moving long guides has the form:

$$
\left.
\begin{aligned}
&m\ddot{x}=P_x-G-X;\\
&m\ddot{y}=P_x\vartheta+2T(\delta_\vartheta\cos\varkappa-\delta_\psi\sin\varkappa)+\\
&\quad+R_{11}^{\text{т}}+R_{12}^{\text{т}}-R_{21}^{\text{т}}-R_{22}^{\text{т}}+Y_{\text{в}};\\
&m\ddot{z}=-P_x\psi+2T(\delta_\psi\cos\varkappa+\delta_\vartheta\sin\varkappa)+\\
&\quad+R_1^{\text{б}}-R_2^{\text{б}}+Z_{\text{в}};\\
&J_{x1}\ddot{\gamma}=4Tr_y\delta_\gamma+(R_{11}^{\text{т}}-R_{12}^{\text{т}}-R_{21}^{\text{т}}+R_{22}^{\text{т}})r_1+M_{\text{в}x_1};\\
&J_{y1}\ddot{\psi}=2T(x_{\text{т}}-x_{\text{дв}})\delta_\psi+\\
&\quad+[(R_{11}^{\text{т}}+R_{12}^{\text{т}})l_1+(R_{21}^{\text{т}}+R_{22}^{\text{т}})l_2]\sin\varkappa-\\
&\quad-(R_1^{\text{б}}l_1+R_2^{\text{б}}l_2)\cos\varkappa+M_{\text{в}y_1};\\
&J_{z1}\ddot{\vartheta}=2T(x_{\text{т}}-x_{\text{дв}})\delta_\vartheta+\\
&\quad+[(R_{11}^{\text{т}}+R_{12}^{\text{т}})l_1+(R_{21}^{\text{т}}+R_{22}^{\text{т}})l_2]\cos\varkappa+\\
&\quad+(R_1^{\text{б}}l_1+R_2^{\text{б}}l_2)\sin\varkappa+M_{\text{в}z_1}.
\end{aligned}
\right\}
\qquad (3.2)
$$

As in the case of a free launch of a rocket, to these equations it is necessary to add the control system equations describing the deflections of the control elements δ_γ, δ_ψ, δ_ϑ depending on the parameters of rocket motion.

The solution of the system of equations obtained in this way is more complex than the systems of (3.1), and also, as a rule, it is found with the aid of computers by the method of statistical testing.

3.2. STAGE SEPARATION AND NOSE SECTION SEPARATION

Stage Separation

The phase of motion of a rocket from the moment of the issuance of the main command for shutting down of the engine system of the previous stage to the moment, when the separated part cannot affect the subsequent flight of the rocket, we will call the stage separation phase. For the stage separation of multistage rockets set up according

to a "tandem" layout, two basic setups of separating systems can be employed:[1]

1) cold separation (or separation by braking), in which the separating part is braked by special means after breaking the connection between the stages, and the main engine of the subsequent stage is started after a safe interval is attained between the stages;

2) hot (or fire) stage separation, in which the engine of the subsequent stage is started before the breaking of the connections between the stages and the separating part is repelled by the gas jet of the subsequent stage engine.

With any separation system it is necessary to ensure the continuous controllability of a rocket during the separation phase. The method of carrying out of this specification depends upon the type of control elements.

Cold stage separation (Fig. 3.4) is possible and more acceptable for rockets, the control of which is accomplished with the aid of special controlling engines. The controlling engines of the subsequent stage can be activated before shutting down the controlling engines of the separating part. In this case continuous rocket controllability is ensured during the separation phase.

Hot stage separation (Fig. 3.5) is possible in principle on any rockets with sequential stage connection, however it requires special design of the adapter between the stages and of the rear section of the following stage. Hot separation is advantageous for the rockets, the control of which is connected with main engine operation (jet vane control, control of main engine combustion chanber oscillation, control of the blowing of generator gas into the supercritical part of the engine nozzle). With such control elements the continuous controllability of a rocket in the separation phase is possible only during

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[1]Certain intermediate schemes are also possible, however we will not dwell on these.

the starting of the main engine of the subsequent stage before breaking the connections between the stages.

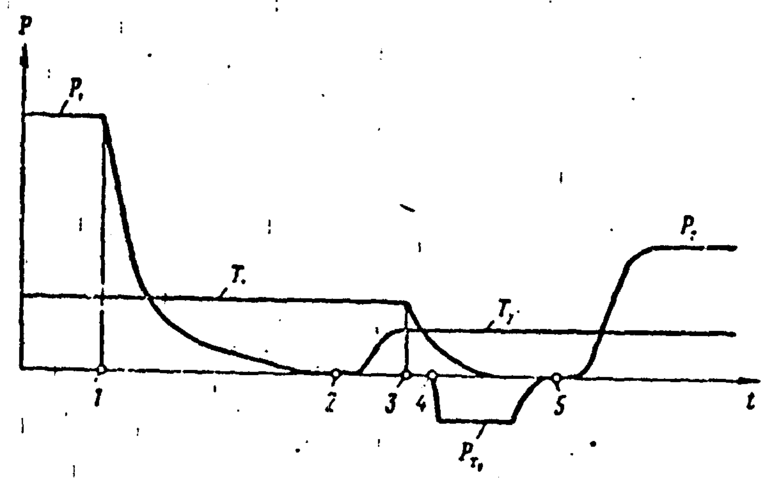


Fig. 3.4. Diagram of cold stage separation. Approximate command sequence for: 1 - shutdown of the main engine of the separating part; 2 - starting of the controlling engine of the subsequent stage; 3 - shutdown of the controlling engine of the separating part; 4 - breaking of the connections between the stages and the starting of the retro-solid-propellant rocket engines; 5 - starting of the main engine of the subsequent stage.

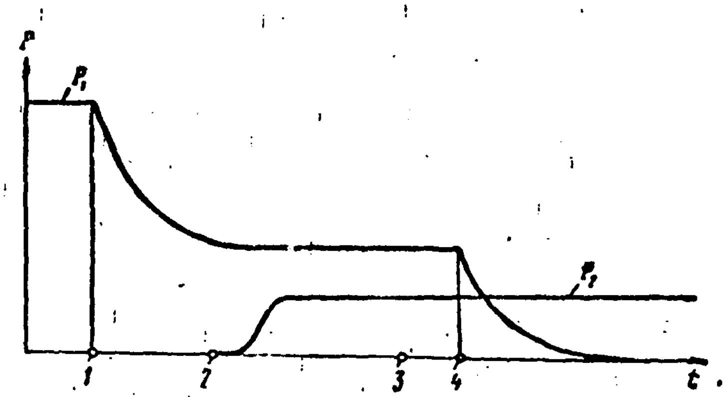


Fig. 3.5. Diagram of the hot separation. The approximate sequence of commands for: 1 - throttling the engine of the separating part; 2 - starting the engine of the subsequent stage; 3 - breaking the connections between the stages; 4 - shutting down the engine of the separating part.

Another requirement for separation systems is the ensuring of reliable separation, by which is understood separation without

colliding of the rocket parts. For reliable separation it is necessary to ensure sufficient energy, used for separating and spreading the rocket parts a safe distance apart, their continuous controllability in the separation phase and correct selection of the moment for breaking the connections.

During separation the deviations in the parameters of the motion of the subsequent stage from the optimum values and the deviations in the parameters of the relative motion of the rocket parts from the rated values should have as small dispersion as possible. This requirement is determinant in selecting an actual scheme of separation.

The cited requirements for separation systems, as it is easy to see, are interconnected. The degree of the complexity of the execution of these requirements depends upon the values of the perturbing forces and moments acting on the parts of the rocket during the separation phase. Thus when developing a rocket and a stage separation system various measures are specified which lead to a reduction in the perturbing forces and moments. In particular, when selecting the flight path of a rocket the values of the dynamic pressure and the angle of attack are limited in the separation phase. In its turn, the selection of a rational separation scheme makes it possible to increase the permissible values of dynamic pressure and angle of attack of the rocket in the separation phase and thus reduce the specifications imposed on the shape of the rocket trajectory.

The process of stage separation can give rise to noticeable losses in the maximum firing range, in connection with which the requirement of reducing these losses is always imposed on separation systems. In order to decrease the reduction in range due to gravitational losses in rocket speed in the separation phase it is necessary to ensure that the separation process occurs with great rapidity. However the abridging of the time of separation requires an increase in the forces, separating the rocket parts, which gives rise to an increase in perturbing forces and moments. Another cause for reduction in range consists in increasing the weight of the rocket due to the weight of the separation system.

Finally, a number of specifications are imposed on the design of separation equipment. Among these it is possible to note compactness and small overall dimensions of separation devices, safety and simplicity in operation and others.

In the light of the above examined specifications let us note the basic pros and cons of cold and hot separation systems. These systems can be made in different variants.

Above, Fig. 3.4 gives one of the possible variants of cold separation, in which for braking the separating part solid-propellant retrorockets (22) are used.

The basic advantages of the cold separation systems are: separation under the effect of small forces with small perturbing forces and moments and small weight of the separation devices (retro-solid-propellant rocket engines with attachment fittings). Included among the deficiencies of this system are: the comparatively complex separation scheme and the reduction in firing range due to prolonged separation time.

One of the possible schemes of hot separation is shown in Fig. 3.5. The advantages of the hot separation systems are: the speed of separation, which practically does not lead to gravitational losses in range; the simplicity of the separation process and the command sequence; the increased reliability of the starting of the engine of the subsequent stage due to the axial acceleration created by the operating engine of the separating part. The deficiencies of hot separation are: large perturbations, obtained by the subsequent stage during separation; the fuel consumption by the engine of the subsequent stage before the breaking of the connections between the stages; the necessity for protecting the parts of the rocket from the effect of the gas jet of the operating engine.

Proceeding from the conditions of ensuring the above examined specifications, the following problem are usually solved in the dynamic designing of a separation system:

1) the basic selection of the scheme and the means for stage separation;

2) the selection of the basic parameters (characteristics) of the separation system;

3) the selection of the sequence of moments of the transmission of instructions by the control system for executing the separation operations;

4) ensuring the reliability of the separation process, the stability and the controllability of the subsequent stage.

The basic method for solving the enumerated problems is the investigation of the relative motion of the separating parts of the rocket.

In composing the equations of motion of the parts of a rocket for investigating the separation process it is necessary to consider the following forces and moments which are acting on the separating parts of the rocket: gravity; the thrust force of the engine systems; the thrust forces of retro engines or nozzles; the forces and moments created by the control engines; aerodynamic forces and moments; the forces and moments from the gas-dynamic effect of the engine jet of the subsequent stage on the separating part (during hot stage separation); the perturbing forces and moments.

The perturbing forces and moments, significently affecting the process of stage separation, are due to the following factors:

- wind effect;

- the errors made in the manufacture and the assembly of the rocket and the engine system;

- the eccentricity of the center of mass of the rocket caused by the design peculiarties of the rocket layout;

- the misalignment of the line of engine thrust effect caused by the elastic deformation of the engine system mounting;

- the thrust differential of the engines (or the combustion chambers of one engine) on the steady-state and transitional operation modes;

- the thrust differential of the retro engines or nozzles;

- the aftereffect [thrust trailoff] impulse of the engines.

The calculating formulas for determining the enumerated perturbing forces and moments are given in [22]. For the sake of simplifying the investigation of the separation process the effect of the liquid propellant in the tanks, the flexural and longitudinal oscillations of the separating parts of a rocket, the variation with time of the mass, inertial moments and the positions of the centers of mass of the separating parts are usually disregarded.

In order to evaluate the parameters of the relative motion of the separating parts of a rocket, it is sufficient, as a rule, to limit oneself to an examination of the longitudinal and lateral motion of the rocket.

For describing the motion of the separating parts of a rocket after the breaking of the connections between them it is convenient to use an inertial coordinate system Oxyz, moving with a velocity, equal to the velocity of the rocket at moment of the beginning of the separation process ($t = 0$). At moment of time $t = 0$ the origin of the coordinate system O coincides with the center of mass of the entire missile; axis Ox coincides with the longitudinal axis of the rocket; axis Oy is directed upward and forms a vertical plane with axis Ox.

Let us designate by $O_1x_1y_1$ and $O_2x_2y_2$ the coordinate systems, connected with the separating part and the subsequent stage respectively and formed by the standard rules.

The actual form of the equations of motion of the separating parts

of a rocket depends upon the setup of the separation system and the flight conditions at the time of separation. Thus let us show the basic peculiarities of investigating a separation process as illustrated by dynamic separation schemes formed by taking the above given assumptions into account. Furthermore, let us assume that at the initial moment of separation the angles of attack and the angular velocity of the rocket are equal to zero. The remaining assumptions will be apparent from the systems of forces acting on the separating parts, and the explanations for the equations of motion.

During cold stage separation let us assume that the retro-solid-propellant rocket engines begin instantaneously operating in a steady-state mode and the reactions at the site of the joining of the stages from the moment of the starting of the solid-propellant rocket engines are equal to zero. Then the equations of motion of the separating parts of the rocket in the plane of pitch taking into account all that was stated above for the systems of acting forces shown in Figs. 3.4 and 3.6, have the form:

For the Separating Part

$$\left.\begin{aligned} m_1\ddot{x}_1 &= (-P_{T1} - X_1 + X_{s1}) - (Y_1 + Y_{s1})\vartheta_1 - G_1 \sin\vartheta_0; \\ m_1\ddot{y}_1 &= (-P_{T1} - X_1 + X_{s1})\vartheta_1 + (Y_1 + Y_{s1}) - G_1 \cos\vartheta_0; \\ J_{z1}\ddot{\vartheta}_1 &= M_{z1} + M_{sz_1}; \end{aligned}\right\} \qquad (3.3a)$$

For the Subsequent Stage

$$\left.\begin{aligned} m_2\ddot{x}_2 &= (P_{22} - X_2) - (Y_2 + Y_{s2})\vartheta_2 - G_2 \sin\vartheta_0; \\ m_2\ddot{y}_2 &= (P_{22} - X_2)\vartheta_2 + (Y_2 + Y_{s2}) + 2T_2\delta_{\vartheta_2} - G_2 \cos\vartheta_0; \\ J_{z2}\ddot{\vartheta}_2 &= M_{z2} + M_{yz_2} + M_{sz_2}; \\ \delta_{\vartheta_2} &= \delta_{\vartheta_2}(\vartheta_2, y_2). \end{aligned}\right\} \qquad (3.3b)$$

Here ϑ_0 - the angle of pitch of the rocket relative to the local horizon at the initial moment of separation; ϑ_1, ϑ_2 - the angles of deflection of the longitudinal axes of the separating part (1) and the subsequent stage (2) from axis Ox of the inertial coordinate system; $P_2(t)$ - the thrust of the engine system of the subsequent stage; $P_{T1}(t)$ - the thrust of the retro engine; T_2 - the thrust of the

subsequent stage controlling engine; $X_{в1}, Y_{в1}, Y_{в2}, M_{вz_1}, M_{вz_2}$ - the perturbing forces and moments acting on the separating parts of the rocket; M_{z1} and M_{z2} - aerodynamic moments; $M_{yz_2} = 2T_2(x_{т2} - x_{кв2})\delta_{\vartheta_2}$ - controlling moment.

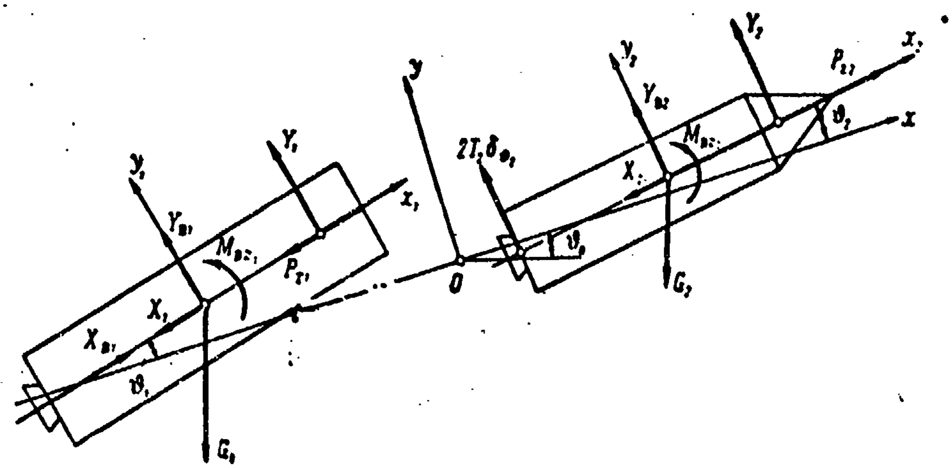


Fig. 3.6. Diagram of forces in an investigation of cold stage separation.

As a result of solving of the system of equations of (3.3) with the aid of a digital computer it is possible to calculate the relative motion of the separating parts of the rocket, which makes it possible to select the basic parameters of the separation system and to ensure the necessary reliability of the separation process.

Usually the problem of determining retro impulse for a given safe distance between the parts of a rocket at the end of separation is solved: $x_{отн}(t_к) = x_1(t_к) - x_2(t_к)$.

At a given distance $x_{отн}(t_к)$ the necessary magnitude of breaking impulse I_t depends on the duration of separation $t_к$. The greater is $t_к$, the smaller is I_T and the weight of the retro solid-propellant rocket engines and therefore the smaller is the reduction in range due to the installation of the solid-propellant rocket engines. On the other hand, with an increase in $t_к$ the losses in range due to gravitation increase. Thus such a value of I_T is selected, at which the reduction in range is the smallest.

In order to check the reliability of separation, we usually plot the trajectory of the relative motion of the most dangerous point of the plane of joining of the objects and on the basis of the obtained results we draw a conclusion about the possibility of the separation of the objects without colliding.

An investigation of the separation process also specifies an evaluation of the stability and the controllability of the subsequent stage during the separation phase, for which the maximum angles of deflection of the ϑ_{2max} and of the controlling engines $\delta_{\vartheta_2 max}$ are determined.

For obtaining the most complete and the most reliable conclusions about the reliability of separation and about the stability and the controllability of the subsequent stage an investigation of the relative motion of the parts of the rocket must be carried out taking into account the statistical characteristics of the perturbations.

For hot separation the effect of the gas-dynamic forces on the separating parts of a rocket is very characteristic. The gas-dynamic forces caused by the effect of the gas jet of the engine of the subsequent stage on the separating part, is conveniently examined as the geometric sum of the axial and lateral components (Fig. 3.7).

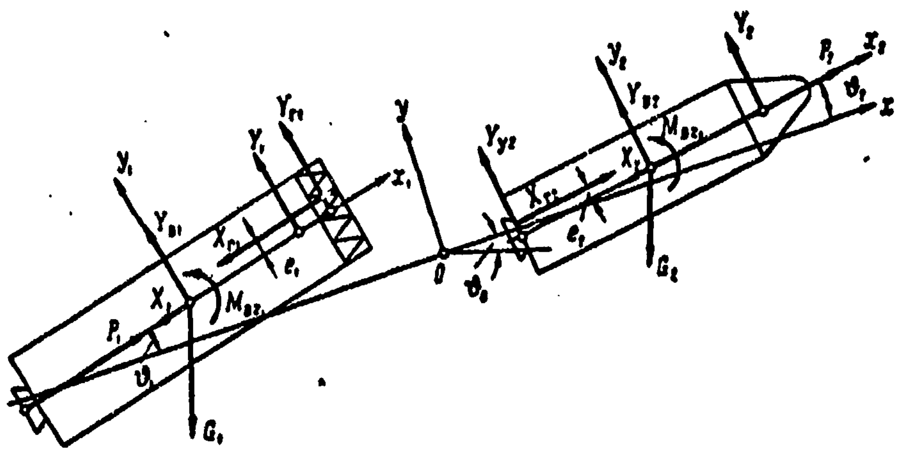


Fig. 3.7. Diagram of the forces in investigating the hot stage separation.

Axial gas-dynamic force X_{r1} is caused by the pressure of the engine jet on the separating part. The lateral displacement and the turning of the part being separating relative to the subsequent stage, and also the perturbations of the gas jet itself as a result of the deflection of the control elements give rise to the eccentricity e_1 of the axial force relative to the longitudinal axis of the separating part. These phenomena cause the transverse gas-dynamic force Y_{r1}.

The effect of the reflected gas jet on the subsequent stage can be described by axial force X_{r2} with eccentricity e_2.

A characteristic feature of gas-dynamic forces is their comparatively great value. Thus, axial force X_{r1} in the initial moments of separation is comparable with the thrust force of the engine system of the subsequent stage P_2. This fact is also used for the rapid separation of stages. However the moment of axial gas-dynamic force creates the conditions for the acute turning (tumbling) of the separating part and of its collision with the engines of the subsequent stage.

With a given rocket design the magnitudes of the gas-dynamic forces and the lines of their action $(X_{r1}, Y_{r1}, X_{r2}, e_1, e_2)$ are determined by the relative position of the separating parts of the rocket, i.e., by the parameters $x_{отн}=x_2-x_1$, $y_{отн}=y_2-y_1$ and $\vartheta_{отн}=\vartheta_2-\vartheta_1$. It is very difficult to determine these dependences by calculation. Usually they are determined by experiments or they are obtained by analogy with existing rockets.

The equations of motion of rocket parts after the breaking of connections in hot separation for the case examined by us (see Figs. 3.5 and 3.7) take the form:

<u>For the Separating Part</u>

$$\left.\begin{aligned}
m_1\ddot{x}_1&=(P_1-X_1-X_{r1})-(Y_1+Y_{н1}+Y_{r1})\vartheta_1-G_1\sin\vartheta_0;\\
m_1\ddot{y}_1&=(P_1-X_1-X_{r1})\vartheta_1+(Y_1+Y_{н1}+Y_{r1})-G_1\cos\vartheta_0;\\
I_{z1}\ddot{\vartheta}_1&=M_{z1}+M_{нz_1}+X_{r1}e_1+Y_{r1}(x_{r1}-x_{дr_1});
\end{aligned}\right\}\qquad(3.4a)$$

For the Subsequent Stage

$$\left.\begin{aligned}
m_2\ddot{x}_2 &= (P_2 - X_2 + X_{r2}) - (Y_2 + Y_{n2} + Y_{y2})\vartheta_2 - G_2 \sin\vartheta_0;\\
m_2\ddot{y}_2 &= (P_2 - X_2 + X_{r2})\vartheta_2 + (Y_2 + Y_{n2} + Y_{y2}) - G_2 \cos\vartheta_0;\\
J_{z1}\ddot{\vartheta}_2 &= M_{z2} + M_{yz_1} + M_{nz_1} - X_{r2}e_2;\\
\delta_{\vartheta 2} &= \delta_{\vartheta 2}(\vartheta_2,\ y_2).
\end{aligned}\right\} \quad (3.4b)$$

It is assumed here that the controlling moment of the subsequent stage M_{yz}, is the moment of force Y_{y2}, created by the deflection of the gas jets of the main engines, for example, by the blowing of generator gas into the supersonic part of the engine nozzle. This force and moment depend on the angle δ_{ϑ}, of deflection of a certain control element. The last equation of system (3.4b) is the equation of the corresponding channel of the control system of the subsequent stage.

When selecting the basic parameters of a hot separation system, besides evaluating the reliability of the separation of the rocket parts, and also the stability and the controllability of the subsequent stage, the sequence of the moments of transmission of instructions is determined. Stage separation begins, when the acceleration of the subsequent stage exceeds the acceleration of the separating part, i.e., when $\ddot{x}_{\text{отн}} = \ddot{x}_2 - \ddot{x}_1 > 0$. From this condition the moment of the transmission of the command for connection breaking is determined. With the given transitional engine characteristics $P_1(t)$ and $P_2(t)$ the beginning of stage separation depends on the selection of the moment of the starting of the engine of the subsequent stage with respect to the moment of the shutdown of the engine of the separating part. A too early beginning of stage separation leads to a reduction in range due to the unused thrust impulse of the separating part, and a too late beginning can make the starting of the engine of the subsequent stage difficult due to the absence of longitudinal acceleration. Finally the time sequence of the moments of transmission of instructions is established on the basis of the results of investigations of the reliability of separation, of the stability and controllability of the subsequent stage and of the reduction in range.

Nose Section Separation

The phase of motion of a rocket from the moment of the transmission of the main instruction for the shutdown of the engines of the subsequent stage of a rocket to the departure of the nose section a sufficient distance from the rocket body we will call the nose section of [ГЧ = NS] separation phase.

Separation of the NS can be accomplished:

- by the braking of the body of the separating stage with special braking elements (solid-propellant rocket engines, retro-nozzles operating on pressurized gases in the tanks of the separating stage);

- by repelling the NS and the body of the separating part with thrusters (spring, pneumatic and pyrotechnic);

- by accelerating the NS with special engines. Specifications are imposed on separation systems to ensure;

- minimum values of perturbations on the velocity of the NS, affecting the dispersion of its points of impact;

- sufficiently small values of angular velocity of the NS, causing the appearance of large angles of attack upon reentry into the atmosphere;

- reliable separation of the NS.

The realization of these specifications ensures small dispersion of the points of impact of the NS and normal operation of all the systems of the NS equipment.

In the dynamic design of a NS separation system the following problems are usually solved:

- selection of the operation mode and the method of shutting down the engine system at the end of the powered-flight phase;

- selection of the scheme and means of separation;

- selection of the basic parameters of the separation system;

- selection of the sequence of instructions for the separation phase;

- ensuring the reliability of the separation process (the absence of collisions of the NS with the rocket body).

The energy characteristics of the separation equipment are selected so as to ensure reliable breaking of the various connections between the NS and the rocket body taking aftereffect thrust into account and to impart to the separating parts of the rocket a specific relative velocity, excluding the possibility of the overtaking of the NS by the rocket body (in the case of the incomplete compensation for aftereffect impulse).

For reducing the dispersion of the points of impact of the NS due to variance in the aftereffect [thrust trailoff] impulse two-stage engine shutdown can be employed. After the preliminary instruction for engine shutdown the feed of propellant is reduced and thrust force is accordingly reduced to a certain intermediate value, and only after the main instruction is the engine shutdown (Fig. 3.8a). This measure leads to a noticeable reduction in the aftereffect impulse and therefore, to a reduction in the magnitude of its variance and also to a reduction in the effect of the time errors in carrying out the main instruction.

Besides two-stage engine shutdown, other measures for reducing t the variance in aftereffect impulse are employed. For instance, Vernier engines are mounted on a rocket, i.e., engines with small thrust which create comparatively small accelerations of the order of 0.5g [12]. As such engines, evidently, it is possible to use the controlling engines.

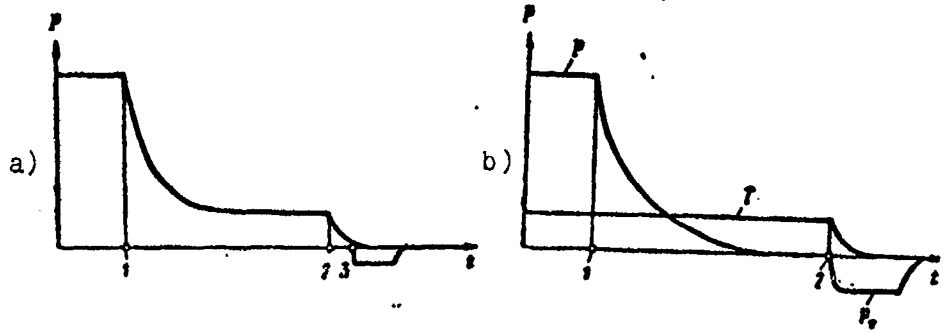


Fig. 3.8. Possible schemes of nose section separation: a) throttling of the rocket engine and separation of the NS by a thruster: 1 - preliminary instruction; 2 - main instruction; 3 - breaking of the connections between the NS and the rocket body and repelling of the NS by the thruster; b) using engines with low thrust for reducing the aftereffect impulse and braking the rocket body with solid-propellant rocket engines: 1 - preliminary instruction for shutting down the engine system (cutting off the main engine); 2 - main instruction for shutting down the engine system (cutting off the controlling engine, breaking the connections between the NS and the rocket body, starting the retro-solid-propellant rocket engines).

To eliminate the effect of aftereffect impulse on the separation of the NS from the rocket body it is expedient simultaneously with the carrying out of the main instruction to activate the engines braking the rocket body (see Fig. 3.8b).

For investigating the process of the separation of the NS from the rocket body the same dynamic scheme in principle is used, as in investigating the process of stage separation by braking (see Fig. 3.6 and the equations of 3.3). Of course, the actual form of the equations of motion of the NS and the rocket body depends on the design features of the separation equipment and the conditions of the problem in question. By solving more or the less complex equations of motion of the NS and the rocket body, the reliability of separation is evaluated and the perturbations of the velocity and angluar velocity of rotation acquires after the termination of the separation process as a result of the effect of various perturbations, are determined.

For a NS with a disoriented reentry into the atmosphere the initial angular velocity has as significant effect on the angle attack of NS

upon its reentry into the atmosphere and thus, on maximum transverse accelerations. Thus in designing a separation system for such NS considerable attention is given to limiting those components of the angular velocity of the NS along the body axes which lead to the appearance of large angles of attacks of the NS during reentry into the atmosphere. The initial angular velocity of a NS also effects the possibility of NS detection and selection by the antimissile system of the enemy.

For a NS with oriented reentry into the atmosphere the initial angular velocity determines the weight of the working medium, necessary for damping the angular velocity of the NS, and thus decreases to a greater or lesser extent the maximum firing range.

When using the rotation of a NS around the longitudinal axis for orientating the NS in space the angular velocities of pitch and yaw reduce orientation accuracy.

The angular velocity which is received by a NS after separation from a rocket body, is due, in the first place, to the errors in the angular stabilization of the rocket (with respect to angular velocity) in the moment of the beginning of separation and, in the second place, to the perturbations acting on the NS during separation.

The composition of the perturbing factors is ascertained taking the actual structural layouts of the NS and the separation system into account. As an example of possible perturbing factors it is possible to indicate the variance in the forces of the separation mechanisms, eccentric application of the force of a thruster relative to the center of mass of the NS, the thrust differential in the separation engines, the variance in the explosive bolt impulses, the asymmetry of the plug-type connector setup, etc.

For an approximate evaluation of angular velocities instead of solving the equations of motion of the NS in the separation phase it is possible to use the formula

$$\omega_i = \frac{1}{J_i} \int_0^{t_s} M_{ij}(t)\,dt, \qquad (3.5)$$

where ω_i and J_i - angular velocity and the inertial moment of the NS relative to the i-th axis; M_{ij} - the perturbing moment relative to the i-th axis due to the j-th perturbing factor.

The obtained statistical characteristics of the components of the angular velocity of the NS along body axes ω_{x1}, ω_{y1}, ω_{z1} are used for determining the angles of attack during disoriented reentry of the NS into the atmosphere, and also for evaluating the consumption of the working medium for damping the angular velocity of NS with oriented reentry into the atmosphere.

CHAPTER IV

THE BALLISTICS OF AN UNGUIDED NOSE SECTION

The flight path of a nose section [ГЧ = NS] from the moment of its separation from the body of the last stage until its impact onto the surface of the earth can be arbitrarily divided into two phases: nonatmospheric and atmospheric. The height of the arbitrary limit of the atmosphere depends on the problem being solved, the characteristics of the NS, the flight range, etc. In the nonatmospheric flight phase the NS moves practically only under the effect of gravity. In the atmospheric phase, besides gravity, aerodynamic forces are also acting on the NS.

The ballistics of NS has been called upon to solve the following basic problems:

1) determining the loads acting on a NS in the atmosphere, which is necessary for calculating the strength of a NS;

2) determining the dispersion of the impact points of a NS due to perturbations in the atmospheric phase;

3) determining the parameters of the motion of a NS at characteristic points in the trajectory for the problem being solved.

The loads on a NS, the parameters of its motion and the dispersion of the impact points mainly depend on the conditions of the reentry of the center of mass of the NS into the atmosphere (the velocity and the angle of inclination of the velocity vector to the local horizon),

the magnitude of the maximum angle of attack at this moment, the characteristics of the NS (aerodynamic, weight, geometric and centering [c.g]), the atmospheric parameters and the random variances in the enumerated parameters and characteristics. The investigation of these dependences makes it possible to formulate specifications for the conditions of reentry into the atmosphere and for the characteristics of a NS when developing rockets.

The mathematical models of motion and the fundamental elements of these models used in NS ballistics taking the flight conditions and the characteristics of the problems being solved into account constitute the main content of this chapter. Common to all the models of NS motion being examined is the simplifying assumption, that a NS is an aerodynamically axisymmetric body, the ellipsoid of inertia of which is an ellipsoid of revolution.

4.1. THE MOTION OF A NOSE SECTION IN THE NON-ATMOSPHERIC PHASE OF ITS TRAJECTORY

The accepted division of the flight path of a NS into t[illegible] sections is arbitrary because in actuality aerodynamic for[illegible] acting in the non-atmospheric phase. However in this ph[illegible] magnitude is negligibly small in comparison with gra[illegible] disregarded are the gravitational moments, the at[illegible]raction forces of the celestial bodies, light pressure, electro[illegible]gnetic forces, etc., in view of their insignificant effect on the motion of a NS. As a result of the absence of forces dependent on the parameters of the rotary motion of a NS, in the non-atmospheric phase of the trajectory the motion of the center of mass of a NS and its rotation around the center of mass are investigated separately.

The Motion of the Center of Mass of a Nose Section

As is known [2], the main factors affecting the agreement of the optimum firing range with the actual, are the refinement of the control system and the calculational accuracy of the NS trajectory. Thus pinpoint accuracy in calculating the trajectory of a NS for determining the aiming data is absolutely necessary. Such accuracy is

satisfied by the system of equations of (2.97), (2.98), (2.100), (2.108) which for the motion of the center of mass of a NS, if angle Ψ is reckoned clockwise, takes the form:

$$\left.\begin{aligned}
&\dot{V}=-g_r\sin\Theta-g_\omega(\cos\varphi_u\cos\Psi\cos\Theta+\sin\varphi_u\sin\Theta);\\
&\dot{\Theta}=-\frac{g_r}{V}\cos\Theta-\frac{g_\omega}{V}(-\cos\varphi_u\cos\Psi\sin\Theta+\\
&\quad+\sin\varphi_u\cos\Theta)+\frac{V}{r}\cos\Theta+2\omega_3\cos\varphi_u\sin\Psi;\\
&\dot{\Psi}=\frac{g_\omega}{V}\,\frac{\cos\varphi_u\sin\Psi}{\cos\Theta}+\frac{V}{r}\operatorname{tg}\varphi_u\sin\Psi\cos\Theta-\\
&\quad-2\omega_3(\cos\varphi_u\cos\Psi\operatorname{tg}\Theta-\sin\varphi_u);\\
&\dot{\varphi}_u=\frac{V}{r}\cos\Psi\cos\Theta;\\
&\dot{\lambda}=\frac{V}{r}\,\frac{\sin\Psi\cos\Theta}{\cos\varphi_u};\\
&\dot{r}=V\sin\Theta.
\end{aligned}\right\}\qquad(4.1)$$

The accuracy of the calculations of the trajectory of a NS by these equations is determined only by the accuracy, with which the components forces of gravity g_r and g_ω are known.

Another problem of ballistics is determining the parameters of the motion of the center of mass of a NS during its reentry into the atmosphere. In solving this problem the non-centrality of the terrestrial gravitational field and the non-sphericity of the terrestrial surface can be disregarded. Thus, it is convenient instead of integrating the equations of (4.1) to make use of the known results of elliptical theory being used for the absolute motion of a NS (i.e., motion relative to a certain inertial coordinate system), and then to go to motion relative to the earth, having calculated the rotation of the earth by introducing appropriate corrections into the values of velocity V, slope angle of trajectory Θ, azimuth Ψ and longitude λ. For this let us examine the motion of a NS relative to an "absolute" coordinate system $O_3x_ay_az_a$, not participating in the diurnal rotation of the earth. Let us place the origin of the

coordinates O_3 at the center of the earth; let us direct axis O_3x_a along the axis of rotation of the earth toward the north pole; let us place axis O_3y_a in the meridianal plane, passing through the launch point at the moment of launch. We will consider the terrestrial gravitational field central. Let us designate the parameters of motion relative to the absolute coordinate system by V_a, Θ_a, Ψ_a, λ_a. Then the system of equations of (4.1) takes the form.

$$\left.\begin{aligned}
&\dot{V}_a=-g\sin\Theta_a;\\
&\dot{\Theta}_a=-\frac{g}{V_a}\cos\Theta_a+\frac{V_a}{r}\cos\Theta_a;\\
&\dot{\Psi}_a=\frac{V_a}{r}\operatorname{tg}\varphi_u\sin\Psi_a\cos\Theta_a;\\
&\dot{\varphi}_u=\frac{V_a}{r}\cos\Psi_a\cos\Theta_a;\\
&\dot{\lambda}_a=\frac{V_a}{r}\,\frac{\sin\Psi_a\cos\Theta_a}{\cos\varphi_u};\\
&\dot{r}=V_a\sin\Theta_a,
\end{aligned}\right\}\qquad(4.2)$$

where it is possible to take: $g = 9.81\,(R/r)^2$ m/s^2 – acceleration due to gravity; R = 6 371 210 m – the mean radius of the earth.

Using Fig. 4.1, let us determine the connections between the parameters of absolute (V_a Θ_a Ψ_a) and relative (V, Θ, Ψ) motion. The horizontal component of velocity $V\cos\Theta$ is geometrically added to linear velocity $\omega_3 r\cos\phi_u$ due to the rotation of the earth. Thus, the modulus of the horizontal component of the absolute velocity of the center of mass of a NS is equal to

$$V_{a.\text{гор}}=\sqrt{(V\cos\Theta\cos\Psi)^2+(V\cos\Theta\sin\Psi+\omega_3 r\cos\varphi_u)^2}.$$

Since the vertical component of velocity $V\sin\Theta$ does not change due to the rotation of the earth, the absolute velocity of the center of mass is equal to

$$V_a=\sqrt{(V\sin\Theta)^2+(V\cos\Theta\cos\Psi)^2+(V\cos\Theta\sin\Psi+\omega_3 r\cos\varphi_u)^2}.\qquad(4.3)$$

The angle of slope of vector $\bar{V}_a$ to the local horizon is equal to

$$\Theta_a = \arcsin \frac{V \sin \Theta}{V_a}. \tag{4.4}$$

Angle Ψ_a in an absolute coordinate system is determined from equations:

$$\left.\begin{aligned} \cos \Psi_a &= \frac{V \cos \Theta \cos \Psi}{\sqrt{(V \cos \Theta \cos \Psi)^2 + (V \cos \Theta \sin \Psi + \omega_3 r \cos \varphi_{\text{ц}})^2}}; \\ \sin \Psi_a &= \frac{V \cos \Theta \sin \Psi + \omega_3 r \cos \varphi_{\text{ц}}}{\sqrt{(V \cos \Theta \cos \Psi)^2 + (V \cos \Theta \sin \Psi + \omega_3 r \cos \varphi_{\text{ц}})^2}}. \end{aligned}\right\} \tag{4.5}$$

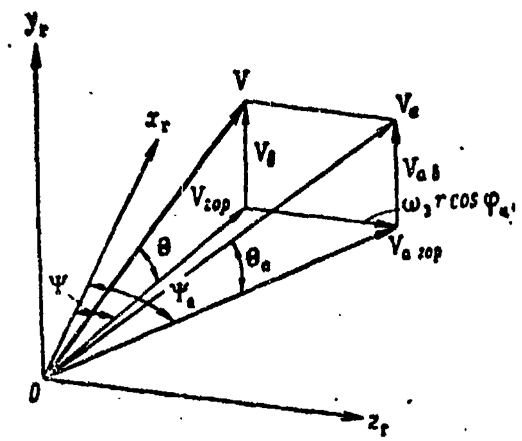


Fig. 4.1. For determining the parameters of absolute motion of the center of mass of a nose section.

Since because of the rotation of the earth the point of intersection of radius-vector r with the surface of the earth is displaced along the parallel,

$$\varphi_{\text{ц.а}} = \varphi_{\text{ц}}. \tag{4.6}$$

Let us determine the longitude of the center of mass of a NS in an absolute coordinate system by formula

$$\lambda_a = \lambda + \omega_3 (t - t_{\text{н}}). \tag{4.7}$$

If in the first two equations of system (4.2) we replace coordinates V_a and Θ_a with the aid of kinematic relationships:

$$\dot{r} = V_a \sin \Theta_a;$$
$$r\dot{\chi} = V_a \cos \Theta_a,$$

where κ – the angle included between radius-vector r and radius-vector $r_н$ at the beginning of the unpowered-flight phase, then after some transformations we will obtain certain equations of the elliptical theory:

$$\left.\begin{aligned} \ddot{r}-r\dot{\chi}^2&=-g; \\ \frac{d}{dt}(r^2\dot{\chi})&=0. \end{aligned}\right\} \tag{4.8}$$

As a result of integrating the equations of (4.8) we obtain the equation of the flight path in the form of an equation of a conical section

$$r=\frac{p}{1-e\cos(\chi_в-\chi)}, \tag{4.9}$$

where $\kappa_в$ – the angle corresponding to the peak of the trajectory;

$p=r\varepsilon\cos^2\Theta_a^2$ – a parameter of the cross section

$e=\sqrt{1-(2-\varepsilon)\varepsilon\cos^2\Theta_a}$ – the eccentricity of the cross section:

$$\varepsilon=\frac{V_a^2 r}{fM}.$$

For a NS e < 1, i.e., the trajectory of a NS is elliptical.

Knowing the parameters of the motion of the center of mass of a NS relative to the earth at the beginning of the unpowered-flight phase

$$t_н,\ V_н,\ \Theta_н,\ \Psi_н,\ r_н,\ \varphi_{ц.н},\ \lambda_н, \tag{4.10}$$

let us now determine the parameters of motion of the center of mass of a NS at the moment of reentry into the atmosphere, i.e., at the height of the arbitrary limit of the atmosphere h_0.

From the formulas of (4.3), (4.4), (4.5) and (4.7) let us find the parameters of the absolute motion of the center of mass of a NS at the beginning of the unpowered-flight phase: $V_{а.н}$, $\Theta_{а.н}$, $\Psi_{а.н}$, $\lambda_{а.н}$. Then using the formulas of the elliptical theory let us

determine the parameters of motion of a NS at altitude $h_0 = r_0 - R$:

$$V_{a0} = \sqrt{V^2_{a.н} + 2fM\left(\frac{1}{r_0} - \frac{1}{r_н}\right)}; \qquad (4.11)$$

$$\Theta_{a0} = -\arccos\left(\frac{V_н r_н \cos\Theta_{a.н}}{V_{a0} r_0}\right); \qquad (4.12)$$

$$\chi_0 = 2\,\mathrm{arctg}\,[\{\varepsilon_н r_0 \,\mathrm{tg}\,\Theta_{a.н} + \sqrt{(\varepsilon_н r_0 \,\mathrm{tg}\,\Theta_{a.н})^2 +}$$
$$\overline{+[2r_0(1+\mathrm{tg}^2\,\Theta_{a.н}) - (r_н + r_0)\varepsilon_н](r_н - r_0)\varepsilon_н}\} : \{2r_0(1+\mathrm{tg}^2\,\Theta_{a.н}) -$$
$$-(r_н + r_0)\varepsilon_н\}]; \qquad (4.13)$$

$$t_0 - t_н = \frac{r_н}{V_{a.н}} \frac{\varepsilon_н \cos\Theta_{a.н}}{2-\varepsilon_н}\Big\{\mathrm{tg}\,\Theta_{a.н} -$$
$$-\mathrm{tg}\,\Theta_{a0} - \frac{1}{\cos\Theta_{a.н}\sqrt{(2-\varepsilon_н)\varepsilon_н}}\Big[\arcsin\frac{1-\varepsilon_0}{e} +$$
$$+\arcsin\frac{1-\varepsilon_н}{e}\Big] + \frac{\pi}{\sqrt{1-e^2}}\Big\}. \qquad (4.14)$$

After this from spherical triangle ABC (Fig. 4.2) let us obtain the spherical coordinates of a NS in absolute motion at the moment of reentry into the atmosphere.

$$\varphi_{ц0} = \arcsin(\sin\varphi_{ц.н}\cos\chi_0 + \cos\varphi_{ц.н}\sin\chi_0\cos\Psi_{a.н}). \qquad (4.15)$$

Longitude λ_{a0} varying within the limits of from 0° to 360°, is determined by formulas:

$$\left.\begin{aligned} \sin(\lambda_{a0} - \lambda_{a.н}) &= \frac{\sin\chi_0 \sin\Psi_{a.н}}{\cos\varphi_{ц0}};\\ \cos(\lambda_{a0} - \lambda_{a.н}) &= \frac{\cos\chi_0 - \sin\varphi_{ц.н}\sin\varphi_{ц0}}{\cos\varphi_{ц.н}\cos\varphi_{ц0}}. \end{aligned}\right\} \qquad (4.16)$$

Azimuth Ψ_{a0} which also varies within the limits of from 0° to 360° is determined by formulas:

$$\left.\begin{aligned} \sin(180 - \Psi_{a0}) &= \frac{\cos\varphi_{ц.н}\sin\Psi_{a.н}}{\cos\varphi_{ц0}};\\ \cos(180 - \Psi_{a0}) &= \sin\Psi_{a.н}\sin(\lambda_{a0} - \lambda_{a.н}) \times\\ &\times \sin\varphi_{ц.н} - \cos\Psi_{a.н}\cos(\lambda_{a0} - \lambda_{a.н}). \end{aligned}\right\} \qquad (4.17)$$

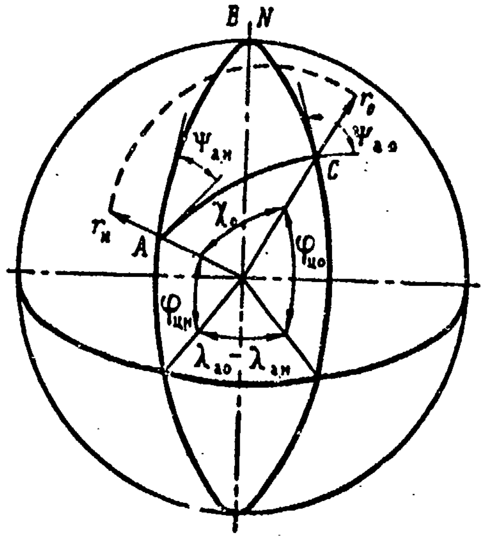


Fig. 4.2. For determining the coordinates (during absolute motion) of the center of mass of a nose section at the moment of reentry into the atmosphere.

Finally, using formulas, analogous to the formulas of (4.3), (4.4), (4.5), and (4.7), let us determine the parameters V_0, Θ_0, Ψ_0 of the motion of the center of mass of a NS relative to the rotating earth:

$$V_0=\sqrt{(V_{а0}\cos\Theta_{а0}\cos\Psi_{а0})^2+(V_{а0}\cos\Theta_{а0}\sin\Psi_{а0}-\omega_3 r\cos\varphi_{ц0})^2+(V_{а0}\sin\Theta_{а0})^2}; \tag{4.18}$$

$$\lambda_0=\lambda_{а0}-\omega_3(t_0-t_н); \tag{4.19}$$

$$\Theta_0=\arcsin\frac{V_{а0}\sin\Theta_{а0}}{V_0}. \tag{4.20}$$

Angle Ψ_0 is determined by the formulas

$$\left.\begin{aligned}\cos\Psi_0&=\frac{V_{а0}\cos\Theta_{а0}\cos\Psi_{а0}}{\sqrt{(V_{а0}\cos\Theta_{а0}\cos\Psi_{а0})^2+(V_{а0}\cos\Theta_{а0}\sin\Psi_{а0}-\omega_3 r\cos\varphi_{ц0})^2}};\\ \sin\Psi_0&=\frac{V_{а0}\cos\Theta_{а0}\sin\Psi_{а0}-\omega_3 r\cos\varphi_{ц0}}{\sqrt{(V_{а0}\cos\Theta_{а0}\cos\Psi_{а0})^2+(V_{а0}\cos\Theta_{а0}\sin\Psi_{а0}-\omega_3 r\cos\varphi_{ц0})^2}}.\end{aligned}\right\} \tag{4.21}$$

The Motion of a Nose Section Around Its Center of Mass

The motion of a NS around its center of mass during the non-atmospheric phase of the trajectory is investigated for the purpose of determining the parameters of this motion during reentry into the atmosphere.

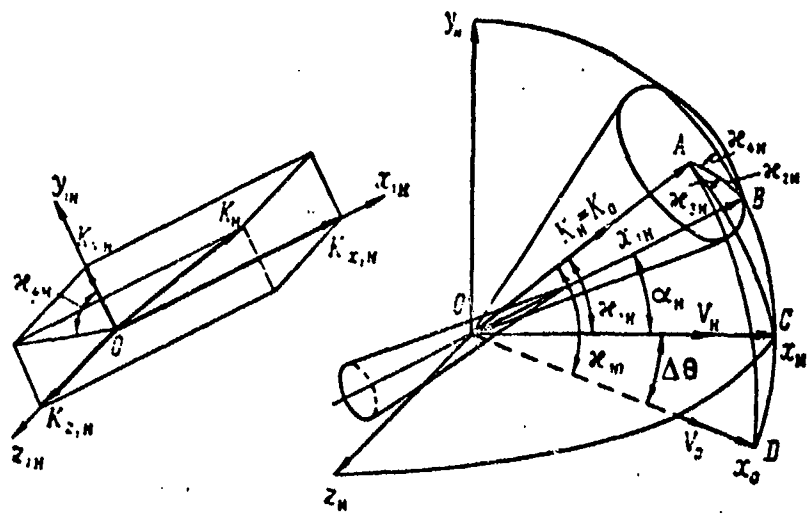


Fig. 4.3. The parameters of motion of a nose section around its center of mass during the non-atmospheric phase of the trajectory.

In connection with the fact that NS usually have an axisymmetric shape, their main inertial moments J_{y1} and J_{z1} differ insignificantly. Thus in analyzing the motions of a NS around its center of mass both in the non-atmospheric and in the atmospheric phase of the trajectory it is assumed that the inertial moments J_{y1} and J_{z1} are equal to each other.

We will obtain the equations of motion of a NS around its center of mass in the non-atmospheric phase of the trajectory, having set the moments of force equal to zero and $J_{y1} = J_{z1}$ in the dynamic Euler equations (2.100):

$$\left.\begin{aligned} &J_{x1}\dot{\omega}_{x1}=0;\\ &J_{z1}\dot{\omega}_{y1}-(J_{z1}-J_{x1})\omega_{z1}\omega_{x1}=0;\\ &J_{z1}\dot{\omega}_{z1}-(J_{x1}-J_{z1})\omega_{x1}\omega_{y1}=0.\end{aligned}\right\} \tag{4.22}$$

This case of motion of a rigid body around its center of mass, called regular precession, is well studied in classical mechanics [4]. The longitudinal axis of a NS in flight in the non-atmospheric phase of the trajectory moves uniformly along the surface of a right

circular cone (cone of precession) whose axis coincides with the vector of angular momentum K preserving constant direction in space (Fig. 4.3). The direction of a vector of angular momentum of a NS, the half-angle of a cone of precession and the angular velocity of motion of the longitudinal axis along the surface of a cone of precession (the angular velocity of precession) are determined by the initial conditions corresponding to the moment of separation of the NS from a rocket body: by the orientation of the longitudinal axis and the vector of angular velocity.

Let us introduce the following designations (see Fig. 4.3):

κ_1 – the angle included between the vectors of angular momentum $\bar{K}$ and of velocity $\bar{V}$;

κ_2 – the half-angle of a cone of precession;

κ_3 – the angle included between the planes of angles κ_1 and κ_2;

κ_3' – the angle included between the planes of angles κ_{10} and $\kappa_{2\text{н}}$;

κ_4 – the angle included between the plane of angle κ_2 and the plane of firing.

The modulus of angular momentum is determined by the values of the projections of angular velocity on the axis of a body coordinate system:

$$K = \sqrt{J_{x1}^2 \omega_{x1}^2 + J_{z1}^2 (\omega_{y1}^2 + \omega_{z1}^2)}. \tag{4.23}$$

Let us determine the orientation of angular momentum with respect to plane Ox_1y_1 proceeding on the assumption that at the moment of separation of a NS from a rocket this plane is parallel to the plane of firing:

$$\left.\begin{aligned} \sin \kappa_{4\text{н}} &= -\frac{\omega_{z_1\text{н}}}{\sqrt{\omega_{y_1\text{н}}^2 + \omega_{z_1\text{н}}^2}}; \\ \operatorname{tg} \kappa_{4\text{н}} &= \frac{\omega_{z_1\text{н}}}{\omega_{y_1\text{н}}}. \end{aligned}\right\} \tag{4.24}$$

The components of angular momentum K_{x1} and $K_{экв}$ in the non-atmospheric phase of the trajectory do not vary and are equal to:

$$K_{x1}=J_{x1}\omega_{x1}; \tag{4.25}$$

$$K_{экв}=J_{x1}\sqrt{\omega_{y1}^2+\omega_{z1}^2}. \tag{4.26}$$

The parameters of motion of a NS around its center of mass at the moment of reentry into the atmosphere ($h = h_0$) we will determine in the following sequence.

1. Angles $\varkappa_{1н}$ and $\varkappa_3$ (angle BAD) we will obtain from spherical triangles ABC and ABD:

$$\varkappa_{1н}=\arccos(\cos\varkappa_2\cos\alpha_н-\sin\varkappa_2\sin\alpha_н\cos\varkappa_{4н}); \tag{4.27}$$

$$\left.\begin{aligned}\sin\varkappa_3'&=\frac{\sin(\alpha_н+\Delta\Theta)\sin\varkappa_{4н}}{\sin\varkappa_{10}};\\ \cos\varkappa_3'&=\frac{\cos(\alpha_н+\Delta\Theta)-\cos\varkappa_{10}\cos\varkappa_2}{\sin\varkappa_{10}\sin\varkappa_2},\end{aligned}\right\} \tag{4.28}$$

where

$$\varkappa_2=\arccos\frac{K_{x1}}{K}=\text{const}.$$

2. Angle $\varkappa_{10}$ at the moment of reentry into the atmosphere we will obtain from spherical triangle ABD:

$$\varkappa_{10}=\arccos[\cos\varkappa_2\cos(\alpha_н+\Delta\Theta)-\sin\varkappa_2\sin(\alpha_н+\Delta\Theta)\cos\varkappa_{4н}], \tag{4.29}$$

where

$$\Delta\Theta=\Theta_н-\Theta_0+\chi_0; \tag{4.30}$$

χ_0 – the angular flight range of a N.S.

Then

$$K_{x0}=K\cos\varkappa_{10}. \tag{4.31}$$

3. Angle $\varkappa_{30}$ we will obtain proceeding from the fact that the longitudinal axis of a NS Ox_1 moves along a cone of precession with constant angular velocity

$$\dot{\varkappa}_3 = \frac{K}{J_{z1}} . \qquad (4.32)$$

Then

$$\varkappa_{30} = \varkappa_{3н} + \dot{\varkappa}_3 (t_0 - t_н), \qquad (4.33)$$

where $t_0 - t_н$ – the flight time of a NS in the non-atmospheric phase of its trajectory, determined by formula (4.14).

Thus, we obtained the values of angles κ_{10}, κ_2, κ_{30} at altitude of the reentry into the atmosphere.

Knowing these angles, it is possible to determine from spherical triangle ABC (Fig. 4.4) the initial angle of attack

$$\alpha_0 = \arccos(\cos\varkappa_{10}\cos\varkappa_2 + \sin\varkappa_{10}\sin\varkappa_2\cos\varkappa_{30}). \qquad (4.34)$$

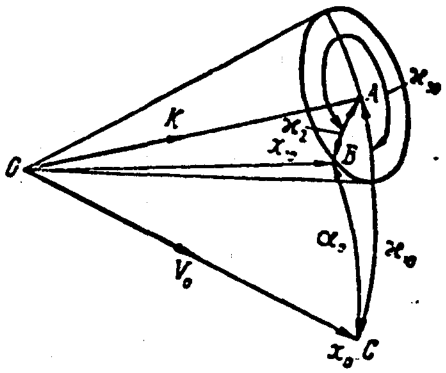


Fig. 4.4. For determining the angle of attack of a NS at the moment of reentry into the atmosphere.

Disregarding angular velocity κ_1 as compared with κ_3, we obtain that angular velocity α_0 is equal to

$$\dot{\alpha}_0 = \frac{\sin\varkappa_{10}\sin\varkappa_2\sin\varkappa_{30}}{\sin\alpha_0}\dot{\varkappa}_{30}. \qquad (4.35)$$

Thus, by assigning the initial conditions of the motion of the center of mass: $V_н$, $\Theta_н$, $\Psi_н$, $r_н$, $\phi_{ц.н}$, $\lambda_н$ and the motions around the center of mass $\alpha_н$, $\mu_н$, $\nu_н$, $\omega_{x_1 н}$, $\omega_{y_1 н}$, $\omega_{z_1 н}$, at the moment of the beginning of the unpowered-phase of the trajectory, it is possible by the obtained formulas to determine initial conditions α_0 and α_0

for solving the equation of motion of a NS around its center of mass in the atmospheric phase of the trajectory.

As the calculations by formula (4.34) show, the angle of attack of a NS upon reentry into the atmosphere in the general case can take any values depending on the relationship between the components of angular velocity of a NS at the moment of its separation from the last stage of a rocket.

4.2. THE MOTION OF A NOSE SECTION IN THE ATMOSPHERIC PHASE OF THE TRAJECTORY

General Characteristics of the Motion of a NS in the Atmosphere

The atmospheric phase of the trajectory begins at the arbitrary limit of the atmosphere, whose altitude depends upon the problem being solved, the characteristics of the NS, the flight range, etc. Thus, for instance, the beginning of the noticeable effect of the atmosphere on the parameters of motion of a NS during long-range firing corresponds to heights of about 80-100 km. In connection with this an altitude, equal to 80 km [10], [26], is usually taken as the arbitrary boundary.

Initially as a result of the effect of comparatively small (in magnitude) aerodynamic moments the precessional motion is disturbed and the statically stable NS begins to carry out three-dimensional oscillations around its center of mass. As a result of the rather rapid increase in the restoring and damping aerodynamic moments the amplitude of the oscillations noticeably decreases with the decrease in altitude. It is conditionally possible to consider that with an amplitude of the oscillations of the angle of attack, smaller than 1°-2°, a NS is stabilized, i.e., its axis is oriented along the velocity vector of flight to within 1°-2°. The altitude of the beginning of the stabilized flight of a NS depends upon its characteristics and the conditions of its reentry into the atmosphere and can be found within various limits.

The conditions of motion of the center of mass of a NS in the atmosphere substantially differ from the conditions of the motion of a rocket in the powered-flight phase. Thus, in the dense layers of the atmosphere the dynamic head and the longitudinal acting on a NS differ by tens of times, the transverse acceleration – by hundreds of times. The flight of a NS is also characterized by considerable heating of its surface.

For calculating the strength of a NS it is necessary to know the maximum loads acting on a NS in flight. The basic data for determining these loads are the peak values of accelerations (axial and transverse), the amplitude of the angle of attack and the dynamic pressure. These characteristics with assigned design parameters of a NS are determined by the conditions of the reentry of the center of mass of a NS into the dense layers of the atmosphere, and the latter – by the shape of the flight path of the rocket in the powered-flight phase, by the limits of the firing range and by the geophysical conditions of rocket launch.

With a decrease in the angle of attack and in the velocity of reentry of the center of mass into the atmosphere the longitudinal and transverse loads acting on the NS decrease. For the purpose of limiting the loads acting on a NS special measures are taken in designing a NS for reducing the maximum angles of attack of a NS at the moment of reentry into the atmosphere. The problem of ballistics, apart from preparing data for calculating strength, consists in evaluating the effect of the components of the angular velocity of a NS at the moment of separation from a rocket on the magnitude of the angle of attack upon reentry into the atmosphere and in preparing recommendations for eliminating adverse combinations of these components.

In accordance with the indicated problems of the ballistics of a NS methods of calculating the trajectories of NS in the atmosphere, determining the limiting flight modes employed in evaluating the strength of a NS, and calculating the deflection of a NS in the atmospheric phase are examined below.

Initial Equations of Motion and Their Simplification

During the motion of a NS in the atmospheric phase of the trajectory the flight range and duration are comparatively small, in connection with which the earth can be examined as a non-rotating sphere with a central gravitational field.

Due to the heating of the surface of a NS during flight in the atmosphere ablation of the heatshield covering by the oncoming flow of air takes place, as a result of which the mass, shape and dimensions of the NS and thus, the inertial moments, the position of the center of mass and the aerodynamic characteristics of the NS vary during the course of flight. In investigating the motion of a NS let us consider that the reactive forces and moments due to the ablation of the heatshield coating are negligibly small.

The equations of motion of (2.97), (2.98), (2.99) with respect to the motion of the center of mass of a NS in the atmosphere, taking into account the above accepted assumptions, are converted to the form:

$$\left.\begin{aligned}
&\dot{V}=-\frac{c_x q S}{m}-g\sin\Theta;\\
&\dot{\Theta}=\frac{c_y q S}{mV}\sin\mu-\frac{g}{V}\cos\Theta+\frac{V}{r}\cos\Theta;\\
&\dot{\Psi}=-\frac{c_y q S}{mV\cos\Theta}\cos\mu+\frac{V}{r}\operatorname{tg}\varphi_u\sin\Psi\cos\Theta;\\
&\dot{\varphi}_u=\frac{V}{r}\cos\Psi\cos\Theta;\\
&\dot{\lambda}=\frac{V}{r}\,\frac{\sin\Psi\cos\Theta}{\cos\varphi_u};\\
&\dot{h}=V\sin\Theta;\\
&\dot{L}=\dot{\varphi}_u R\cos\psi+\dot{\lambda}R\sin\psi\cos\varphi_u;\\
&\dot{z}=-\dot{\varphi}_u R\sin\psi+\dot{\lambda}R\cos\psi\cos\varphi_u,
\end{aligned}\right\}\qquad(4.36)$$

where L – the distance covered by the NS on the arc of the great circle in the plane of firing; Ψ – the azimuth of the velocity vector of the NS, reckoned clockwise; ψ – the azimuth of firing; z – the

deflection of the center of mass of the NS from the plane of firing.

The equations of the rotary motion of an object around its center of mass (2.100) and (2.108) for the case of the flight of a NS in the atmosphere take the form:

$$\left.\begin{aligned}
&J_{x1}\dot{\omega}_{x1}=M_{x1}+M_{\text{Д}x1};\\
&J_{y1}\dot{\omega}_{y1}-(J_{z1}-J_{x1})\omega_{z1}\omega_{x1}=M_{y1}+M_{\text{Д}y1};\\
&J_{z1}\dot{\omega}_{z1}-(J_{x1}-J_{y1})\omega_{x1}\omega_{y1}=M_{z1}+M_{\text{Д}z1};\\
&\dot{\alpha}=\omega_{y1}\cos\nu-\omega_{z1}\sin\nu-\Omega_y\cos\mu-\Omega_z\sin\mu;\\
&\dot{\mu}=\omega_{y1}\frac{\sin\nu}{\sin\alpha}+\omega_{z1}\frac{\cos\nu}{\sin\alpha}-\Omega_x+\Omega_y\operatorname{ctg}\alpha\sin\mu-\\
&\qquad-\Omega_z\operatorname{ctg}\alpha\cos\mu;\\
&\dot{\nu}=\omega_{x1}-\omega_{y1}\operatorname{ctg}\alpha\sin\nu-\omega_{z1}\operatorname{ctg}\alpha\cos\nu-\\
&\qquad-\Omega_y\frac{\sin\mu}{\sin\alpha}+\Omega_z\frac{\cos\mu}{\sin\alpha},
\end{aligned}\right\} \qquad (4.37)$$

where M_{x1}, M_{y1}, M_{z1} – the moments of the aerodynamic forces (see the formulas of (2.101); $M_{\text{Д}x_1}$, $M_{\text{Д}y_1}$, $M_{\text{Д}z_1}$ – the aerodynamic damping moments determined by the formulas of (1.28).

In solving the basic problems of the ballistics of unguided NS it is usually sufficient to examine the particular case of the motion of a NS in the atmospheric phase of the trajectory, when the longitudinal and transverse axes of the NS Ox_1 and Oz_1 are moving in the plane of firing.

The equations of such motion, which is customarily called longitudinal, can be obtained from the equations of (4.36) and (4.37), if the considerations presented in Sect. 2.4 are taken into account, and, in particular, if it is assumed that $\mu = 90°$ and $\nu = 0°$. Then the parameters of motion of a NS in the plane of firing can be determined by solving the following nonlinear system of equations:

$$\left.\begin{aligned}
\dot{V} &= -\frac{c_x qS}{m} - g\sin\Theta;\\
\dot{\Theta} &= \frac{c_y qS}{mV} - \frac{g}{V}\cos\Theta + \frac{V}{r}\cos\Theta;\\
\dot{h} &= V\sin\Theta;\\
\dot{L} &= R\frac{V}{r}\cos\Theta;\\
\dot{\alpha} &= \omega_{y1} - \frac{c_y qS}{mV} + \frac{q}{V}\cos\Theta;
\end{aligned}\right\} \qquad (4.38)$$

$$\dot{\omega}_{y1} = \frac{c_n qS(x_T - x_d)}{J_{z1}} + \frac{m_{z\omega_z} qSl}{J_{z1}}.$$

It should be noted that during motion in the atmosphere the trajectory of the center of mass of a NS due to the effect of lift is periodically deflected from a certain center line (Fig. 4.5) by the frequency of the oscillations of the angle of attack.

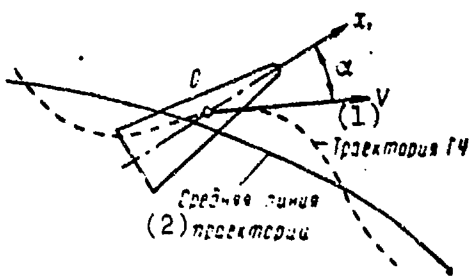


Fig. 4.5. The effect of the lift force of a nose section on the trajectory of its motion in the atmospheric phase.

KEY: (1) Trajectory of a NS; (2) Center line of the trajectory.

Let us examine further possible simplifications of the equations of motion of a NS in the atmosphere. Let us reduce the two latter equations from system (4.38) to one equation relative to the angle of attack α. In this case let us make the following simplifications:

1) disregarding the effect of flight path curvature due to the force of gravity on the variation in the angle of attack; let us set in the 5th equation of system (4.38) $\frac{g}{V}\cos\Theta\approx 0$;

2) the dependence of damping moment on angular velocity we shall consider to be linear, assuming

$$m_{zy_1} qSl = m_{y1}^{\omega y_1} \frac{qSl^2}{V} \dot{\omega}_{y1};$$

3) taking into account that the angle of attack varies many times more rapidly than the parameters of the motion of the center of mass and that coefficient c_y with sufficient accuracy can be considered proportional to the angle of attack when $\alpha < 60°$, let us calculate

$$\frac{d}{dt}\left(\frac{c_y qS}{mV}\right) \approx \frac{c_y^\alpha qS}{mV} \frac{d\alpha}{dt};$$

4) let us disregard the effect on the angular acceleration of a NS of component

$$\frac{m_{y1}^{\omega y1} qSl^2}{J_{z1} V} \frac{c_y qS}{mV} \text{ as compared with } \frac{c_n qS (x_T - x_d)}{J_z}.$$

Then we will obtain a second order equation which describes the oscillation of the angle of attack

$$\ddot{\alpha} - m_{y1}^{\dot{\alpha}} \frac{qSl^2}{J_{z1} V} \dot{\alpha} - c_n(\alpha) \frac{qS (x_T - x_d)}{J_{z1}} = 0, \qquad (4.39)$$

where $m_{y1}^{\dot{\alpha}} = m_{y1}^{\omega y1} - c_y^\alpha \frac{J_{z1}}{ml^2}$ – the coefficient of damping taking into account the flight path curvature due to the lift force of the NS.

The system of equations of motion of a NS in the atmospheric phase of the trajectory can now be written in the following form:

$$\left.\begin{aligned}
\dot{V} &= -\frac{c_x(\alpha) qS}{m} - g \sin\Theta;\\
\dot{\Theta} &= \frac{c_y(\alpha) qS}{mV} - \frac{g}{V}\cos\Theta + \frac{V}{r}\cos\Theta;\\
\dot{h} &= V \sin\Theta;\\
\dot{L} &= R\frac{V}{r}\cos\Theta;\\
\ddot{\alpha} - m_{y1}^{\dot{\alpha}} &\frac{qSl^2}{J_{z1} V}\dot{\alpha} - c_n(\alpha)\frac{qS (x_T - x_d)}{J_{z1}} = 0.
\end{aligned}\right\} \qquad (4.40)$$

The solution of this system of equations requires comparatively large expenditures of time because the frequency of the oscillations of the angle of attack of an NS is great (can attain 10 Hz) and during the numerical integration of the system of equations of (4.40) it is necessary to select an integration step ten times finer than in integrating the equations of motion of a NS with a zero angle of attack. For this reason the simplification of the system of equations of (4.40) is used by separating it into the equations of motion of the center of mass and into the equation of the oscillations of the angle of attack. In this case, in order to take into account the effect of the oscillations of the angle of attack on the motion of the center of mass of the NS, the aerodynamic coefficients for angle of attack α_{cp} averaged for the period of oscillations are calculated.

If the oscillations of the angle of attack are considered to be harmonic, i.e.

$$\alpha = A\sin\omega t,$$

and the aerodynamic coefficients equal to: $c_x = c_{x0} + k\alpha^2$; $c_y = c_y^{\alpha}\alpha$, then

$$c_x(\alpha_{cp}) = \frac{4}{T}\int_0^{\frac{T}{4}} (c_{x0} + kA^2\sin^2\omega t)\,dt =$$

$$= c_{x0} + k\frac{A^2}{2} = c_x\left(\frac{A}{\sqrt{2}}\right);$$

$$c_y(\alpha_{cp}) = \frac{1}{T}\int_0^{T} c_y^{\alpha}\sin\omega t\,dt = 0 = c_y(0).$$

Thus, $\alpha_{cp} = A/\sqrt{2}$ in calculating c_x and $\alpha_{cp} = 0$ in calculating c_y. The appropriate system of equations of the motion of the center of mass takes the simpler form:

$$\left.\begin{aligned}\dot{V}&=-\frac{qS}{m}c_x(\alpha_{cp})-g\sin\Theta;\\ \dot{\Theta}&=-\frac{g}{V}\cos\Theta+\frac{V}{r}\cos\Theta;\\ \dot{h}&=V\sin\Theta;\\ \dot{L}&=\frac{R}{r}V\cos\Theta\end{aligned}\right\} \quad (4.41)$$

and can be solved independently of the equation of the oscillations of the angle of attack, if the amplitude of these oscillations is given.

Let us now examine one of the methods of the approximate solution of the equation of the oscillations of the angle of attack (4.39), making it possible to determine amplitude A and frequency ω of the oscillations of the NS.

For angles of attack $\alpha \leq 60°$ the coefficient of the normal force of the NS can be with sufficient accuracy considered as a linear function of the angle of attack: $c_n=c_n^{\alpha}\alpha$. Then the equation of the oscillations (4.39) is written in the form

$$\ddot{\alpha}-m_{z1}^{\dot{\alpha}}\frac{qSl^2}{VJ_{z1}}\dot{\alpha}-c_n^{\alpha}(x_T-x_d)\frac{qS}{J_{z1}}\alpha=0. \quad (4.42)$$

Such a linear differential equation of the second order with variable coefficients has the following approximate solution:

$$\alpha=A_0\sqrt{\frac{\omega_0}{\omega}}\exp\left(\frac{Sl^2}{2J_{z1}}\int_0^t\frac{q}{V}m_{z_1 cp}^{\dot{\alpha}}dt\right)\sin\left(\int_0^t\omega\,dt+\xi\right). \quad (4.43)$$

where A_0 and ω_0 – the initial values of the amplitude of the angle of attack and of the frequency:

$\omega=\sqrt{\dfrac{-c_n^{\alpha}qS(x_T-x_d)}{J_{z1}}}$ – the frequency of the oscillations of the angle of attack; ξ – the initial phase; $m_{z_1 cp}^{\dot{\alpha}}$ – the coefficient of

damping (computed for angle of attack $\alpha_{cp} = \frac{A}{\sqrt{2}}$) averaged for the oscillation period.

Thus, the amplitude of the angle of attack at moment of time t is determined by the formula

$$A(t) = A_0 \sqrt{\frac{\omega_0}{\omega}} \exp\left(\frac{Sl^2}{2J_{z1}} \int_0^t \frac{q}{V} m_{z_1 cp}^{\dot{z}} dt\right). \qquad (4.44)$$

Calculating the Transverse Displacement of the Center of Mass of a Nose Section

Let us examine the equations of motion of a NS taking into account the displacement of the center of mass relative to the longitudinal axis. Such a displacement of the center of mass y_T leads to an increase in the balance angle of attack $\Delta\alpha_T$, whose magnitude can be determined from the condition of equilibrium of the moments of axial force X_1 and the additional transverse force ΔY_1 (Fig. 4.6):

$$Y_1^{\alpha}\Delta\alpha_T(x_T - x_d) - X_1 y_T = 0.$$

We will hence obtain

$$\Delta\alpha_T = \frac{y_T}{x_T - x_d}\frac{c_\tau}{c_n^{\alpha}}. \qquad (4.45)$$

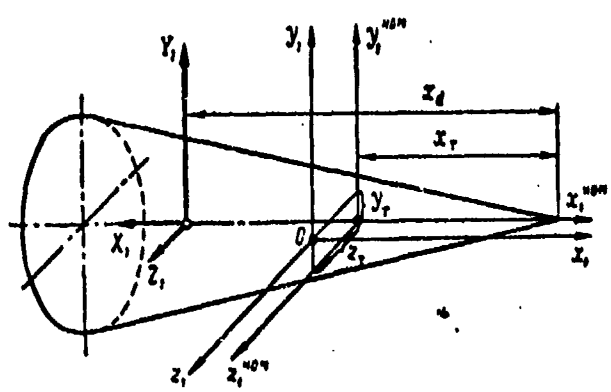


Fig. 4.6. For determining the additional aerodynamic moment acting on a nose section as a result of the displacement of its center of mass relative to the longitudinal axis.

In order to take into account the effect of the transverse displacement of the center of mass on the motion of the NS, it is necessary to introduce into the dynamic Euler equations projections of the additional aerodynamic moment brought about by the indicated displacement of the center of mass. Besides this, in the general case the calculation of the turning of the main central axes of inertia due to the transverse displacement of the center of mass can be required.

Let the center of mass of a NS have the coordinates x_T, y_T, z_T relative to the optimum body axes (see Fig. 4.6). In this case the projections of the additional aerodynamic moment on the axes of the body coordinate system will be equal to:

$$\left.\begin{aligned} M_{Tx_1} &= Y_1 z_T - Z_1 y_T; \\ M_{Ty_1} &= X_1 z_T; \\ M_{Tz_1} &= -X_1 y_T. \end{aligned}\right\} \tag{4.46}$$

Let us determine the components of aerodynamic force X_1, Y_1, Z_1, using the components of total aerodynamic force $\bar{R}$ along semi-body axes [see formula (2.92)] and the direction cosines between the semi-body and body axes. Then the expressions of the additional moments relative to the body coordinate axes $Ox_1y_1z_1$, caused by the displacement of the center of mass of the NS, take the following form:

$$\left.\begin{aligned} M_{Tx_1} &= c_n qS(-z_T \sin\nu + y_T \cos\nu); \\ M_{Ty_1} &= c_\tau qS z_T; \\ M_{Tz_1} &= -c_\tau qS y_T. \end{aligned}\right\} \tag{4.47}$$

Substituting these moments in the first three equations of system (4.37), we obtain the equations of motion of a NS taking into account the transverse displacement of the center of mass of the NS.

In the equation of the two-dimensional oscillation of a NS (4.39) due to the displacement of the center of mass an additional term appears

$$\doteq \frac{c_\tau qS}{J_{z1}} \sqrt{y_T^2 + z_T^2}.$$

Taking Wind into Account

Usually wind velocity is assigned a vertical W_B and horizontal $W_{\Gamma op}$ components and a horizontal component direction – wind azimuth ψ_W. The angle reckoned clockwise from the northern direction of the meridian to the direction from which the wind blows is usually called wind azimuth (Fig. 4.7). The vertical component of wind velocity directed upward is considered positive, that directed downward – negative.

Taking the velocity of the center of mass of the NS $\bar{V}$ as the absolute velocity, wind velocity $\bar{W}$ as drift velocity, and the velocity of the center of mass of the NS relative to the air taking wind into account (the so-called "airspeed") $\bar{V}_W$ as the relative velocity, we can write

$$\bar{V}=\bar{V}_W+\bar{W}. \tag{4.48}$$

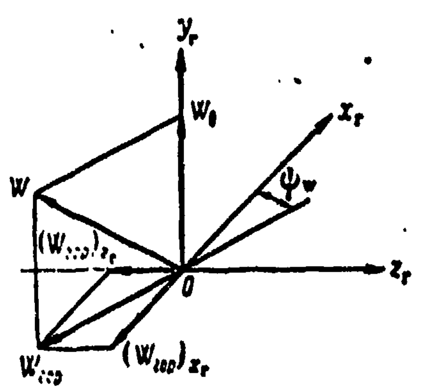


Fig. 4.7. The wind velocity vector and its projection on the axes of a geographical coordinate system.

The direction of velocity $\bar{V}$ in a terrestrial geographical coordinate system is given by two angles: by azimuth Ψ and by the angle of inclination to the horizon Θ. By analogy the direction of airspeed we will assign the angles Ψ_W and Θ_W (when there is no wind, angles Ψ_W and Θ_W coincide with angles Ψ and Θ).

For determining angles Ψ_W and Θ_W let us project the vectorial equality (4.48) in the horizontal and vertical plane (Fig. 4.8). From Fig. 4.8a it is evident that

$$V_{W\,rop}=\sqrt{V_{rop}^2+W_{rop}^2+2V_{rop}W_{rop}\cos(\Psi-\psi_W)}; \qquad (4.49)$$

$$\operatorname{tg}\Psi_W=\frac{V_{rop}\sin\Psi+W_{rop}\sin\psi_W}{V_{rop}\cos\Psi+W_{rop}\cos\psi_W};$$

$$\sin\Psi_W=\frac{V_{rop}\sin\Psi+W_{rop}\sin\psi_W}{V_{W\,rop}}, \qquad (4.50)$$

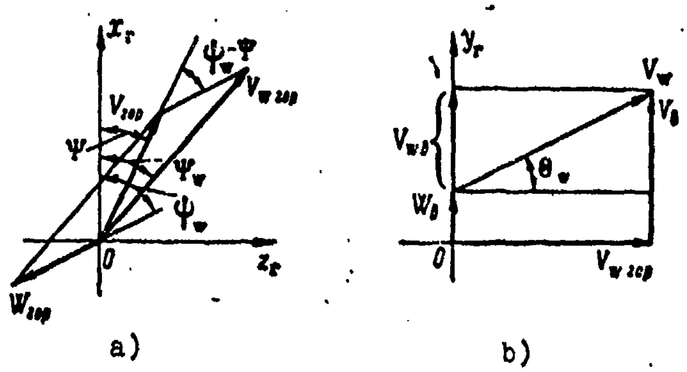


Fig. 4.8. For determining the angles of orientation of the airspeed vector relative to geographical coordinate axes: a) horizontal plane; b) vertical plane.

where

$$V_{rop}=V\cos\theta.$$

From Fig. 4.8b it follows that:

$$V_W=\sqrt{V_{rop}^2+W_{rop}^2+2V_{rop}W_{rop}\cos(\Psi-\psi_W)+(V_в-W_в)^2}; \qquad (4.51)$$

$$\theta_W=\arcsin\frac{V_в-W_в}{V_W}, \qquad (4.52)$$

where

$$V_в=V\sin\theta.$$

Let us determine the position of vector $\bar{V}_W$ in a wind coordinate system with the aid of angles ξ and η, where ξ – the angle included between vectors $\bar{V}$ and $\bar{V}_W$; η – the angle between the plane determined by vectors $\bar{V}$ and $\bar{V}_W$ and by coordinate plane Oxy (Fig. 4.9). From

spherical triangle ABC we find:

$$\xi = \arccos[\sin\Theta_W \sin\Theta + \cos\Theta_W \cos\Theta \cos(\Psi_W - \Psi)]; \quad (4.53)$$

$$\sin\eta = \frac{\sin(\Psi_W - \Psi)\cos\Theta_W}{\sin\xi};$$

$$\cos\eta = \frac{\sin\Theta_W - \sin\Theta\cos\xi}{\cos\Theta\sin\xi}. \quad (4.54)$$

Let us determine the angle of attack of a NS taking into account the wind effect α_W (the angle included between the longitudinal axis of the NS and the airspeed vector $\overline{V}_W$), for which let us examine Fig. 4.10. From spherical triangle ABC we will obtain

$$\cos\alpha_W = \cos\alpha\cos\xi + \sin\alpha\sin\xi\cos\zeta, \quad (4.55)$$

where

$$\zeta = 180° - \mu - (90° - \eta) = 90° + \eta - \mu.$$

Knowing the angle of attack α_W and velocity V_W, it is possible to determine the axial and normal aerodynamic forces:

$$\left.\begin{aligned} X_{1W} &= c_\tau(\alpha_W, M_W) q_W S; \\ Y_{1W} &= c_n(\alpha_W, M_W) q_W S, \end{aligned}\right\} \quad (4.56)$$

where

$$M_W = \frac{V_W}{a}; \quad q_W = \frac{\rho V_W^2}{2}. \quad (4.57)$$

Normal force Y_{1W} is directed perpendicular to axis Ox_1 in the plane of the air angle of attack α_W, constituting with the plane of angle α angle ι (ι = 0-180°). Let us determine angle ι from spherical triangle ABC:

$$\left.\begin{aligned} \sin\iota &= \sin\xi\frac{\sin\zeta}{\sin\alpha_W}; \\ \cos\iota &= \frac{\cos\xi - \cos\alpha_W\cos\alpha}{\sin\alpha_W\sin\alpha}. \end{aligned}\right\} \quad (4.58)$$

The components of aerodynamic force with respect to axes of a semi-body coordinate system are equal to:

$$\left.\begin{aligned} X'_W &= X_{1W}; \\ Y'_W &= -Y_{1W}\sin\iota; \\ Z'_W &= -Y_{1W}\cos\iota. \end{aligned}\right\} \quad (4.59)$$

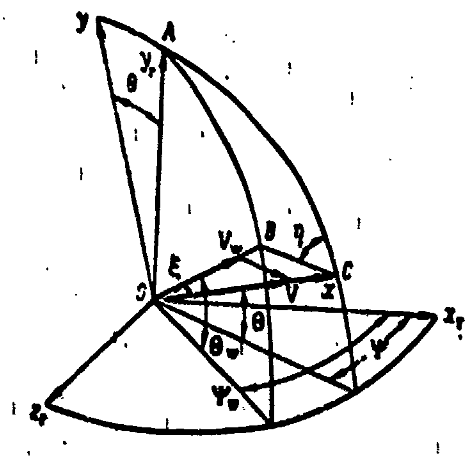


Fig. 4.9. The orientation of the airspeed vector relative to a semi-wind coordinate system.

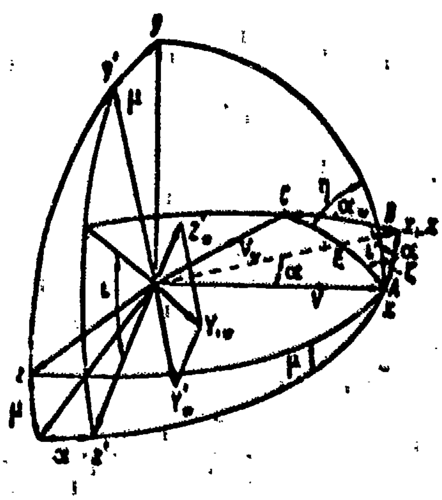


Fig. 4.10. For determining the angle of attack of a nose section taking wind effect into account.

Let us determine the projections of the aerodynamic forces taking wind effect into account on the axes of a semi-wind coordinate system, using direction cosines written in matrix form (2.71):

$$\left.\begin{aligned} X &= X_{1W}\cos\alpha - Y_{1W}\cos\iota\sin\alpha; \\ Y &= X_{1W}\sin\alpha\sin\mu - Y_{1W}\sin\iota\cos\mu + Y_{1W}\cos\iota\cos\alpha\sin\mu; \\ Z &= -X_{1W}\sin\alpha\cos\mu - Y_{1W}\sin\iota\sin\mu - \\ &\quad - Y_{1W}\cos\iota\cos\alpha\cos\mu. \end{aligned}\right\} \quad (4.60)$$

Let us express the projections of the aerodynamic forces taking wind effect into account on body axes through their components with respect to semi-body axes. Then we will obtain:

$$\left.\begin{aligned} X_1 &= X_{1W}; \\ Y_1 &= -Y_{1W}\sin\iota\cos\nu - Y_{1W}\cos\iota\sin\nu; \\ Z_1 &= Y_{1W}\sin\iota\sin\nu - Y_{1W}\cos\iota\cos\nu. \end{aligned}\right\} \quad (4.61)$$

These components of aerodynamic force cause the aerodynamic moment whose projections on the axes of a body coordinate system taking expressions (4.46) into account have the form:

$$\left.\begin{aligned} M_{x1} &= Y_1 z_r - Z_1 y_r; \\ M_{y1} &= X_1 z_r - Z_1(x_r - x_d); \\ M_{z1} &= -X_1 y_r + Y_1(x_r - x_d). \end{aligned}\right\} \quad (4.62)$$

It is also necessary to determine aerodynamic damping moments taking wind effect into account:

$$\left.\begin{aligned} M_{dx_1} &= m_{x1}^{\omega_{x1}}(M_W)\frac{q_W S D^2}{V_W}; \\ M_{dy_1} &= m_{y1}^{\omega_{y1}}(M_W)\frac{q_W S h}{V_W}. \end{aligned}\right\} \quad (4.63)$$

Substituting the obtained expressions of aerodynamic forces and moments (4.60), (4.62) and (4.63) into equations (2.97) and (2.100), we obtain the dynamic equations of motion of a NS taking wind effect and the displacement of the center of mass of the NS into account. These equations together with the kinematic relationships (2.98), (2.99), and (2.108) make up the system of equations of the motion of

a NS in the atmosphere taking wind effect and the displacement of the center of mass of the NS relative to the longitudinal axis into account. From such a general system in particular cases simpler systems of equations can be obtained. The assumptions made in this case, are determined by the conditions of the actual problem being solved.

Initial Conditions of Motion

For solving differential equations of the three-dimensional of motion of a NS, for example equations (4.36) and (4.37), it is necessary to know twelve parameters of the motion of a NS at an initial moment of time. These initial conditions are taken from the results of the calculations of the parameters of the motion of a rocket in the powered-flight phase of the trajectory and of the process of the separation of a NS from the last stage. For a two-dimensional case of motion of a NS in the atmospheric phase of the trajectory [the equations of (4.38) or (4.40)] the number of the initial conditions of motion is reduced to six.

In firing a rocket over various distances from minimum to maximum under various geographical launch conditions the parameters of the motion of the center of mass of the NS at the moment of reentry into the atmosphere form a certain region of the initial conditions of motion.

If it is assumed that the reentry of the NS into the atmosphere occurs at a certain arbitrary altitude h_0, and the initial distance of the atmospheric phase of the trajectory corresponding to this altitude, is set equal to zero ($L_0 = 0$), then we will obtain the region of the initial conditions V_0, Θ_0 of the motion of the center of mass of the NS at altitude h_0.

Figure 4.11 depicts one of the possible regions of the initial conditions of motion of the center of mass of the NS at altitude of reentry into the atmosphere. In the given region of initial conditions V_0, Θ_0 line 1-2 corresponds to firing over minimum range, line 3-4 – over maximum, line 2-3 corresponds to firing eastwards, 1-4 – to firing westwards.

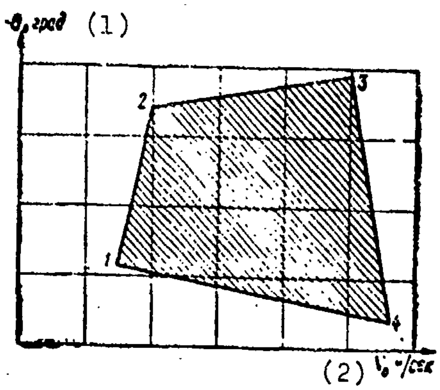


Fig. 4.11. The region of the initial conditions of motion of the center of mass of a nose section upon reentry into the atmosphere.
KEY: (1) Deg; (2) m/s.

In the case of two-dimensional oscillations of a NS in the atmosphere the initial conditions of the oscillations of the NS are given by the two parameters: α_0 and $\dot{\alpha}_0$. Instead of these two parameters it is possible to assign only one parameter α_0, because for any pair of initial conditions of oscillations α_0 and $\dot{\alpha}_0 \neq 0$ it is always possible to select such α_0' and $\dot{\alpha}_0'=0$, that the amplitude of the angles of attack of NS will be the same as under the initial conditions α_0 and $\dot{\alpha}_0 \neq 0$. Thus in solving practical problems, for example in investigating maximum transverse overloads or maximum deviations in the parameters of motion of a NS due to oscillations of the angle of attack, it is convenient to give the initial conditions of the oscillations in the form $\alpha_0=\alpha_0'$ and $\dot{\alpha}_0'=0$, so that with all possible values of α_0 and $\dot{\alpha}_0$ the angles of attack of a NS do not emerge with the given probability B beyond the limits of the envelope of the angles of attack obtained when $\alpha_0=\alpha_0'$ and $\dot{\alpha}_0 = 0$.

For this certain values of α_0', are assigned, envelope of angles of attack A(t) is determined and the probability of the appearance of angles of attack, not exceeding value A(t) is found. If the obtained probability is small, a new value of α_0', is selected, smaller than the previous one, the indicated probability is again determined, etc.

For determining probability $B(\alpha_0')$ of the appearance of angles of attack, not exceeding the calculated values of A(t), it is possible to propose various methods. Let us examine two of these: the method based on using the distributive laws of random values α_0 and $\dot{\alpha}_0$ at

the moment of reentry into the atmosphere (the method of deriving the regions of the adverse initial conditions of oscillations), and the method of statistical testing.

The Method of Deriving the Regions of Adverse Initial Conditions of Oscillations

Let us call the adverse initial conditions of the oscillations those initial conditions of α_0 and $\dot{\alpha}_0$, in which the angles of attack emerge at corresponding altitudes beyond the limits of envelope A, obtained for the accepted initial conditions of $\alpha_0^{\bullet}$, $\dot{\alpha}_0^{\bullet} = 0$ (when $h = h_0^{\bullet}$). All the remaining combinations of α_0 and $\dot{\alpha}_0$ are favorable.

The determination of the probability of the appearance of favorable initial conditions of oscillations is carried out in the following sequence:

1) the parameters of the motion of a NS in the atmospheric phase of the trajectory for the accepted calculated initial conditions of oscillations $\alpha_0^{\bullet}$, $\dot{\alpha}_0^{\bullet} = 0$, $q = q_0^{\bullet}$ are calculated and dependences A(t), h(t), V(t) are determined;

2) from the region of the possible initial conditions of oscillations at the moment of reentry into the atmosphere at an altitude of h_0 a number of values of α_0, $\dot{\alpha}_0$ is assigned and, integrating equation (4.39), such values of α_0, $\dot{\alpha}_0$ are selected which lead to oscillations in the angle of attack with amplitude A(t). Such initial conditions of α_0, $\dot{\alpha}_0$ are equivalent to the initial conditions $\alpha_0^{\bullet}$ and $\dot{\alpha}_0^{\bullet} = 0$.

A locus of equivalent initial conditions of oscillations forms in the region of the possible values of parameters in α_0 and $\dot{\alpha}_0$ lines limiting the region of the adverse initial conditions of the oscillations. The approximate form of the region of adverse initial conditions at altitude h_0 is depicted in Fig. 4.12 (region D of the adverse initial conditions of oscillations is shaded);

3) the probability P of the fact that the initial conditions of α_0, $\dot{\alpha}_0$ fall into region (D) of the adverse initial conditions of oscillations is determined:

$$P=\int\limits_D\int f(\alpha_0,\ \dot{\alpha}_0)\,d\alpha_0\,d\dot{\alpha}_0,$$

where $f(\alpha_0,\ \dot{\alpha}_0)$ – the distribution density of the probabilities of random variables α_0 and $\dot{\alpha}_0$.

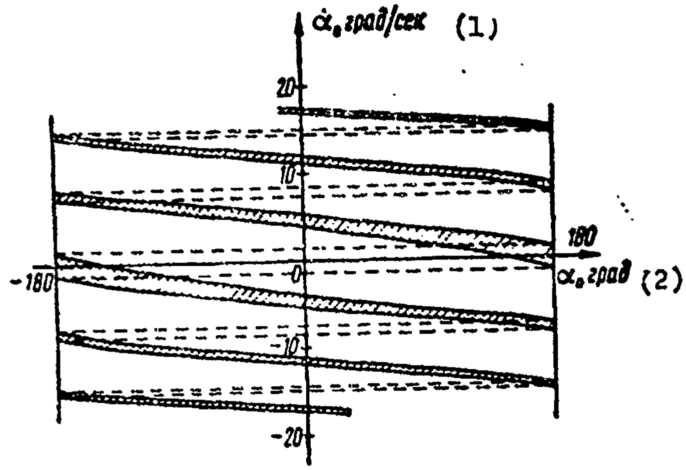


Fig. 4.12. The approximate form of a region (shaded) of adverse initial conditions of oscillations of a nose section upon reentry into the atmosphere.

KEY: (1) deg/s; (2) deg.

Let us assume that the angle of attack of a NS at altitude h_0 obeys the law of uniform probability density $f(\alpha_0) = 1/360°$ (i.e., the NS performs two-dimensional motion and in the course of unpowered flight makes several revolutions), and the angular velocity – the normal distribution law:

$$f(\dot{\alpha}_0)=\frac{1}{\sigma_{\dot{\alpha}0}\sqrt{2\pi}}\,e^{-\frac{(\dot{\alpha}_0-m_{\dot{\alpha}_0})^2}{2\sigma^2_{\dot{\alpha}0}}},$$

and random variables α_0 and $\dot{\alpha}_0$ are independent, i.e.,

$$f(\alpha_0,\ \dot{\alpha}_0)=f(\alpha_0)\,f(\dot{\alpha}_0).$$

Then the probability of the appearance of adverse initial conditions of oscillations is determined by the formula

$$P=\frac{1}{360\sigma_{\dot\alpha 0}\sqrt{2\pi}}\iint\limits_{D} e^{-\frac{-(\dot\alpha_0-m_{\dot\alpha 0})^2}{2\sigma^2_{\dot\alpha 0}}}\,d\alpha_0 d\dot\alpha_0.$$

In calculations using this formula region D is broken down into a number of elementary regions D_i with width $\Delta\alpha_0$ by straight lines, parallel to axis $O\dot\alpha_0$. Then the probability of adverse initial conditions of oscillations of the NS will be equal to:

$$P=\sum_{i=1}^{n} P_i;$$

$$P_i=\frac{\Delta\alpha_0}{2\cdot 360}\left[\Phi\left(\frac{\dot\alpha_{0\text{в}}-m_{\dot\alpha 0}}{\sigma_{\dot\alpha 0}\sqrt{2}}\right)_i-\Phi\left(\frac{\dot\alpha_{0\text{н}}-m_{\dot\alpha 0}}{\sigma_{\dot\alpha 0}\sqrt{2}}\right)_i\right],$$

where n – the number of elementary regions D_i; Φ – Laplace's function; $\dot\alpha_{0\text{в}}$ and $\dot\alpha_{0\text{н}}$ – the ordinates of the middles of the upper and lower boundaries of elementary region D_i respectively.

With values α_0 and $\dot\alpha_0$, corresponding to a region of favorable initial conditions, the motion of the NS occurs with angles of attack, not exceeding calculated A(t). The probability of such motion is equal to

$$B=1-P.$$

The value of probability B obtained by this method is approximate because the actual distributive law of random initial conditions $f(\alpha_0, \dot\alpha_0)$ can differ from the accepted one. Furthermore, this method does not make it possible to take the random variances of the characteristics of the NS and of the parameters of the atmosphere into account. For determining a more precise value of probability B it is possible to employ other methods, for example the method of statistical testing.

The Method of Statistical Testing

For determining probability B of favorable initial conditions of oscillations of a NS by the method of statistical testing on digital computers N calculations of the parameters of the motion of the NS are carried out with random combinations of parameters of the NS, the atmosphere, wind characteristics and other factors determining the angular motion of the NS. Then the distribution function f(A) of the amplitude of the angles of attack of the NS at certain altitude h is constructed and with this distribution function the probability B of the fact that the calculated value of the amplitude of the angles of attack A(h) will not be exceeded is determined.

For the purpose of reducing the expenditures of machine time it is necessary to use more of the less simplified equations of motion of the NS, introducing various assumptions, which practically do not effect the maximum angles of attack, for example, the assumption that the motion of the NS is two-dimensional.

CHAPTER V.

DISPERSION OF NOSE SECTION IMPACT POINTS

5.1. GENERAL ASPECTS

As a result of the effect of various perturbations the actual trajectory of a rocket and its nose section never coincides with the optimum and the point of impact of a nose section is unavoidably deflected from the precalculated aiming point by a certain random variable. This phenomenon is called dispersion.

In firing against ground-based targets the random deviation of the point of impact of a nose section from a target is characterized by two random variables – by the abscissa and by the ordinate of the point of impact on a certain coordinate plane called the plane of dispersion.

Let us define the plane of dispersion as a plane tangent to the terrestrial ellipsoid drawn through the target. On this plane let us construct the Cartesian coordinate system OLZ (Fig. 5.1). On the figure there are designated: A – the launch point; O – the target; the trajectory AK*O – optimum, corresponding to the calculated time of separation of the NS t_{κ}^{*}.

Leaving the surrounding conditions constant, let us vary the time of separation of the NS; for a certain moment of time $t_{\kappa} = t_{\kappa}^{*} + \Delta t_{\kappa}$ the flight path will be AKO'. The locus of the intersection of the optimum trajectories which are characterized by different moments of

the separation of a NS, with the plane of dispersion will occur in the form of a certain curve OO'. Let us draw in the plane of dispersion through point O an axis, tangent to curve OO', and let us orient it in the direction of an increase in range; let us designate this axis OL and we reckon range error ΔL along it. In connection with the fact that axis OZ is perpendicular to axis OL, small variation in time $t_{\text{н}}$ gives rise to the deflection of the point of impact of a nose section from a target along axis OZ by a magnitude of the second order of smallness as compared with magnitude ΔL, i.e., the derivative of the lateral coordinate of the point of impact of a NS with respect to time $t_{\text{н}}$ is approximately equal to zero. This characteristic is very convenient in dispersion calculations.

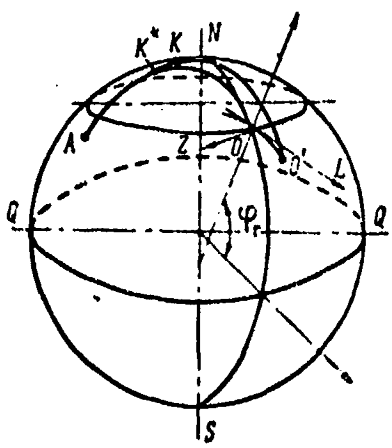


Fig. 5.1. Coordinate system for determining the dispersion of nose section impact points: O – the origin of the coordinates (the impact point of a nose section under optimum conditions, or the target).

The coordinate system OLZ, constructed by the indicated manner, we will call an arbitrary coordinate system.

All the perturbing factors affecting the dispersion of nose sections, can be divided into two groups. Included in the first group are perturbing factors acting in the powered-flight phase of the trajectory, in the second – the perturbing factors acting in the unpowered-flight phase.

In the powered-flight phase of trajectories, where the motion of a rocket is guided, the deviations of the actual values of the

parameters of motion from the calculated values are due, mainly, to control system errors. In the unpowered-flight phase, where the motion of a nose section is unguided, the perturbing factors are the errors made in the manufacture of the NS, differences in the actual composition of the atmosphere from the rated and variances in the initial conditions (upon reentry into the atmosphere) of motion of the NS around its center of mass.

The difference in the conditions of motion of a rocket in the powered-flight phase and of its nose section in the unpowered-flight phase leads to a different approach in determining firing accuracy and in developing measures increasing this accuracy.

For reducing the dispersion of nose section impact points brought about by deviations in actual motion of a rocket from the optimum in the powered-flight phase of the trajectory, the ideal control system should completely take into account the deviations in the parameters of motion of the rocket from the optimum and compensate for these deviations in such a way as to reduce their effect at the end of the flight of the NS to the target to zero. However, unavoidable errors in the operation of individual elements of a control system, caused by manufacturing inaccuracies and by operating conditions, give rise to errors in determining the coordinates and the projections of rocket speed. Furthermore, for the purpose the simplifying equipment (for reducing its weight, cost, increasing its reliability, etc.) the algorithm for processing data concerning the motion of the rocket is frequently simplified, which also gives rise to certain inaccuracies in determining the parameters of motion of a rocket.

It is natural, that success in solving the problems of a NS impacting on a target by an actual control system depends on how completely the control algorithm takes into account all possible factors affecting firing accuracy. Incompleteness in taking into account the perturbing factors affecting firing accuracy by the control system, gives rise to the appearance of so-called "systematic error." This error arises as a result of the fact that parameters, not directly connected with the flight range of a NS – the components

of apparent velocity and apparent path, are controlled by guidance systems instruments. If a certain perturbation caused a variation in trajectory, then deviation in the impact point of the NS appears despite the fact that the parameters controlled by the guidance system at the moment of NS separation are exactly equal to the required values. An example of systematic error is the error caused by the disregarding the effect of lateral deviations in firing range. For reducing systematic errors it is usually necessary to complicate the control algorithm and consequently, the control system itself.

As with any automatic device, a control system has a certain instrumental error, which includes the following components:

measuring equipment errors – zero drift, transmission coefficient inaccuracies, gyroscopic drifts, initial orientation errors;

computers error – roundoff errors, approximation errors, functional unit errors (adders, multipliers, integrators), coordinate transformer errors.

When a control system is made more complex its instrumental error usually increases. Thus, for a correct approach to selecting algorithms for controlling the range and the heading of a NS both instrumental and systematic errors should be taken into account.

5.2. NOSE SECTION DISPERSION, CAUSED BY PERTURBATIONS IN THE UNPOWERED-FLIGHT PHASE OF THE TRAJECTORY

During the motion of nose sections in the unpowered-flight phase of the trajectory the main perturbing factors causing deviations of the points of impact from the optimum, act in the atmospheric section. Included in these, in the first place, are the following:

– variances in the atmosphere parameters;

– wind;

– deviations in the characteristics of the NS (weight, geometric,

centering [c.g.], aerodynamic and others);

– variances in the initial conditions of the angular motion of a NS in the atmosphere.

Let us examine the effect of each of the cited perturbing factors.

Of the atmospheric parameters variances in density and air temperature have a basic effect on impact point dispersion. A peculiarity of these perturbing factors is the fact that in various phases of NS motion in the atmosphere they have, as a rule, different, sometimes even values opposite in sign. For instance, in winter time the density of the atmosphere near the surface of the earth is higher than normal (standard), and at altitudes of more than 6-10 km – below. Negative deviations in atmospheric density at high altitudes cause positive deviations in flight range; positive deviations in density near the surface of the earth – negative deviations in range. The total deviation in flight range due to variance in atmospheric density depends on the characteristics of the NS and on the actual values of the atmospheric parameters.

For calculating nose section dispersion it is possible to give the deviations in the density and the temperature of the atmosphere, which are random functions of altitude, in the form of a canonical expansion (see Sect. 1.2).

Wind effect can cause deviations in NS impact points both with respect to range and direction. Wind velocity and direction are also the random functions of altitude which for calculating NS dispersion can also be given in the form of a canonical expansion.

Let us group the deviations in NS characteristics from the optimum with respect to their effect on the deflections of impact points in the following manner:

– deviations in weight (G), in midsection area (S) and in the coefficient of aerodynamic drag (c_x) from the calculated values

(variance in the so-called ballistic coefficient $\sigma_x = c_x S/G$);

– transverse displacement of the center of mass of a NS relative to the geometric axis of symmetry.

Deviations in the other characteristics of a nose section (for instance, inertial moments, length and others) insignificantly affect impact point dispersion.

An increase in the ballistic coefficient σ_x causes negative deviations in range, a decrease – positive. Transverse displacement of the center of mass of a NS $r_T = \sqrt{y_T^2 + z_T^2}$ causes motion of the NS with a certain balance angle of attack (4.45), the orientation of the plane of which relative to the plane of firing depends on the angle of spin ν at a given moment of time. In connection with the fact that during the motion of unguided nose sections the value of angle ν is arbitrary, the deviations in the impact point due to transverse displacement of the center of mass can be both with respect to range and direction.

For a NS, rotating around the longitudinal axis, the effect of the transverse displacement of the center of masses on impact point dispersion is substantially reduced and for a certain value of angular spin it can be practically reduced to zero.

During the motion of a NS with angles of attack arising due to disoriented reentry into the atmosphere, an increase occurs in the aerodynamic drag of the NS, which gives rise to a negative deviation in range, the value of which depends on the amplitude of oscillations of the angle of attack; in this case the smallest negative deviation in range corresponds to motion with zero angles of attack, the greatest negative deviation – to motion with a maximum amplitude of the oscillations of the angle of attack (corresponding to a given probability).

For calculating NS dispersion due to the factors acting in the unpowered-flight phase of the trajectory, it is possible to carry out

numerical integration (with a digital computer) of the system equations of the perturbed motion of a NS of the type of (4.36)-(4.37), individually evaluating the effect of every perturbing factor on the coordinates of the impact point.

Assuming that the indicated perturbing factors are independent of one another, and the deviations in the impact points are proportional to the magnitudes of the perturbations, the maximum values of the deviations in NS impact points are determined by geometric summing of the maximum deviations caused by each perturbing factor:

$$\left.\begin{aligned}\Delta_{n}L &= \sqrt{[\Delta L(\Delta\rho,\ \Delta T)]^2+[\Delta L(W)]^2+[\Delta L(a_x)]^2+[\Delta L(r_\tau)]^2+[\Delta L(\alpha_0,\ \dot{\alpha}_0)]^2};\\ \Delta_{n}Z &= \sqrt{[\Delta Z(W)]^2+[\Delta Z(r_\tau)]^2}.\end{aligned}\right\} \quad (5.1)$$

For a stricter and more exact determination of the deviations in the coordinates of NS impact points due to the factors acting in the unpowered-flight phase of the trajectory, it is possible to use other statistical methods, for example the method of statistical testing.

5.3. INERTIAL CONTROL OF THE FLIGHT RANGE AND DIRECTION OF A NOSE SECTION

Formulation of the Problem

In investigating an inertial system for guiding a rocket it is advantageous to examine it relative to an inertial coordinate system. In this case the dependence of the parameters of the motion of a rocket on the measurable components of acceleration is considerably simplified and the analysis of guidance system errors is facilitated.

Let us place the origin of the inertial coordinate system at the center of the earth, and let us orient the coordinate axes parallel to the corresponding axes of an initial launch coordinate system. Between the parameters of rocket motion, assigned relative to inertial

and launch coordinate systems, the following relationships occur:

$$\bar{r}_c = \bar{r} - \bar{R}_0;$$
$$\bar{V} = \bar{V}_a - \bar{\omega}_3 \times \bar{r},$$

where $\bar{r}_c$, $\bar{r}$ – the radius-vectors determining the position of the center of mass of the rocket relative to the launch point and relative to the center of the earth, respectively; R_0 – the radius-vector determining the position of the launch point relative to the center of the earth; $\bar{V}$, $\bar{V}_a$ – the velocity vectors of the center of mass of the rocket determined relative to launch and inertial coordinate systems, respectively; $\bar{\omega}_3$ – the angular velocity vector of the diurnal rotation of the earth.

Let us define firing range as the distance measured along the arc of the great circle between the launch point and the intersection of the descending leg of the flight path of the rocket with the surface of the terrestrial ellipsoid.

As is known, firing range is uniquely determined by the parameters of rocket motion relative to a launch coordinate system at moment $t_к$ of NS separation

$$L = L[x(t_к), y(t_к), z(t_к), V_x(t_к), V_y(t_к), V_z(t_к)]$$

or by the parameters of rocket motion relative to an inertial coordinate system at the moment of NS separation and by the duration $t_к$ of the rocket flight up to this moment

$$L = L[\xi(t_к), \eta(t_к), \zeta(t_к), V_\xi(t_к), V_\eta(t_к), V_\zeta(t_к), t_к]. \qquad (5.2)$$

Subsequently to reduce notation let us also use the following designations for the parameters of motion:

$$q_1 = \xi,\ q_2 = \eta,\ q_3 = \zeta,\ \dot{q}_1 = V_\xi,\ \dot{q}_2 = V_\eta,\ \dot{q}_3 = V_\zeta.$$

If in expression (5.2) it is assumed that the parameters of the end of the powered-flight phase are equal to their calculated (optimum) values:

$$\left.\begin{aligned} q_i(t_\kappa)&=q_i^*(t_\kappa^*);\\ \dot{q}_i(t_\kappa)&=\dot{q}_i^*(t_\kappa^*);\\ t_\kappa&=t_\kappa^*,\end{aligned}\right\} \tag{5.3}$$

that we will obtain the calculated firing range

$$L^*=L[q_i^*(t_\kappa^*),\ \dot{q}_i^*(t_\kappa^*),\ t_\kappa^*]. \tag{5.4}$$

In fulfilling the conditions of (5.3) the separation of the NS can be accomplished at calculated moment of time t_κ^*. However, in actual flight due to the effect on the rocket of the perturbing factors the parameters of motion at the calculated moment of time t_κ^* will differ from the calculated parameters and, thus,

$$L[q_i(t_\kappa^*),\ \dot{q}_i(t_\kappa^*),\ t_\kappa^*]\neq L^*.$$

Condition (5.4) apparently, does not determine the values of each of the magnitudes $q_i(t_\kappa)$, $\dot{q}_i(t_\kappa)$, t_κ; it only requires, that for achieving range L^* their set satisfy relationship (5.4); generally speaking, there can be an infinite number of such sets. On the other hand, for each actual trajectory of a powered-flight phase because of the unique conditions only one set $(q_i(t_\kappa),\ \dot{q}_i(t_\kappa),\ t_\kappa)$ corresponds to calculated range L^*. These considerations indicate one of the methods of controlling firing range: it is necessary to stop the powered-flight phase of the trajectory, more precisely speaking, to separate the NS, at moment $t = t_\kappa$, when function $L[q_i(t),\ \dot{q}_i(t),\ t]$ reaches the desired value of L^*. The realization of the indicated method can be accomplished with the aid of a certain system of measuring and computing devices determining the current values of $q_i(t)$ and $\dot{q}_i(t)$ with subsequent calculation of function $L[q_i(t),\ \dot{q}_i(t),\ t]$ and by continuous comparison of its current value L with assigned L^*. At the moment of time, when the equality is fulfilled

$$L[q_i(t),\ \dot{q}_i(t),\ t]=L^*, \tag{5.5}$$

instruction is supplied for the separation of the nose section.

Let us now formulate a general statement of the problem of selecting the time for NS separation. This moment should be determined from the current values of the measured parameters of motion of the center of mass of the rocket. With a computer of greater or lesser complexity it is possible to calculate the value of certain function J from the current parameters of motion and to separate the NS, when this value becomes equal to the required value. Function J, with the aid of which the moment of NS separation is determined, we will subsequently call the *controlling functional*, or simply the *functional*. (In a number of works the terms "ballistic function", or "controlling function" are also used).

The value of the controlling functional at a certain moment of time should be directly connected with the magnitude of the firing error which would arise, if NS separation occurred at this moment of time. The control system should emit the signal for NS separation when the functional attains the value corresponding to the zero (practically minimal) value of the mentioned error.

One of the possible controlling functionals is the firing range itself expressed by the current coordinates of the rocket and the projections of its velocity:

$$J_L = L[q_i(t), \dot{q}_i(t), t]. \tag{5.6}$$

Using this expression, it is possible to represent firing error in the form

$$\Delta L = L[q_i(t_K), \dot{q}_i(t_K), t_K] - L^* = J_L - J_L^*. \tag{5.7}$$

The control equations (5.5) and the firing error (5.7) correspond to functional (5.6). If we separate the NS at the moment of the fulfillment of condition (5.5), then, naturally, there will be no range error due to perturbation in the powered-flight phase.

However, the possibilities of plotting functional (5.6) are limited, especially, by the limited operating speed of computing equipment. Thus, the functional is usually used which is obtained as

a result of the expansion of function $L[q_i(t_к), \dot{q}_i(t_к), t_к]$ into a Taylor series in the vicinity of the calculated values of its arguments.

Let $q_i^*(t)$ be the calculated variation in the i-th parameter of motion with respect to time and $t_к^*$ be the calculated moment of time of NS separation; $q_i(t)$ – the actual variation in the i-th parameter and $t_к$ – the actual moment of NS separation (Fig. 5.2).

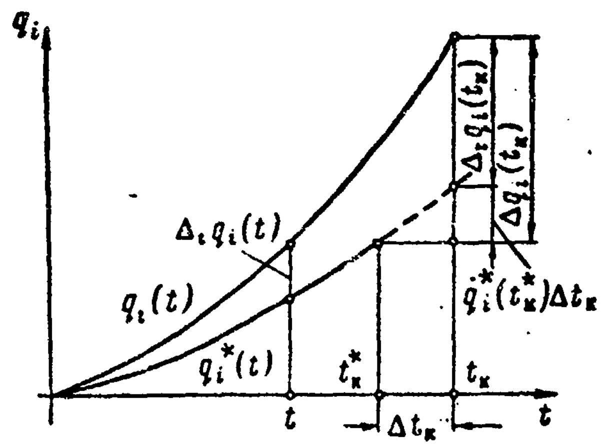


Fig. 5.2. For determining the complete and isochronal variations in the parameters of rocket motion.

The difference

$$\Delta q_i(t_к) = q_i(t_к) - q_i^*(t_к^*) \tag{5.8}$$

we will call the total variation in parameter q_i.

Analogously

$$\Delta \dot{q}_i(t_к) = \dot{q}_i(t_к) - \dot{q}_i^*(t_к^*). \tag{5.8a}$$

With an accuracy of terms of the second order of smallness relative to the total variations we have

$$\Delta L = L - L^* = \frac{\partial L}{\partial \xi}\Delta\xi + \frac{\partial L}{\partial \eta}\Delta\eta + \frac{\partial L}{\partial \zeta}\Delta\zeta + \frac{\partial L}{\partial V_\xi}\Delta V_\xi + \frac{\partial L}{\partial V_\eta}\Delta V_\eta + \frac{\partial L}{\partial V_\zeta}\Delta V_\zeta + \frac{\partial L}{\partial t}\Delta t_к, \tag{5.9}$$

where $\Delta t_{\text{к}} = t_{\text{к}} - t^*_{\text{к}}$ – the variation in the moment of time of NS separation.

The partial derivatives in this expression (ballistic coefficients) are determined for the calculated values of variables $\xi^*(t^*_{\text{к}})$, $\eta^*(t^*_{\text{к}}), \ldots, V^*_{\zeta}(t^*_{\text{к}})$, $t^*_{\text{к}}$ (usually by calculations on a digital computer.

Analogous to expression (5.9) the deflection ΔZ of the point of impact of the NS from the firing plane is written

$$\Delta Z = \frac{\partial Z}{\partial \xi}\Delta\xi + \frac{\partial Z}{\partial \eta}\Delta\eta + \frac{\partial Z}{\partial \zeta}\Delta\zeta + \frac{\partial Z}{\partial V_{\xi}}\Delta V_{\xi} + \\ + \frac{\partial Z}{\partial V_{\eta}}\Delta V_{\eta} + \frac{\partial Z}{\partial V_{\zeta}}\Delta V_{\zeta}. \tag{5.10}$$

Taking the expressions for variations (5.8) into account, let us write the conditions for the equality to zero of relationship (5.9) and (5.10) in the form

$$\Delta L = J_L - J^*_L = 0 \text{ или } J_L = J^*_L; \tag{5.11}$$

$$\Delta Z = J_Z - J^*_Z = 0 \text{ или } J_Z = J^*_Z, \tag{5.12}$$

where

$$\left.\begin{aligned} J_L &= \sum_{i=1}^{3} \frac{\partial L^*}{\partial q_i} q_i(t) + \sum_{j=1}^{3} \frac{\partial L}{\partial \dot{q}_j} q_j(t) + \frac{\partial L}{\partial t} t; \\ J^*_L &= \sum_{i=1}^{3} \frac{\partial L}{\partial q_i} q^*_i(t^*_{\text{к}}) + \sum_{j=1}^{3} \frac{\partial L}{\partial \dot{q}_j} q^*_j(t^*_{\text{к}}) + \frac{\partial L}{\partial t} t^*_{\text{к}}; \end{aligned}\right\} \tag{5.13}$$

$$\left.\begin{aligned} J_Z &= \sum_{i=1}^{3} \frac{\partial Z}{\partial q_i} q_i(t) + \sum_{j=1}^{3} \frac{\partial Z}{\partial \dot{q}_j} \dot{q}_j(t); \\ J^*_Z &= \sum_{i=1}^{3} \frac{\partial Z}{\partial q_i} q^*_i(t^*_{\text{к}}) + \sum_{j=1}^{3} \frac{\partial Z}{\partial \dot{q}_j} \dot{q}^*_j(t^*_{\text{к}}). \end{aligned}\right\} \tag{5.14}$$

Functional J_L we will call the *functional flight range control,* and J_Z – the *functional of flight heading control.* Values J_L^* and J_Z^* we will call the adjustment values of the corresponding funtionals.

If the difference in the actual motion of a rocket from the calculated motion is small, i.e., the variations in (5.8) are small, then upon separation of the NS at a moment of time which is the root of the equation (5.11), the deflection of the NS from the target with respect to range due to perturbations in the powered-flight phase will be a magnitude of the second order of smallness.

It is possible to always find such a moment of time of NS separation, when equality (5.11) is fulfilled; generally speaking it is not possible to attain the fulfillment of equality (5.12), only by varying the time of NS separation. In order that this equality is fulfilled, it is necessary at the moment of NS separation to impart to the rocket a certain lateral component of velocity. The required value of this velocity component can be obtained by supplying the corresponding signals to the lateral stabilization channel.

For calculating functionals (5.11) and (5.12) during flight it is necessary to know the components of the velocity vector and the rocket coordinates. The determining of these data in a launch coordinate system with the aid of measuring devices set up on the earth (as is done in radio-command guidance systems), is completely feasible with the required accuracy. However, when using inertial guidance systems these parameters cannot be directly measured. Thus when developing inertial systems it is very important to select a controlling functional of a rather simple type, ensuring the required accuracy of range and firing direction control.

Let us examine one of the possible ways of simplifying controlling functionals, suitable for any type of the guidance system.

As can be seen from relationship (5.9) and (5.10), the deviation of the NS impact point with respect to range ΔL depends not only on the variations in the parameters of motion of the center of mass of

a rocket in the firing plane ($\Delta\xi$, $\Delta\eta$, ΔV_ξ, ΔV_η), but also on the variations in the parameters of the lateral motion of the rocket ($\Delta\zeta$, ΔV_ζ). The deviation of the impact point with respect to direction ΔZ, in turn, depends not only on the variations in the parameters of lateral motion ($\Delta\zeta$, ΔV_ζ), but also on variations in $\Delta\xi$, $\Delta\eta$, ΔV_ξ, ΔV_η. In spite of this, in rocket control systems with regulated thrust a system of independent range and firing direction control can be used.

Let us examine the order of magnitudes of ballistic derivatives for the case of the firing of a ballistic missile a distance of about 10 000 km:

$$\frac{\partial L}{\partial V_\xi} \approx 5000-6000 \text{ s}, \qquad \frac{\partial L}{\partial \xi} \approx 1-2;$$

$$\frac{\partial L}{\partial V_\eta} \approx 1500-2500 \text{ s}, \qquad \frac{\partial L}{\partial \eta} \approx 2-10;$$

$$\frac{\partial L}{\partial V_\zeta} \approx 100-200 \text{ s}, \qquad \frac{\partial L}{\partial \zeta} \approx 0{,}1-0{,}5.$$

As can be noted, the range derivatives for the parameters of lateral motion $\frac{\partial L}{\partial V_\zeta}$, $\frac{\partial L}{\partial \zeta}$ are substantially less than the corresponding range derivatives for the projections of velocities and for the coordinates characterizing the motion of a rocket in the firing plane. Analogously derivatives $\frac{\partial Z}{\partial V_\xi}$, $\frac{\partial Z}{\partial V_\eta}$, $\frac{\partial Z}{\partial \xi}$, $\frac{\partial Z}{\partial \eta}$ are substantially less than $\frac{\partial Z}{\partial V_\zeta}$ и $\frac{\partial Z}{\partial \zeta}$.

Independent control becomes possible due to the rather exact operation of the systems controlling apparent velocity, and normal and lateral stabilization of the motion of the rocket center of mass. In this case the variations in the parameters of motion of the rocket center of mass at the moment of NS separation remain within such limits, which for ensuring assigned firing accuracy there is no need in the controlling functional to consider the effect of variations $\Delta\zeta$, ΔV_ζ on range error, and variations $\Delta\xi$, $\Delta\eta$, ΔV_ξ, ΔV_η on heading error.

As a result flight range is determined by NS separation at the moment of time, when the following equality is fulfilled

$$J_L(t)=J_L^*, \tag{5.15}$$

where

$$\left.\begin{aligned} J_L &= \frac{\partial L}{\partial \xi}\xi(t)+\frac{\partial L}{\partial \eta}\eta(t)+\frac{\partial L}{\partial V_\xi}V_\xi(t)+\frac{\partial L}{\partial V_\eta}V_\eta(t)+\frac{\partial L}{\partial t}t;\\ J_L^* &= \frac{\partial L}{\partial \xi}\xi^*(t_\kappa^*)+\frac{\partial L}{\partial \eta}\eta^*(t_\kappa^*)+\frac{\partial L}{\partial V_\xi}V_\xi^*(t_\kappa^*)+\\ &\quad+\frac{\partial L}{\partial V_\eta}V_\eta^*(t_\kappa^*)+\frac{\partial L}{\partial t}t_\kappa^*. \end{aligned}\right\} \tag{5.16}$$

In this case the necessary heading of the NS is ensured by the fulfillment at the moment of its separation of the following condition imposed on the lateral component of velocity:

$$J_z(t_\kappa)=J_Z^*, \tag{5.17}$$

where

$$\left.\begin{aligned} J_Z(t_\kappa) &= \frac{\partial Z}{\partial \zeta}\zeta(t_\kappa)+\frac{\partial Z}{\partial V_\zeta}V_\zeta(t_\kappa);\\ J_Z^* &= 0. \end{aligned}\right\} \tag{5.18}$$

In the simplest guidance system the heading of a rocket and its nose section is assigned by the prelaunch orientation of the corresponding measuring elements and is maintained with the required accuracy by the rocket lateral stabilization system (see Sect. 1.8); the corresponding principles of lateral motion control have the form

$$V_\zeta(t)=0; \quad \zeta(t)=0. \tag{5.19}$$

Let us now reduce control equation (5.15) to the form which makes it possible to obtain the information necessary for the calculations directly from the inertial measuring devices.

The Equation of Range Control in Apparent Parameters of Motion

The operating principle of inertial measuring systems is based on the measurement of accelerations and the utilization of the inertial properties of gyroscopes. The direction in space of the axes of a certain inertial coordinate system is fixed with the aid of gyroscopes. By measuring the projections of rocket acceleration for any directions in inertial space and integrating the measured values, it is possible to obtain the projections of the velocity of the rocket and the components of the path which the rocket has covered and therefore, the coordinates of the rocket.

As is known, inertial accelerometers can measure the projections of the so-called apparent, but not of the actual acceleration of that point of the rocket, in which they are located. The apparent acceleration vector of any point is called the vector of the difference between the acceleration relative to an inertial coordinate system and the acceleration due to gravity:

$$\dot{\bar{w}}=\dot{\bar{V}}-\bar{g}. \tag{5.20}$$

Standard single-axis accelerometers measure the projection of the apparent acceleration vector $\dot{\bar{w}}$ in the direction of their axis of sensitivity $\bar{\lambda}$, i.e., value

$$\dot{w}_\lambda=\dot{V}_\lambda-g_\lambda, \tag{5.21}$$

where $\dot{V}_\lambda$ – the projection on axis $\bar{\lambda}$ of the acceleration of the accelerometer housing relative to an inertial coordinate system; g_λ – projection on the same axis of acceleration due to gravity. The indicated condition is due to the effect of gravity which on the basis of Einstein's General Theory of Relativity cannot be distinguished from inertia. This gives rise to the fact that acceleration due to gravity is recorded by an accelerometer as the acceleration directed opposite to the projection of the gravity vector on the axis of sensitivity of the instrument.

Besides accelerometers, for inertial guidance purposes integrating accelerometers are also used. Integrating gyros are broadly employed in them. The rate of angular precession of an integrating gyro ω is proportional to the component of apparent acceleration along the axis of precession $\dot{w}_{\lambda}$. The output signal of the integrating gyro characterizing the angle of precession

$$\varphi=\int_0^t \omega(\tau)\,d\tau=K\int_0^t \dot{w}_{\lambda}(\tau)\,d\tau=Kw_{\lambda}, \tag{5.22}$$

is proportional to the component of apparent velocity along axis $\bar{\lambda}$:

$$w_{\lambda}=\int_0^t \dot{w}_{\lambda}(\tau)\,d\tau \quad (w_{\lambda}(0)=0). \tag{5.23}$$

Iterated integration of the integrating gyro output signal ϕ, accomplished even by another instrument, will give the value of the apparent path with respect to the direction of $\bar{\lambda}$:

$$s_{\lambda}=\int_0^t w_{\lambda}(\tau)\,d\tau \quad (s_{\lambda}(0)=0). \tag{5.24}$$

In accordance with expressions (5.20), (5.21) and (5.24) it is possible to introduce the concepts of apparent velocity vector and apparent path vector by representing these vectors in the form:

$$\left.\begin{aligned} \bar{w}(t)&=\int_0^t \dot{\bar{w}}(\tau)\,d\tau=\bar{V}(t)-\int_0^t \bar{g}(\tau)\,d\tau; \\ \bar{s}(t)&=\int_0^t \bar{w}(\tau)\,d\tau=\int_0^t \bar{V}(\tau)\,d\tau-\int_0^t\int_0^t \bar{g}(\tau)\,d\tau\,dt \end{aligned}\right\} \tag{5.25}$$

when $\quad \bar{w}(0)=0 \quad и \quad \bar{s}(0)=0.$

The coordinates and the components of the velocity of the center of mass of a rocket can be calculated, if in the equation of motion of the center of mass determining absolute acceleration:

$$\ddot{\bar{r}}=\dot{\bar{V}}=\dot{\bar{w}}+\bar{g}(\bar{r}), \tag{5.26}$$

where $\bar{r}$ – the radius-vector of the center of mass of the rocket; the right side and the initial conditions (the coordinates and the projections of the velocity of the launch point are known. With the aid of three accelerometers (or integrating gyros), oriented relative to the inertial axes and mounted at the center of mass of the rocket, it is possible to calculate the three components of apparent acceleration $\dot{\bar{w}}$ (or velocity $\bar{w}$). For calculating components of acceleration due to gravity dependence $\bar{g}(\bar{r})$ should be given.

Equation (5.26) can be solved by one of the numerical methods (by the iterative method, etc.). For automatic computation of function $\bar{r}(t)$ it is possible to use the circuit (Fig. 5.3) which is called an automatic compensation circuit. This circuit is rather complex, thus it is advantageous to use a number of simplifications. Thus, for rockets moving along trajectories, close to optimum, function $\bar{g}(t)$ can be calculated first. In this case the autocompensation circuit becomes open with program input of the correction for acceleration due to gravity (Fig. 5.4).

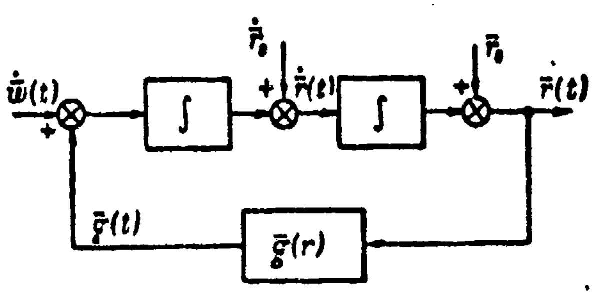


Fig. 5.3. The closed circuit for taking gravitational acceleration into account.

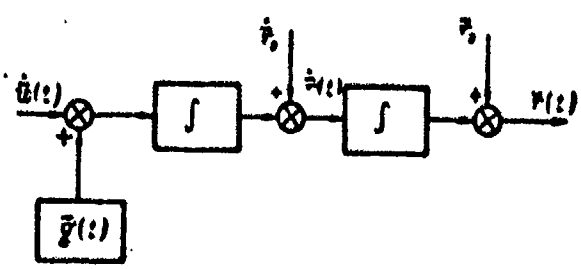


Fig. 5.4. The open circuit for taking gravitational acceleration into account.

Determining the variations in the actual parameters of motion $\Delta q_i(t)$ and $\Delta\dot{q}_i(t)$ with the aid of inertial measuring systems with the use of autocompensation circuits complicates and increases the price of the control system. At the same time it turns out, that with rather small deviations in the perturbed motion of a rocket from the calculated it is possible to go over to the apparent parameters of motion, directly obtained by the inertial measuring system. For this the concept of *isochronous variations* in the parameters of motion of the rocket is introduced.

Examining dependences $q_i(t)$ and $q_i^*(t)$ at any moment of time $t(0 \leq t \leq t_к^*)$, let us define *isochronous variation* $q_i(t)$ at moment of time t as

$$\Delta_t q_i(t) = q_i(t) - q_i^*(t). \tag{5.27}$$

In particular, at moment $t_к$ we have

$$\Delta_t q_i(t_к) = q_i(t_к) - q_i^*(t_к). \tag{5.28}$$

Let us establish the connection between complete and isochronous variations at arbitrary moment of time $t_к$, for which let us extrapolate dependence $q_i^*(t)$ for a certain moment of time, which somewhat exceeds value $t_к^*$, having assumed, for example, that the engine at moment of time $t_к^*$ was not shut down. In accordance with expression (5.8) we have

$$\Delta q_i(t_к) = q_i(t_к) - q_i^*(t_к^*) = q_i(t_к) - q_i^*(t_к) + q_i^*(t_к) - q_i^*(t_к^*). \tag{5.29}$$

Considering $\Delta t_к$ to be a small value and disregarding magnitudes of the second order of smallness, we obtain

$$q_i^*(t_к) - q_i^*(t_к^*) = \dot{q}_i^*(t_к^*)\,\Delta t_к. \tag{5.30}$$

Thus, from expressions (5.28), (5.29) and (5.30) it follows (see Fig. 5.2)

$$\Delta q_i(t_к) = \Delta_t q_i(t_к) + \dot{q}_i^*(t_к^*)\,\Delta t_к. \tag{5.31}$$

The total variation in parameter $\dot{q}_i$ is expressed by isochronous variation in a similar manner:

$$\Delta\dot{q}_i(t_\kappa)=\Delta_t\dot{q}_i(t_\kappa)+\ddot{q}_i^*(t_\kappa^*)\,\Delta t_\kappa. \tag{5.32}$$

Let us now transform (taking isochronous variations into account) the expression for ΔL, which corresponds to the simplified control equation (5.15):

$$\Delta L=\frac{\partial L}{\partial\xi}\Delta\xi+\frac{\partial L}{\partial\eta}\Delta\eta+\frac{\partial L}{\partial V_\xi}\Delta V_\xi+\frac{\partial L}{\partial V_\eta}\Delta V_\eta+\frac{\partial L}{\partial t}\Delta t_\kappa. \tag{5.33}$$

Substituting expressions (5.31) and (5.32), we obtain

$$\begin{aligned}\Delta L=&\frac{\partial L}{\partial V_\xi}\Delta_t V_\xi(t_\kappa)+\frac{\partial L}{\partial V_\eta}\Delta_t V_\eta(t_\kappa)+\frac{\partial L}{\partial\xi}\Delta_t\xi(t_\kappa)+\\&+\frac{\partial L}{\partial\eta}\Delta_t\eta(t_\kappa)+\left[\frac{\partial L}{\partial V_\xi}\dot{V}_\xi^*(t_\kappa^*)+\frac{\partial L}{\partial V_\eta}\dot{V}_\eta^*(t_\kappa)+\right.\\&\left.+\frac{\partial L}{\partial\xi}V_\xi^*(t_\kappa^*)+\frac{\partial L}{\partial\eta}V_\eta^*(t_\kappa^*)+\frac{\partial L}{\partial t}\right]\Delta t_\kappa.\end{aligned} \tag{5.34}$$

Considering dependence (5.21), let us represent expression (5.34) in the form

$$\begin{aligned}\Delta L=&\frac{\partial L}{\partial V_\xi}\Delta_t V_\xi(t_\kappa)+\frac{\partial L}{\partial V_\eta}\Delta_t V_\eta(t_\kappa)+\frac{\partial L}{\partial\xi}\Delta_t\xi(t_\kappa)+\\&+\frac{\partial L}{\partial\eta}\Delta_t\eta(t_\kappa)+\left[\frac{\partial L}{\partial V_\xi}\dot{w}_\xi^*(t_\kappa^*)+\frac{\partial L}{\partial V_\eta}\dot{w}_\eta^*(t_\kappa^*)\right]\Delta t_\kappa+\\&+\left[\frac{\partial L}{\partial V_\xi}g_\xi^*(t_\kappa^*)+\frac{\partial L}{\partial V_\eta}g_\eta^*(t_\kappa^*)+\frac{\partial L}{\partial\xi}V_\xi^*(t_\kappa^*)+\frac{\partial L}{\partial\eta}V_\eta^*(t_\kappa^*)+\frac{\partial L}{\partial t}\right]\Delta t_\kappa.\end{aligned} \tag{5.35}$$

The expression in the second square brackets in dependence (5.35) is identically equal to zero as a range derivative for time of flight in the unpowered-flight phase of the trajectory. Then

$$\begin{aligned}\Delta L=&\frac{\partial L}{\partial V_\xi}\Delta_t V_\xi(t_\kappa)+\frac{\partial L}{\partial V_\eta}\Delta_t V_\eta(t_\kappa)+\frac{\partial L}{\partial\xi}\Delta_t\dot{\xi}(t_\kappa)+\frac{\partial L}{\partial\eta}\Delta_t\eta(t_\kappa)+\\&+\left[\frac{\partial L}{\partial V_\xi}\dot{w}_\xi^*(t_\kappa^*)+\frac{\partial L}{\partial V_\eta}\dot{w}_\eta^*(t_\kappa^*)\right]\Delta t_\kappa.\end{aligned} \tag{5.36}$$

Let us examine the isochronous variations of the components of acceleration:

$$\Delta_t \dot{V}_\xi = \Delta_t \dot{w}_\xi + \Delta_t g_\xi;$$
$$\Delta_t \dot{V}_\eta = \Delta_t \dot{w}_\eta + \Delta_t g_\eta.$$

The appearance of isochronous variations in apparent acceleration $\Delta_t \dot{w}_\xi$ and $\Delta_t \dot{w}_\eta$ is directly connected with the deviation in forces of nongravitational origin and the mass of the rocket from the calculated values; the isochronous variations in acceleration due to gravity $\Delta_t g_\xi$ and $\Delta_t g_\eta$ caused by the fact that the trajectory of perturbed flight is higher (or lower than) the calculated trajectory. For rockets, whose perturbed trajectories are close to the calculated ones, the isochronous variations in acceleration due to gravity are small. In this case it is possible to take:

$$\begin{aligned} &\Delta_t V_\xi(t) \approx \Delta_t w_\xi(t); \\ &\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ &\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ &\Delta_t \eta(t) \approx \Delta_t s_\eta(t). \end{aligned} \tag{5.37}$$

The assumption made makes it possible to write expression (5.36) in the form

$$\Delta L = \frac{\partial L}{\partial V_\xi} \Delta_t w_\xi(t_\kappa) + \frac{\partial L}{\partial V_\eta} \Delta_t w_\eta(t_\kappa) + \frac{\partial L}{\partial \xi} \Delta_t s_\xi(t_\kappa) + \\ + \frac{\partial L}{\partial \eta} \Delta_t s_\eta(t_\kappa) + \left[\frac{\partial L}{\partial V_\xi} \dot{w}_\xi^*(t_\kappa^*) + \frac{\partial L}{\partial V_\eta} \dot{w}_\eta^*(t_\kappa^*)\right] \Delta t_\kappa. \tag{5.38}$$

Let us hence eliminate unknown value Δt_κ, taking into account that in accordance with expressions (5.8), (5.31) and (5.32)

$$\left.\begin{aligned} \Delta w_\xi(t_\kappa) = \Delta_t w_\xi(t_\kappa) + \dot{w}_\xi^*(t_\kappa^*) \Delta t_\kappa = w_\xi(t_\kappa) - w_\xi^*(t_\kappa^*); \\ \Delta w_\eta(t_\kappa) = \Delta_t w_\eta(t_\kappa^*) + \dot{w}_\eta^*(t_\kappa^*) \Delta t_\kappa = w_\eta(t_\kappa) - w_\eta^*(t_\kappa^*). \end{aligned}\right\} \tag{5.39}$$

Then instead of expression (5.38) we will obtain

$$\Delta L=\frac{\partial L}{\partial V_\xi}w_\xi(t_k)+\frac{\partial L}{\partial V_\eta}w_\eta(t_k)+\frac{\partial L}{\partial \xi}\Delta_t s_\xi(t_k)+$$
$$+\frac{\partial L}{\partial \eta}\Delta_t s_\eta(t_k)-\frac{\partial L}{\partial V_\xi}w_\xi^*(t_k^*)-\frac{\partial L}{\partial V_\eta}w_\eta^*(t_k^*). \tag{5.40}$$

The control equation in this case can be represented in the form

$$\frac{\partial L}{\partial V_\xi}w_\xi(t)+\frac{\partial L}{\partial V_\eta}w_\eta(t)+\frac{\partial L}{\partial \xi}\Delta_t s_\xi(t)+\frac{\partial L}{\partial \eta}\Delta_t s_\eta(t)=$$
$$=\frac{\partial L}{\partial V_\xi}w_\xi^*(t_k^*)+\frac{\partial L}{\partial V_\eta}w_\eta^*(t_k^*) \tag{5.41}$$

or

$$J_t=J^*(t_k^*), \tag{5.42}$$

where

$$J(t)=\frac{\partial L}{\partial V_\xi}w_\xi(t)+\frac{\partial L}{\partial V_\eta}w_\eta(t)+\frac{\partial L}{\partial \xi}\Delta_t s_\xi(t)+\frac{\partial L}{\partial \eta}\Delta_t s_\eta(t); \tag{5.43}$$

$$J^*(t_k^*)=\frac{\partial L}{\partial V_\xi}w_\xi^*(t_k^*)+\frac{\partial L}{\partial V_\eta}w_\eta^*(t_k^*). \tag{5.44}$$

The control equation is now reduced to a form, which makes it possible to realize it, without resorting to complex calculations on board the rocket.

Instrument Execution of the Range Control Equation

Let us examine the basic system of instrument execution of the equation of firing range control as illustrated by equation (5.41). The basis of the system (Fig. 5.5) is the gyrostabilized platform [ГСП = GSP]; the axes of the inertial coordinate system assigned by it are directed along the axes of the initial launch system. Two integrating gyroscopes [ГИ = IG] are mounted on the GSP, whose axes of sensitivity are directed along the axes Oξ and Oη. Furthermore, there is the storage device [ЗУ = SD], into which the values necessary for the calculations are introduced: ballistic derivatives,

computed values $s^*_\xi(t)$ and $s^*_\eta(t)$ programmed with respect to time and the computed values of the controlling functional. The values, input into the storage device, depend on the geophysical conditions of firing.

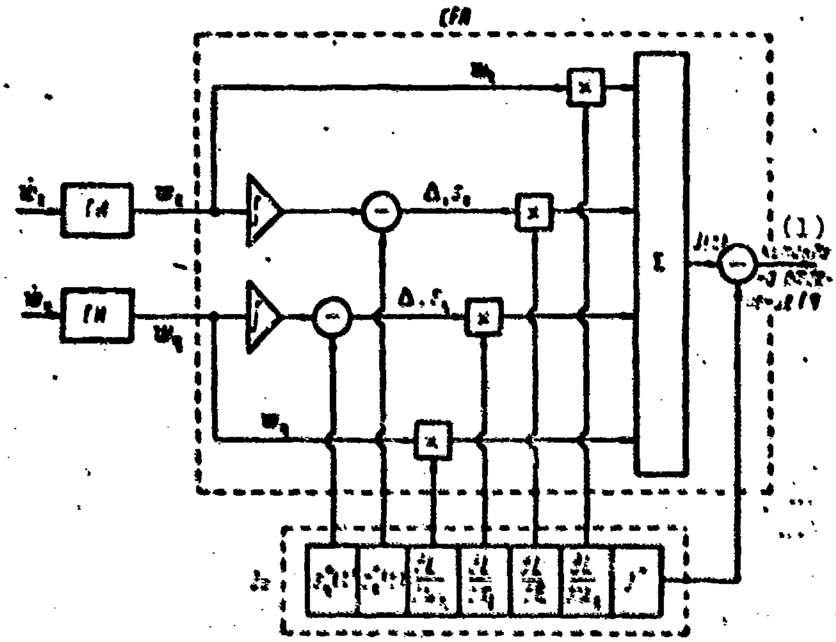


Fig. 5.5. Diagram of the instrumental execution of the control equation.
KEY: (1) Instruction for NS separation.

There is also a computer [CPП = C], containing:

– integrators which carry out iterative integration of the IG readings for the purpose the obtaining the apparent coordinates;

– subtractors which shape the isochronous variations $\Delta_t s_\xi$, $\Delta_t s_\eta$;

– multipliers;

– an adder;

– a comparator, continously carrying out during the time of the

powered-flight phase comparison of the current values of the controlling functional J(t) with its calculated value J*. At moment, when the equality J(t) = J*, is fulfilled, the computer shapes the instruction for engine shutdown and nose section separation.

Let us now examine the possible ways of simplifying instrumental execution of the equation of firing range control.

Of practical importance is the possibility of reducing the number of automatic range control elements by the proper selection of the orientation of the axes of sensitivity of the accelerometers of the integrating gyro accelerometers. It turns out, that it is possible to reduce by half the number of integrating elements as compared with the method of setting up the control equation presented above. For this purpose the axes of sensitivity of the accelerometers (integrating gyro accelerometers) should have a special orientation – parallel to certain directions, constant for the actual case of firing.

In order to determine these directions to set up the appropriate functional, let us examine the projections of apparent velocity w_ξ, w_η as components of a certain vector $\overline{w}$, which is characterized by a modulus $|\overline{w}| = \sqrt{w_\xi^2 + w_\eta^2}$ and by argument $\arg \overline{w} = \operatorname{arctg} w_\eta / w_\xi$.

Let us similarly construct vector $\overline{\Delta_t s}$ with modulus $|\overline{\Delta_t s}| = \sqrt{\Delta_t s_\xi^2 + \Delta_t s_\eta^{\prime 2}}$ and argument $\arg \overline{\Delta_t s} = \operatorname{arctg} \frac{\Delta_t s_\eta}{\Delta_t s_\xi}$.

In a similar manner it is also possible to examine ballistic derivatives as projections of vector $\overline{\Lambda}$ with modulus $|\overline{\Lambda}| = \sqrt{\left(\frac{\partial L}{\partial V_\xi}\right)^2 + \left(\frac{\partial L}{\partial V_\eta}\right)^2}$ and argument $\lambda = \operatorname{arctg} \frac{\partial L / \partial V_\eta}{\partial L / \partial V_\xi}$ and of vector $\overline{M}$ with modulus $|\overline{M}| = \sqrt{\left(\frac{\partial L}{\partial \xi}\right)^2 + \left(\frac{\partial L}{\partial \eta}\right)^2}$ and argument $\mu = \operatorname{arctg} \frac{\partial L / \partial \eta}{\partial L / \partial \xi}$.

Using the designations and the known rule for recording the scalar product of two vectors by their projections introduced in this manner, let us rewrite expression (5.41) in the form

$$\overline{\Lambda} \overline{w}(t) + \overline{M} \overline{\Delta_t s}(t) = \overline{\Lambda} \overline{w}^*(t_\kappa^*). \qquad (5.45)$$

On the other hand, the scalar product can be represented in the form

$$\bar{a}\bar{b} = |\bar{a}| b_a,$$

where b_a – the projection of vector $\bar{b}$ in the direction of $\bar{a}$.

On this basis let us represent control equation (5.45) in the form

$$|\bar{\Lambda}| w_\lambda(t) + |\bar{M}| \Delta_t s_\mu(t) = |\bar{\Lambda}| w_\lambda^*(t_\kappa^*), \qquad (5.46)$$

where

$$\left.\begin{aligned} w_\lambda(t) &= w_\xi(t)\cos\lambda + w_\eta(t)\sin\lambda; \\ \Delta_t s_\mu(t) &= \Delta_t s_\xi(t)\cos\mu + \Delta_t s_\eta(t)\sin\mu. \end{aligned}\right\} \qquad (5.47)$$

Control equation (5.46) can be standardized relative to coefficient $|\bar{\Lambda}|$, as a result of which it takes the form

$$w_\lambda(t) + p\Delta_t s_\mu(t) = w_\lambda^*(t_\kappa^*), \qquad (5.48)$$

where

$$p = \frac{|\bar{M}|}{|\bar{\Lambda}|}.$$

Thus,

$$J_{\lambda-\mu}(t) = w_\lambda(t) + p\Delta_t s_\mu(t) = w_\lambda(t) + p\int_0^t [w_\mu(t) - w_\mu^*(t)]\,dt; \qquad (5.49)$$

$$J_{\lambda-\mu}^{\circ} = w_\lambda^*(t_\kappa^*). \qquad (5.50)$$

The examined modification of the controlling functional is called the λ-μ-functional. It can be shown that the directions, assigned by angles λ and μ, are the optimum ballistic directions in the following sense. The deviations in the apparent velocity of a rocket along the direction of vector $\bar{\Lambda}$ and the deviations in the apparent path along the direction of vector $\bar{M}$ maximally affects the range errors, and the deviations in apparent velocity and apparent path along the normal to the corresponding direction do not cause range errors.

A block diagram of the instrumental execution of the control equation using the λ-μ-functional, is shown in Fig. 5.6. This diagram is simpler than the previous one (see Fig. 5.5) due to the elimination of one of the integrators and the reduction in the volume of the memory. Angles λ and μ of the setting of the axes of sensitivity of integrating relative to axis Oξ of the inertial coordinate system, program $w^*_\mu(t)$, parameter p and the calculated value of the controlling functional $w^*_\lambda(t^*_к)$ are determined by the geophysical firing conditions.

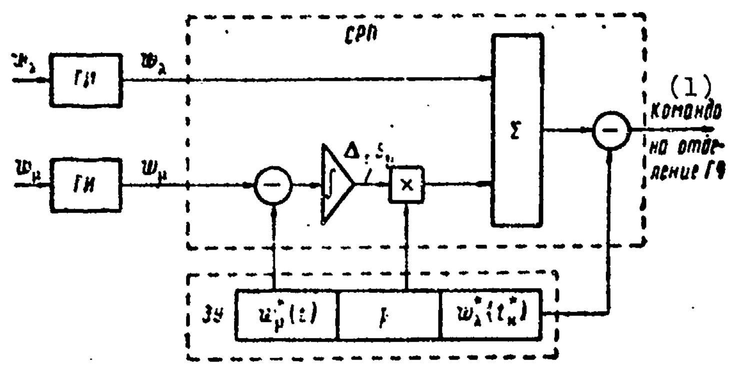


Fig. 5.6. Diagram of the instrumental execution of the control equation using the λ-μ-functional.
KEY: (1) Instruction for NS separation.

5.4. NOSE SECTION IMPACT POINT DISPERSION CAUSED BY CONTROL SYSTEM ERRORS

The Basic Formulas for Calculating Nose Section Impact Point Dispersion

Let us rewrite expression (5.34), taking into account that the isochrounous variations in moments of time $t_к$ and $t^*_к$ are equal to each other to within an accuracy of values of the second order of smallness:

$$\Delta L=\frac{\partial L}{\partial V_\xi}\Delta_t V_\xi(t^*_к)+\frac{\partial L}{\partial V_\eta}\Delta_t V_\eta(t^*_к)+\frac{\partial L}{\partial \xi}\Delta_t \xi(t^*_к)+\frac{\partial L}{\partial \eta}\Delta_t \eta(t^*_к)+$$
$$+\left[\frac{\partial L}{\partial V_\xi}\dot V^*_\xi(t^*_к)+\frac{\partial L}{\partial V_\eta}\dot V^*_\eta(t^*_к)+\frac{\partial L}{\partial \xi}V^*_\xi(t^*_к)+\frac{\partial L}{\partial \eta}V^*_\eta(t^*_к)+\frac{\partial L}{\partial t}\right]\Delta t_к. \quad (5.51)$$

It is readily noted that expression

$$\frac{\partial L}{\partial V_\xi}\dot{V}_\xi^*(t_\kappa^*)+\frac{\partial L}{\partial V_\eta}\dot{V}_\eta^*(t_\kappa^*)+\frac{\partial L}{\partial \xi}V_\xi^*(t_\kappa^*)+ \\ +\frac{\partial L}{\partial \eta}V_\eta^*(t_\kappa^*)+\frac{\partial L}{\partial t}=\frac{dL}{dt}(t_\kappa^*) \tag{5.52}$$

is the value at the moment of NS separation of the total firing range derivative with respect to flight time in the powered-flight phase.

Let us designate the first term in expression (5.51)

$$\Delta_t L(t_\kappa^*)=\frac{\partial L}{\partial V_\xi}\Delta_t V_\xi(t_\kappa^*)+\frac{\partial L}{\partial V_\eta}\Delta_t V_\eta(t_\kappa^*)+ \\ +\frac{\partial L}{\partial \xi}\Delta_t \xi(t_\kappa^0)+\frac{\partial L}{\partial \eta}\Delta_t \eta(t_\kappa^*) \tag{5.53}$$

and let us call it the isochronous firing range deviation. This value $\Delta_t L(t_H^*)$ is the deviation in the impact point of the NS in the case of flight along a perturbed trajectory upon separation of the NS at calculated moment of time t_H^*.

Taking into account what has been said, let us write expression (5.51) in the form

$$\Delta L=\Delta_t L(t_\kappa^*)+\dot{L}(t_\kappa^*)\Delta t_\kappa. \tag{5.54}$$

Deviation Δt_H can be found, by varying the control equation $J(t_H) = J^*(t_H^*)$. Then we will obtain

$$\Delta J(t_\kappa)=\Delta_t J(t_\kappa^*)+\dot{J}^*(t_\kappa^*)\Delta t_\kappa=0, \tag{5.55}$$

where

$$\dot{J}^*(t_\kappa^*)=\left(\frac{dJ^*}{dt}\right)_{t=t_\kappa^*};$$

$\Delta_t J(t_H^*)$ – the isochronous variation in the controlling functional.

Let us hence find deviation

$$\Delta t_\kappa=-\frac{\Delta_t J(t_\kappa^*)}{\dot{J}^*(t_\kappa^*)}. \tag{5.56}$$

Using relationships (5.54) and (5.56), it is possible to represent range deviation in the form

$$\Delta L = \Delta_t L(t_\kappa^*) - \frac{\dot{L}(t_\kappa^*)}{\dot{J}(t_\kappa^*)} \Delta_t J(t_\kappa^*). \tag{5.57}$$

Repeating the arguments carried out, it is possible to obtain an analogous expression for the deviation in NS impact point with respect to direction

$$\Delta Z = \Delta_t Z(t_\kappa^*) + \dot{Z}(t_\kappa^*) \Delta t_\kappa. \tag{5.58}$$

According to the definition of the arbitrary coordinate system $\dot{Z} = 0$ and for this system we have

$$\Delta Z = \Delta_t Z, \tag{5.59}$$

where

$$\Delta_t Z = \frac{\partial Z}{\partial V_\zeta} \Delta_t V_\zeta(t_\kappa^*) + \frac{\partial Z}{\partial \zeta} \Delta_t \zeta(t_\kappa^*), \tag{5.60}$$

if we disregard the effect of the variations in the parameters of motion in the firing plane on the deviations in direction Z.

The Effect of Control System Errors on the Dispersion of the Parameters of Rocket Motion

Let us examine the effect of instrumental and systematic errors of the control system on rocket flight accuracy in the powered-flight phase of the trajectory.

Let us use the following coordinate systems:

– inertial coordinate system $O\xi\eta\zeta$;

– body coordinate system $Ox_1y_1z_1$, characterizing the actual directions of the axes of a rocket in perturbed motion;

— reference coordinate system $Ox_3y_3z_3$, giving the directions of the body axes of the rocket during flight along the optimum trajectory. The directions of the reference axes relative to the axes of the inertial system are shown in Fig. 5.7. Axis Oz_3 is directed parallel to axis $O\zeta$, and axes Ox_3 and Oy_3 are turned relative to axes $O\xi$ and $O\eta$ by programmed angle of pitch ϕ^*. As is evident, the body coordinate system coincides with the reference system during the flight of the rocket along the optimum trajectory.

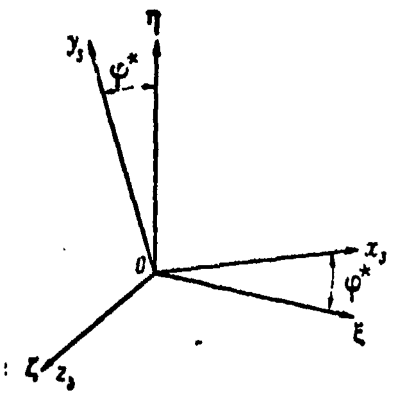


Fig. 5.7. Orientation of the reference coordinate system relative to the inertial system.

Let us find the interrelationship between the components of the apparent acceleration vector determined by the measuring elements of the longitudinal [PHC = RKS], normal [HC = VS] and lateral stabilization [БС = LS] systems and the projections of this vector on the reference coordinate system.

Let the measuring element of the RKS system, for example the longitudinal acceleration integrating gyro which we will call the velocity error sensor [ДРС = VES], be mounted on the rocket in such a way that its axis of sensitivity is oriented in the direction of rocket axis Ox_1. This instrument measures the projection of the apparent acceleration vector on the longitudinal axis of the rocket $\dot{w}_{x1}$ and carries out its integration:

$$w_{x1}=\int_0^t \dot{w}_{x1}(\tau)\,d\tau. \tag{5.61}$$

Let us assume that the measuring elements of the VS and LS systems are mounted on a GSP in such a way that during optimum flight the axis of sensitivity of the VS system measuring element is directed along the Oy_3 axis, and the axis of sensitivity of the LS system measuring element – along the Oz_3. Since the axes of sensitivity of these elements during the whole time of controlled flight are directed perpendicular to the Ox_3 axis, then the effect on the VS and LS systems sensing head readings of the longitudinal component of the apparent acceleration vector $\dot{w}_{x3}$ is eliminated in this way and the deviation of the Ox_1 axis from the reference direction is recorded.

The directions of the reference axes $Ox_3y_3z_3$ are materialized on the rocket by the directions of the axes $Ox_\Gamma y_\Gamma z_\Gamma$ of the GSP and by the programmed turning of the base of the angle of pitch sensor. In optimum flight the axes of the GSP $Ox_\Gamma y_\Gamma z_\Gamma$ are directed parallel to the corresponding axes $O\xi\eta\zeta$ of the inertial (initial launch) coordinate system (Fig. 5.8).

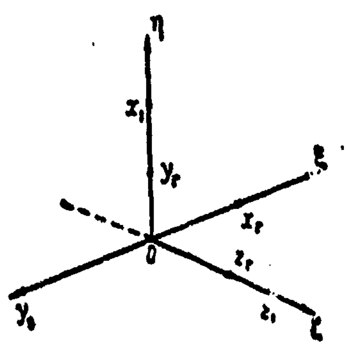


Fig. 5.8. Orientation of the axes of the gyro-stabilized platform $Ox_\Gamma y_\Gamma z_\Gamma$ relative to the inertial (initial launch) coordinate system $O\xi\eta\zeta$ in optimum flight $Ox_1y_1z_1$ – the body axes of the rocket at launch.

In actual flight the directions of the GSP axes $Ox_2y_2z_2$ in the general case do not coincide with the directions of the axes of the inertial coordinate system. The deviation of the Ox_2, Oy_2, Oz_2 axes from the optimum direction are caused by:

– aiming errors (turning around axis Oy_2 of the GSP suspension);

– errors in setting the GSP at the moment of launch, i.e., errors in the GSP actuating system (turns around the Ox_2 and Oz_2 axes);

– gyroscopic drifts during flight.

In perturbed flight the error in the execution of the direction of reference axis Ox_3 is determined by:

– errors in the setting of the GSP at the moment of launching;

– errors in the assigning of the angle of pitch which are made up of errors of the program unit and errors in assigning and reproducing the program;

– GSP drifts around the axis of pitch Oz_2.

The deviation in the direction of the axis of sensitivity of the VS system measuring element from the direction of the Oy_3 axis is caused by the same errors.

The deviation in the direction of the axis of sensitivity of the LS system measuring element from the direction of the Oz_3 axis is caused by:

– aiming errors;

– errors in the setting of the LS system measuring element relative to the aiming prism platform;

– GSP drifts around the Ox_r axis.

When errors exist in the orientation of the RKS, VS, and LS systems measuring elements errors occur in the measurement of the components of apparent acceleration $\dot{w}_{x3}$, $\dot{w}_{y3}$, $\dot{w}_{z3}$. For determining these errors let us examine Fig. 5.9. The direction of the axis of sensitivity of the VS system measuring element is determined by

axis Oy_2, and its orientation errors – by angles α and β; the direction of the axis of sensitivity of the LS system measuring element is determined by axis Oz_2, and its orientation errors – by angles γ and χ.

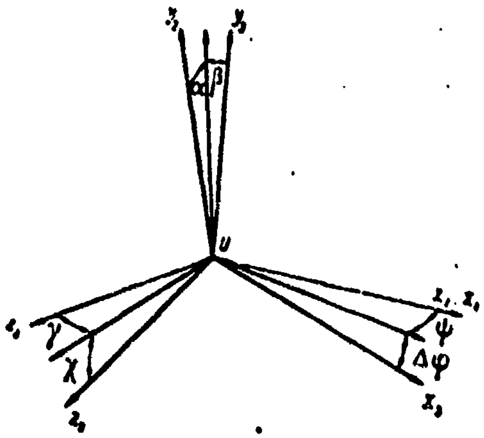


Fig. 5.9. The orientation of the $Ox_2y_2z_2$ coordinate system, executed as a reference system, relative to the $Ox_3y_3z_3$ reference system.

Table 5.1 gives the direction cosines of the Ox_1, Oy_2, Oz_2 axes relative to the Ox_3, Oy_3, Oz_3 axes, determined to within an accuracy of second order smallnesses (such accuracy is entirely sufficient for practical purposes). From Table 5.1 it follows that projections of the apparent acceleration vector on the Ox_1, Oy_2, Oz_2 axes and on the Ox_3, Oy_3, Oz_3 axes are found in the following dependence:

$$\dot{w}_{x1}=\dot{w}_{x3}+\dot{w}_{y3}\Delta\varphi-\dot{w}_{z3}\psi; \quad (5.62)$$

$$\dot{w}_{y2}=-\dot{w}_{x3}\beta+\dot{w}_{y3}+\dot{w}_{z3}\alpha; \quad (5.63)$$

$$\dot{w}_{z2}=+\dot{w}_{x3}\gamma-\dot{w}_{y3}\chi+\dot{w}_{z3}. \quad (5.64)$$

Table 5.1.

(1)Оси	Ox_1	Oy_2	Oz_2
Ox_3	1	$-\beta$	γ
Oy_3	$\Delta\varphi$	1	$-\chi$
Oz_3	$-\psi$	α	1

KEY: (1) axes.

The projections of the apparent acceleration vector $\dot{w}_{y3}$, $\dot{w}_{z3}$ and angular deflections $\Delta\phi$, ψ, α, β, γ, χ in optimum flight are equal to zero, and in actual flight they arise due to the effect of perturbing factors. All these values are small control system errors. Thus products of the type $\dot{w}_{y3}\Delta\phi$, $\dot{w}_{z3}\psi$, $\dot{w}_{z3}\alpha$ can be disregarded as second order of smallness values. Then the projections of the apparent acceleration vector on the reference axes $Ox_3y_3z_3$ are determined by the following expressions:

$$\dot{w}_{x3} = \dot{w}_{x1}; \tag{5.65}$$

$$\dot{w}_{y3} = \dot{w}_{y2} + \dot{w}_{x3}\beta; \tag{5.66}$$

$$\dot{w}_{z3} = \dot{w}_{z2} - \dot{w}_{x3}\gamma. \tag{5.67}$$

Hence we will obtain the deviations of the projections (in question) of the apparent acceleration vector from their optimum values:

$$\Delta_t\dot{w}_{x3} = \Delta_t\dot{w}_{x1}; \tag{5.68}$$

$$\Delta_t\dot{w}_{y3} = \Delta_t\dot{w}_{y2} + \dot{w}_{x3}\beta; \tag{5.69}$$

$$\Delta_t\dot{w}_{z3} = \Delta_t\dot{w}_{z2} - \dot{w}_{x3}\gamma. \tag{5.70}$$

Let us now find the values of the deflections from the optimum values of the projections of the apparent acceleration vector on the axes of an inertial coordinate system. The indicated values can be easily obtained, using known formulas for transforming from a body to an inertial coordinate system:

$$\Delta_t\dot{w}_{\xi} = \Delta_t\dot{w}_{x3}\cos\varphi^{\circ} - \Delta_t\dot{w}_{y3}\sin\varphi^{\circ}; \tag{5.71}$$

$$\Delta_t\dot{w}_{\eta} = \Delta_t\dot{w}_{x3}\sin\varphi^{\circ} + \Delta_t\dot{w}_{y3}\cos\varphi^{\circ}; \tag{5.72}$$

$$\Delta_t\dot{w}_{\zeta} = \Delta_t\dot{w}_{z3} \tag{5.73}$$

and relationships (5.68)-(5.70).

We will finally obtain:

$$\Delta_t\dot{w}_\xi=\Delta_t\dot{w}_{x1}\cos\varphi^*-\Delta_t\dot{w}_{y2}\sin\varphi^*-\dot{w}_{x3}\beta\sin\varphi^*; \quad (5.74)$$

$$\Delta_t\dot{w}_\eta=\Delta_t\dot{w}_{x1}\sin\varphi^*+\Delta_t\dot{w}_{y2}\cos\varphi^*+\dot{w}_{x3}\beta\cos\varphi^*; \quad (5.75)$$

$$\Delta_t\dot{w}_\zeta=\Delta_t\dot{w}_{z2}-\dot{w}_{x3}\gamma. \quad (5.76)$$

Integrating expressions (5.74)-(5.76), we obtain the isochronous variations in the projections of the apparent velocity vector on the axes of an inertial coordinate system:

$$\Delta_t w_\xi=\int_0^t\Delta_t\dot{w}_{x1}\cos\varphi^* d\tau-\int_0^t\Delta_t\dot{w}_{y2}\sin\varphi^* d\tau-\int_0^t\dot{w}_{x3}\beta\sin\varphi^* d\tau; \quad (5.77)$$

$$\Delta_t w_\eta=\int_0^t\Delta_t\dot{w}_{x1}\sin\varphi^* d\tau+\int_0^t\Delta_t\dot{w}_{y2}\cos\varphi^* d\tau+\int_0^t\dot{w}_{x3}\beta\cos\varphi^* d\tau; \quad (5.78)$$

$$\Delta_t w_\zeta=\int_0^t\Delta_t\dot{w}_{z2} d\tau-\int_0^t\dot{w}_{x3}\gamma d\tau. \quad (5.79)$$

Iterative integration will give the isochronous variations in the apparent path for axes of the inertial coordinate system:

$$\Delta_t s_\xi=\int_0^t\Delta_t w_\xi d\tau; \quad (5.80)$$

$$\Delta_t s_\eta=\int_0^t\Delta_t w_\eta d\tau; \quad (5.81)$$

$$\Delta_t s_\zeta=\int_0^t\Delta_t w_\zeta d\tau. \quad (5.82)$$

Equations (5.77)-(5.82) are integrated under zero initial conditions.

Assuming that the effect of gnavity on a rocket in perturbed and optimum flights is practically identical (such an assumption is permissible because the deviations in perturbed motion from optimum are rather small), it is possible to approximately assume:

$$\Delta_t \dot{V}_\xi \approx \Delta_t \dot{w}_\xi; \quad \Delta_t V_\xi \approx \Delta_t w_\xi; \quad \ldots, \Delta_t \zeta \approx \Delta_t s_\zeta. \tag{5.83}$$

In equations (5.77)-(5.82) dependences $\dot{w}_{x3}(t)$ and $\phi^*(t)$ correspond to flight under optimum conditions. The remaining parameters $\Delta_t\dot{w}_{x1}(t)$, $\Delta_t\dot{w}_{y2}(t)$, $\Delta_t\dot{w}_{z2}(t)$, $\beta(t)$m $\gamma(t)$ are determined by the effect of the perturbations. The methods for determining them for the basic types of control system errors are examined below.

1. The instrumental error of the RKS system meter is mainly determined by the variance in the transmission coefficient of the meter and thus is proportional to the measured value, i.e.,

$$\Delta_t \dot{w}_{x1}(t) = n\dot{w}_{x3}(t),$$

where n — a dimensional coefficient.

2. The error in activating the RKS system meter is taken into account in the initial conditions, i.e., it is assumed that

$$(\Delta_t w_{x1})_{t=0} = c.$$

The error in assigning the apparent velocity program is taken into account in an analogous manner.

3. The systematic error in assigning the angle of pitch $\Delta\phi$ program is one of the components of angle β. Among the other constant components of angle β are: the angle of pitch program unit error, the error in actuating the GSP around the pitch axis, the error in setting the VS system measuring element.

Aiming error, the errors in actuating the GSP around the yaw and spin axes, the error in setting the LS system measuring element are, in an analogous manner, taken into account as constant components of angle γ.

4. The error in activating the RKS program τ.

As can be seen from Fig. 5.10, this error leads to a shift in curve $w_{x1}(t)$ along the abscissa axis by constant value τ. Expanding $w_{x1}(t) = w^*_{x1}(t-\tau)$ into a series by degrees of τ, we obtain as a first approximation $\Delta_t w_{x1}(t) = -\tau \dot{w}^*_{x1}(t)$.

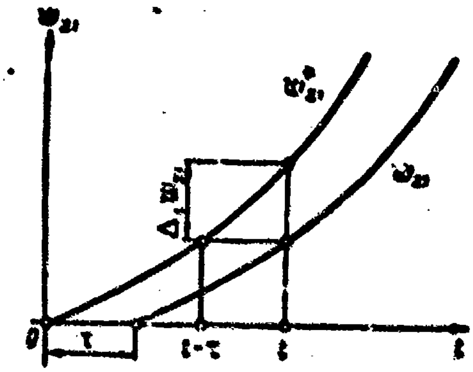


Fig. 5.10. The effect of the error in activating the PKS program on the longitudinal component of apparent velocity.

The error in activating the pitch program is taken into account in an analogous manner

$$\beta(t) = -\tau \dot{\varphi}^*(t).$$

5. The OSP drifts due to the effect of constant moments can be assumed proportional to time:

$$\beta(t) = \dot{\varphi}_r t;$$
$$\psi(t) = \dot{\psi}_r \sin \varphi^* t;$$
$$\gamma(t) = \dot{\gamma}_r \cos \varphi^* t,$$

where $\dot{\varphi}_r$, $\dot{\psi}_r$, $\dot{\gamma}_r$ – respectively the rates of OSP drifts around the pitch, yaw, and spin axes.

6. OSP drifts due to the statistical lack of balance of the gyro-blocks.

It is necessary to note the following characteristic of drift of the GSP gyroscopes. For gyroscopes mounted for stabilizing a GSP equipped with stabilizing engines, the perturbing moments with respect to the axes of sensitivity of the gyroscopes are compensated for by stabilizing moments and do not give rise to drifts. However the harmful effect of the perturbing moments with respect to the axis of precession is preserved and produces drifts of the GSP relative to the axis of sensitivity.

Let us examine the expression for a corresponding error as illustrated by a pitch gyro-block installed at angle $\phi_{\Gamma Б}$ to the Ox_r axis (Fig. 5.11). In this case the component of apparent acceleration causing drift, is equal to $\dot{w}_{x1} \sin(\phi_{\Gamma Б} - \phi^*)$, and the magnitude of drift is determined by the expression

$$\vartheta(t) = \frac{\dot{\varphi}_g}{g} \int_0^t \dot{w}_{x1} \sin(\varphi_{\Gamma Б} - \varphi^*) d\tau,$$

where $\dot{\phi}_g$ - the rate of drift of the GSP around the pitch axis under the effect of acceleration due to gravity.

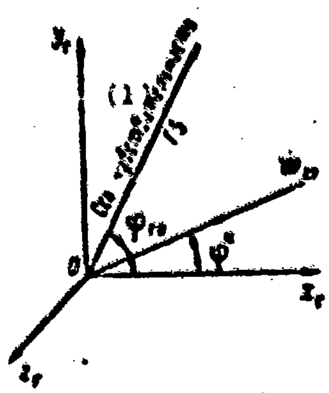


Fig. 5.11. For determining the drift of the GSP gyroscopes.
KEY: (1) Axis of sensitivity.

The GSP drifts around the yaw and spin axes are determined in an analogous manner.

Calculating Dispersion

Examining expression (5.57) as a recording of a certain realization of the random deviations of the parameters of motion, let us transform it, having clearly distinguished the systematic errors, the instrumental errors of the GSP and the other instrumental errors:

$$\Delta L=\Delta_t L-\frac{\dot{L}}{\dot{J}}\Delta_t J=\left(\Delta_{t\,\text{и}} L-\frac{\dot{L}}{\dot{J}}\Delta_{t\,\text{и}} J\right)+\Delta_{t\,\text{г}} L-\frac{\dot{L}}{\dot{J}}\Delta_{t\,\text{н}} J=\Delta_{\text{и}} L+\Delta_{\text{г}} L+\Delta_{\text{н}} L. \tag{5.84}$$

Let us write the expression for the actual realization of random deviation for the following in an analogous manner:

$$\Delta Z=\Delta_{\text{и}} Z+\Delta_{\text{г}} Z+\Delta_{\text{н}} Z. \tag{5.85}$$

Terms $\Delta_{\text{и}} L$ and $\Delta_{\text{и}} Z$ in these formulas are due to the instrumental errors of the instruments shaping the main instruction for NS separation, and the instruments of the lateral stabilization system.

As follows from expressions (5.84) and (5.85), the calculation equations for determining the instrumental errors have the form:

$$\Delta_{\text{и}} L=-\frac{\dot{L}}{\dot{J}}\Delta_{t\,\text{и}} J; \tag{5.86}$$

$$\Delta_{\text{и}} Z=\frac{\partial Z}{\partial V_\zeta}\Delta_{t\,\text{и}} V_\zeta+\frac{\partial Z}{\partial \zeta}\Delta_{t\,\text{и}}\zeta, \tag{5.87}$$

where $\Delta_{t\text{и}} J$ – the instrumental error in calculating controlling functional J; $\Delta_{t\text{и}} V_\zeta$, $\Delta_{t\text{и}}\zeta$ – the instrumental errors in determining the lateral component of velocity and the lateral coordinate.

Terms $\Delta_{\text{г}} L$ and $\Delta_{\text{г}} Z$ in formulas (5.84) and (5.85) represent the deviations in the NS impact points due to the instrumental errors of the GSP:

$$\Delta_{\text{г}} L=\Delta_{t\,\text{г}} L=\frac{\partial L}{\partial V_\xi}\Delta_{t\,\text{г}} V_\xi+\frac{\partial L}{\partial V_\eta}\Delta_{t\,\text{г}} V_\eta+\frac{\partial L}{\partial V_\zeta}\Delta_{t\,\text{г}} V_\zeta+\frac{\partial L}{\partial \xi}\Delta_{t\,\text{г}}\xi+\frac{\partial L}{\partial \eta}\Delta_{t\,\text{г}}\eta+\frac{\partial L}{\partial \zeta}\Delta_{t\,\text{г}}\zeta; \tag{5.88}$$

$$\Delta_r Z = \Delta_{tr} Z = \frac{\partial Z}{\partial V_\xi}\Delta_{tr}V_\xi + \frac{\partial Z}{\partial V_\eta}\Delta_{tr}V_\eta + \frac{\partial Z}{\partial V_\zeta}\Delta_{tr}V_\zeta + + \frac{\partial Z}{\partial \xi}\Delta_{tr}\xi + \frac{\partial Z}{\partial \eta}\Delta_{tr}\eta + \frac{\partial Z}{\partial \zeta}\Delta_{tr}\zeta. \tag{5.89}$$

The instrumental errors of the GSP $\Delta_{tr}V_\xi, \ldots, \Delta_{tr}\zeta$ are calculated by formulas (5.77)-(5.82).

The first terms in formulas (5.84) and (5.85) are the systematic errors brought about by the approximate nature (incompleteness) of the controlling functional.

Let functional (5.49) be accepted for range control. Comparing it with exact functional (5.6), we detect the following systematic errors brought about by simplifying the functional.

1. The error due to neglecting terms higher than the first order of the expansion of function L into a Taylor series which can be approximately evaluated using formula

$$\Delta L_R = \sum_{i=1}^{6}\sum_{j=1}^{6}\frac{\partial^2 L}{\partial q_i \partial q_j}\Delta q_i \Delta q_j. \tag{5.90}$$

2. The error due to disregarding the isochronous variations in gravity

$$\Delta L_g = \frac{\partial L}{\partial V_\xi}\int_0^t \Delta_t g_\xi\, dt + \frac{\partial L}{\partial V_\eta}\int_0^t \Delta_t g_\eta\, dt + + \frac{\partial L}{\partial \xi}\int_0^t\int_0^t \Delta_t g_\xi\, dt\, dt + \frac{\partial L}{\partial \eta}\int_0^t\int_0^t \Delta_t g_\eta\, dt\, dt. \tag{5.91}$$

3. The error due to disregarding the effect of deviations in the parameters of lateral motion on firing range

$$\Delta L_Z = \frac{\partial L}{\partial V_\zeta}\Delta_t V_\zeta + \frac{\partial L}{\partial \zeta}\Delta_t \zeta. \tag{5.92}$$

If the direction of flight of the NS is ensured by fulfilling conditions (5.17) or (5.19), then the systematic error for firing direction is defined as

$$\Delta Z_{\text{м}} = \frac{\partial Z}{\partial V_\xi}\Delta_t V_\xi + \frac{\partial Z}{\partial V_\eta}\Delta_t V_\eta + \frac{\partial Z}{\partial \xi}\Delta_t \xi + \frac{\partial Z}{\partial \eta}\Delta_t \eta. \tag{5.93}$$

The formulas given above can be used for calculating individual component errors which are then added up by the rules for summing independent random values, for example,

$$\Delta_{\text{и}} L = \sqrt{\sum_i \Delta_{\text{и}} L_i^2}.$$

The total dispersion due to instrumental and systematic errors of a control system and the perturbations during the unpowered-flight phase of a trajectory we find by formulas:

$$\Delta L = \sqrt{\Delta_{\text{и}} L^2 + \Delta_{\text{т}} L^2 + \Delta_{\text{м}} L^2 + \Delta_{\text{п}} L^2}; \tag{5.94}$$

$$\Delta Z = \sqrt{\Delta_{\text{и}} Z^2 + \Delta_{\text{т}} Z^2 + \Delta_{\text{м}} Z^2 + \Delta_{\text{п}} Z^2}, \tag{5.95}$$

in which $\Delta_n L$ and $\Delta_n Z$ are determined by formulas (5.1).

CHAPTER VI

OPTIMUM ROCKET FLIGHT PATH IN THE POWERED-FLIGHT PHASE

The selection of the control programs is an integral part of rocket design and development. This is due to the fact, that the design, tactical-flight and operational characteristics of a rocket to a significant degree depend on the flight path determined by the programs or by the control algorithms.

The composition of rocket flight control programs can be diverse and depends on the purpose of the rocket, its design characteristics and on the control system. For rockets with controlled thrust the programs of pitch angle and of the projections of apparent velocity in certain direction can be included in the main control programs.

The selection of the optimum configuration of the flight path of ballistic rockets with zero program values of normal and lateral velocities, angles of yaw and roll reduces to the optimization of two control system programs – programs of apparent velocity and pitch angle regulation.

The program of apparent velocity regulation, is equal to

$$w_{x1}(t)=\int_0^t \frac{\dot{G}P_{y\pi}-c_xqS}{m_0-\dot{m}t}\,dt,$$

it is practically completely determined by the basic design and energy characteristics of the rocket – by the thrust-weight ratio, by the

thrust of the engine systems of the stages, by the fuel consumption per second and to a lesser extent by the configuration of the trajectory. For this reason, and also as a result of the rather narrow limits of thrust level regulation the problem of selecting an optimum program of apparent velocity regulation does not have vital importance. Thus for a rocket with assigned design parameters the determining of control programs reduces to the selecting of an optimum pitch angle program which is an independent problem in this case.

A number of works has been dedicated to the solving of the problem of ballistic missile pitch angle program. However, in the overwhelming majority of these the selection of a program is examined disregarding the actual limitations imposed by the technical specifications on the control program, the configuration of the trajectory and flight conditions of the rocket.

The purpose of the present chapter is mainly to present the engineering approach to the selecting of the optimum configuration of the trajectory or, in other words, of the optimum program of pitch angle variation for long-range rockets with liquid-propellant engines.

6.1. SPECIFICATIONS IMPOSED ON A PITCH ANGLE PROGRAM AND THE METHODS FOR SELECTING IT

Such flight-tactical characteristics, as maximum firing range, nose section dispersion, and also skin temperature, rocket body and nose section strength, controllability in the powered-flight phase, etc., depend on the flight path configuration assigned by the pitch angle program. Thus the selection of a rocket pitch angle program should be carried out taking into account the specifications imposed on the flight path configuration, together with the selection of control element effectiveness, body skin thicknesses, by strength calculations, etc. The disregarding of the complex approach to the selection of pitch angle program can lead to a substantial reduction in the tactical-flight characteristics of a rocket.

One of the basic specifications, imposed on a pitch angle program, is the ensuring of maximum firing range. A pitch angle

program satisfying this condition and all the other specifications imposed on it, is called *maximum range program*. Another important specification, imposed on a pitch angle program, is the requirement of ensuring minimum nose section dispersion.

Both requirements – the ensuring of maximum range and minimum dispersion, are almost always incompatible. Minimum dispersion, as a rule, does not correspond to maximum range flight. By reducing range it is possible by an appropriate selection of pitch angle program to diminish dispersion. Thus long-range ballistic missiles can be equipped not with one, but with several programs. One of these is the maximum range program (or a program close to it). It is intended for firing for maximum range or ranges close to it. Another program is the so-called *minimum dispersion program*. This program is used for firing for minimum and intermediate ranges. Nose section dispersion in the rocket flight using this program is less than in flight with a maximum range program.

The flight path of a rocket when using a minimum dispersion program is steeper (less flat) in comparison with a flight for maximum range. There can be several minimum dispersion programs. Each of these has its own sphere of application.

Besides the two indicated specifications, still other specifications are imposed on a pitch angle program, which are determined by the operating conditions, the purpose of rocket, the characteristics of the control system, etc. However these specifications, as a rule do not depend on which program is used – maximum range or minimum dispersion.

Among the mentioned specifications there are also those which are common and typical for long-range ballistic missiles. There are those, which are determined by the specifics of a given missile or its individual systems and are not always mandatory for any missile.

Among the common and typical specifications imposed on a program it is possible, to include, for example, the following.

1. A missile launch should be vertical; the duration of the vertical phase of the trajectory should not be less than assigned, usually determined by the launch conditions.

2. The pitch angle program should be a continuous function of time;[1] the programmed velocities and the accelerations of the turning of the axis of rocket should be acceptable for instrumental execution.

3. The temperatures of the missile body skin and the programmed angles of attack in the high ram-pressure phase should be acceptable from the point of view of strength.

4. In the rocket flight phase in the dense layers of the atmosphere acceptable conditions should be ensured for controllability (with a reduction in the slope of the trajectory maximum ram pressure increases and, as a consequence of this, the perturbations caused by the effect of such perturbing factors, as wind, aerodynamic asymmetry of the configuration, etc., increase).

5. The rocket flight conditions in the stage separation phase should be acceptable for ensuring reliable separation.

6. The parametric domain of the reentry of nose sections into the atmosphere should be acceptable from the point of view of strength, temperature regimes and the operating conditions of the automatic equipment of the nose section.

An example of the particular specifications imposed on a program due to the specifics of a rocket, is the specifications imposed on the pitch angle program control system:

a) the angle included between the line of radio sighting and the plane of the local horizon at the point of the location of the

[1]In this case there is meant the requirement of the continuity of function $\phi(t)$ with its "ideal" (without gaps) assignment in the control-system equipment.

ground-based antenna (angle of elevation β_H) should not be less than permissible $\beta_{H\,min}$, i.e.,

$$\beta_H \geqslant \beta_{H\,min}; \tag{6.1}$$

b) the angle included between the line of radio sighting and the longitudinal axis of the rocket $\beta_б$ should be located within certain limits, i.e., satisfy the inequality

$$\beta_{б\,min} \leqslant \beta_б \leqslant \beta_{б\,max}. \tag{6.2}$$

The enumerated specifications impose more significant limitations on a maximum range program. This is due to the fact that any departures from the optimum program executing the maximum range, caused by the above enumerated requirements, g ve rise to a reduction in maximum range. Among the number of such specifications whose effect on the configuration of the trajectory and the maximum range program is determinant, it is possible to include the specifications ensuring acceptable dispersion, strength and controllability.

Thus, for reducing rocket dispersion it is necessary to increase the slope of the trajectory. With an increase in the slope of the trajectory the values of the partial derivatives of range and lateral deflection with respect to the parameters of rocket motion at the moment of nose section separation decrease and, as a consequence of this, dispersion is decreased; furthermore, the dispersion of nose sections in the atmospheric descent phase of the trajectory is decreased. With an increase in the slope of the trajectory the loads acting on the missile body (mainly, due to the reduction in ram pressure), and the temperature of the body skin (due to the fact that the time of motion in the dense layers is decreased) decrease. Finally, with an increase in the slope of the trajectory the ram pressures and the values of the perturbing moments decrease, which facilitates rocket control. On the other hand, a consequence of an increase in the slope of a trajectory is a reduction in maximum range.

When selecting a program of minimum dispersion the specifications of acceptable temperature regimes, controllability and a number of others becomes superfluous because the trajectory in this case, as a rule, takes precedence over the trajectory of maximum range. Furthermore, when selecting a minimum dispersion program a certain freedom of action in varying the program is possible. The fact is that a certain variation in the program in one or another direction for fulfilling the specifications and limitations, imposed on it, leads, as a rule, to an insignificant variation in dispersion.

In connection with what has been said it can be concluded that the greatest difficulties arise when selecting the maximum range program. Thus a further examination of the requirements imposed on the pitch angle program is carried out with respect to maximum range program.

The necessity for ensuring the first of the above enumerated requirements is due to the simplicity and the convenience of vertical launch. In this connection vertical launching of long-range rockets is generally accepted.

The flight time in the vertical phase of a trajectory t_{B} (or the path covered in the vertical flight phase) can be varied in selecting the flight program. The minimum permissible duration of vertical flight is determined by the conditions of launch safety. The duration of the vertical phase is selected as short as possible because the greater it is, the steeper is the trajectory (the velocity losses in overcoming gravity are increased) and the more difficult it is to accomplish turning of the rocket in the subsequent phase (large angles of attack are required) [2].

The second requirement is due to the possibilities of instrumental execution of pitch angle programs. It is evident that the technical implementation of discontinuous pitch angle programs is generally impossible, because the two angular positions of the longitudinal axis of the rocket differing from one another by a finite angle, correspond to one and the same moment of time. Thus it is possible

to discuss only the very high velocities of the turning of the longitudinal axis of a rocket. The magnitude of this velocity is limited by the technical capabilities of the control-system equipment. With non-fulfillment of the indicated limitation the accuracy of execution by the pitch angle program control system deteriorates. The limiting of the velocity of the turning of the axis of the rocket plays a significant role with a large rocket thrust-weight ratio, in which the duration of the powered-flight phase is short. The necessary velocities of turning of the rocket axis in this case can attain great values.

The pitch angle program has a substantial effect on the magnitude of the transverse loads and the temperature of the body skin. With a reduction in the slope of the trajectory the velocity of movement and the time of the rocket sojourn in the dense layers of the atmosphere increase. A consequence of this is an increase in the temperature of the housing skin and in this connection a reduction in the strength margins. Furthermore, with a reduction in the slope of the trajectory the effect of the perturbing factors (wind, variance in atmospheric density, etc.) on the strength of the housing is increased. And finally with a reduction in the slope of the trajectory in rocket flight in the dense layers of the atmosphere the values of the programmed angles of attack increase and, as a consequence, the values of the transverse loads increase. As a result the thicknesses of the body skins and the "dry" weight of the rocket are increased. The loss in maximum range due to an increase in "dry" weight in this case can be more than the gain in range due to the variation in the program in the atmospheric phase.

For long-range missiles and in particular for intercontinental ballistic missiles the requirement of reducing body skin thickness and "dry" weight almost always limits the domain of the possible values of pitch angle in the dense layers of the atmosphere to the condition of small (close to zero) angles of attack. The requirement of smallness of angles of attack is the more significant, the greater is the thrust-weight ratio of the rocket. In the case of small values of thrust-weight ratio the velocity of the rocket increases slowly

and the rocket passes through the dense layers of the atmosphere at comparatively small velocities. With an increase in thrust-weight ratio the velocity of a rocket increases more rapidly and the rocket passes through the dense layers of the atmosphere at high velocities. In accordance with this the transverse loads caused by the angles of attack, increase with an increase in thrust-weight ratio

The flight path of a rocket in powered-flight phase affects the controllability of the rocket and conditions of stage separation. With a reduction in the slope of the flight path in the powered-flight phase the perturbing forces and moments increase, the required control element effectiveness increases and the conditions of stage separation deteriorate. Furthermore, the controllability of a rocket and the reliability of stage separation depend on the programmed angles of attack, and with an increase in the latter the controllability of the rocket and the reliability of separation deteriorate. Thus for ensuring for rocket controllability and stage separation reliability it is expedient to increase the slope of the trajectory and to decrease the programmed angles of attack during flight in the dense layers of the atmosphere. However in this case, as a rule, the maximum firing range decreases.

The greatest difficulties arise in ensuring rocket controllability at the beginning of the motion of the second stage. This is due to the fact that for reducing the weight of the second stage control elements it is expedient to select the latter from the condition of ensuring controllability in the rarefied layers of the atmospheres, since the greater part of flight of the second stage practically occurs in airless space. For this reason for ensuring reliable stage separation and controllability of the second stage it is necessary to limit the magnitudes of the perturbations acting in the stage separation phase and at the beginning of motion of the second stage. The solution to this problem is more complex for rockets with the low stage separation altitudes ($h < 40$ km), when the aerodynamic perturbation are comparable with the perturbations due to errors in the manufacture and the assembly of a rocket. For rockets with short first stage flight duration the fulfillment of the indicated requirement

can lead to a significant reduction in maximum range.

Significant losses in range connected with ensuring the requirements of reliable stage separation, second stage controllability, acceptable temperature regimes of the body and its strength, give rise to the necessity for selecting the pitch angle program jointly with the selection of the stage separation configuration, control element effectiveness, and skin thicknesses. The necessity of ensuring acceptable temperature regimes of a missile body skin in the first place makes it necessary to consider trajectories executing the maximum range of rockets with a high thrust-weight ratio.

For ensuring the strength and the controllability of a rocket and reliability of stage separation there is imposed on the pitch angle program the requirement of smallness of the angles of attack during rocket flight in the dense layers of the atmosphere. In this phase the pitch angle program is usually taken so that the programmed angles of attack are equal to zero. The loss in maximum range due to the necessity of fulfilling condition $\alpha = 0$, for rockets with a comparatively large powered-flight phase duration is small; with a reduction in the powered-flight phase duration loss in maximum range increases.

The flight paths of a nose section in the atmosphere, overloads, temperature regimes and the strength of the nose section body are mainly determined by the parameters of motion of the center of mass of the nose section at the moment of its reentry into the atmosphere[1] - by the velocity of the nose section V_0 and by the angle of reentry Θ_0. In firing under various geodetic conditions (latitude of the launch point and firing azimuth) for maximum and minimum ranges using the accepted pitch angle programs (maximum range, minimum dispersion) the parameters of motion of the center of mass at the moment of reentry into the atmosphere form a certain region, called the *region of reentry of a nose section into the atmosphere.* In the case when the rocket

[1]The angle of attack at the moment of reentry into the atmosphere which also determines the transverse overloads, is not examined here, since its dependence on the pitch angle program can be disregarded.

is equipped with several programs, this region includes the regions, obtained using each of the indicated programs. The approximate form of the region of reentry of the center of mass of a nose section into the atmosphere using two pitch angle programs (maximum range and minimum range and minimum dispersion) is given in Figure 6.1.

The upper boundary of the given region (line 2-3) corresponds to employment of the minimum dispersion program, the lower line 1-4) – the maximum range program. The boundary of the region, passing through points 1 and 2, corresponds to firing for minimum range using the mentioned pitch angle programs, through points 3 and 4 – to firing for maximum possible range.

With an increase in velocity V_0 and in the absolute value of the reentry angle $|\Theta_0|$ the longitudinal and transverse accelerations increase and, as consequence, the weight of the nose section. The worst (from the point of view of nose section strength and the efficiency of its automatic equipment) conditions of reentry into the atmosphere correspond to point 3 of the given region. On the other hand, with a reduction in the absolute value of angle Θ_0 the required weight of the heatshield covering is increased, which is due to the increase in the flight time of the nose section in the atmosphere.

The boundaries of the region of the parameters of the reentry of the center of mass of the nose section into the atmosphere, as a rule, are due to the maximum permissible values of longitudinal and lateral accelerations. The latter are determined by the strength and by the conditions of normal operation of the automatic equipment of the nose section and, finally, – by the level of the development of technology at the time of the designing of the rocket. The pitch angle program of a ballistic missile should be such, that the parameters of motion of the nose section V_0 and Θ_0 satisfy the assigned region of its reentry into the atmosphere.

Requirements imposed on the pitch angle program with respect to ensuring limited values of the angle of elevation ε_M and the onboard angle ε_6 of the line of radio sighting are caused by the operational

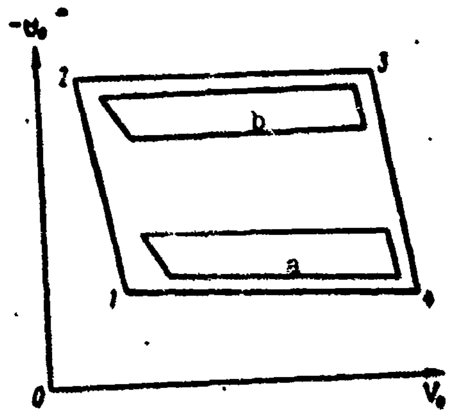


Fig. 6.1. The region of the reentry of a nose section into the atmosphere: a) region of reentry corresponding to the maximum range program; b) region of reentry corresponding to the minimum dispersion program.

characteristics of the radio control system. In particular, the requirement for limiting the range of the possible values of the angle of elevation by the condition $\beta_{\text{н}} \geqslant \beta_{\text{н min}}$ is connected with the fact that with a reduction in the angle of elevation of the line of radio sighting $\beta_{\text{н}}$ the measuring errors of the radio control system in the parameters of motion of the rocket due to an increase in the effect of the heterogeneity of the atmosphere, its turbulence, refraction, etc. on measuring accuracy increase. For this reason, if the possible values of the angle of elevation $\beta_{\text{н}}$, are not limited downward, then the measuring errors of the radio control system of the parameters of motion, and also of the deviation of the nose section impact points from the target when using flat trajectories will be large.

The requirement for limiting the range of variation in the onboard angle of the line of radio sighting is brought about by using narrow onboard and ground-based antenna radiation patterns. (As is known, with an increase in the width of the antenna radiation pattern the region of space is increased, in which the radio waves are propagated and accordingly the probability of detecting of radio emission is increased. Furthermore, with an increase in the width of the radiation pattern the power requirement of the radio transmitter is increased.)

In connection with this the range of the variation in the onboard angle of the line of radio sighting is usually small. A certain part of this range is expended on the oscillations of the rocket around its center of mass. If in the radio control phase programmed pitch turning of the rocket of an order of several degrees is executed, then the remaining (available) range of variation in angle $\beta_{\text{б}}$ is small (less

than 5°). In connection with this in the program of a rocket flight, in which separation of the nose section is accomplished upon instruction from the radio control system limitation of pitch turning of the rocket is specified in the radio control phase. Usually this phase of the trajectory is rectilinear.

One of the requirements, imposed by an autonomous rocket control system on the pitch angle program, is the limitation of the maximum angle of pitch turning of the rocket in the powered-flight phase to a maximum of 85°-90° (in firing for ultra-long ranges the angle of pitch turning can exceed 90°). If the programmed angle of rocket turning exceeds the indicated limit, then the structure of the gyroscopic instruments is complicated, their overall dimensions and weight are increased.

The pitch angle program ensuring execution of all the requirements imposed on it, we will henceforth call the optimum program. For selecting pitch angle programs various methods can be employed.

In general the selection of the optimum pitch angle program is a complex variational problem. The solution to this problem by classical Euler-Lagrange methods is fraught with severe difficulties. Even in the simplest case, when a rocket flight occurs in airless space, these methods lead to the necessity of solving a boundary value problem for a complex system of differential equations. For the atmospheric flight phase the problem becomes still lengthier due to the necessity of selecting a pitch angle program taking into account the requirements demanded by the control system and the rocket design, which were discussed above.

Actually these requirements in the individual phases of a trajectory will so narrow the sphere of the possible variations in the pitch angle programs, that for these phases the solution to the variational problem does not have practical significance. Furthermore, the design execution of the rocket body and the systems, composing it, imposed restrictions on the parameters of motion (for example, on the angle of attack in the sphere of high ram pressures and on the stage separation phase, on ram pressure in the stage separation

phase, etc.). Under these conditions in attempting to solve a problem by classical methods of the calculus of variations such crude assumptions are unavoidable, that the effectiveness of these methods comes to naught.

The considerations presented above led to the development of engineering methods of selecting optimum trajectory, on the basis of which lie the ideas of the direct methods of solving variational problems. One of these methods is examined in Section 6.3.

In conclusion let us note, that there are great prospects in the development of methods of selecting the optimum trajectory of rocket motion for the new methods of the theory of optimum processes which make it possible to find solutions of optimization problems taking into account the limitations of the possible values of motion parameters or rocket characteristics [19], [25], [28].

6.2. OPTIMUM PITCH ANGLE PROGRAM IN THE NON-ATMOSPHERIC PHASE OF A TRAJECTORY

The flight range of a nose section is determined by the parameters of motion of the center of mass of a rocket at the moment of the separation of the nose section from the last stage of the rocket, i.e., at the moment of the introduction of the main instruction:

$$L=L(q_i). \tag{6.3}$$

The parameters of motion of the center of mass of a rocket at the moment of the introduction of the main instruction q_i (i = 1-6), in turn, are determined by the pitch angle program $\phi(t)$:

$$q_i=q_i[\varphi(t)] \quad (i=1\div6).$$

Thus, the flight range is determined by the program of $\phi(t)$:

$$L=L[\varphi(t)].$$

The necessary condition of the extremum of functional (6.3) can be written in the form

$$\delta L(q_i)=\sum_{i=1}^{6}\frac{\partial L}{\partial q_i}\,\delta q_i=0. \tag{6.4}$$

Here $\partial L/\partial q_i$ - the ballistic range derivatives with respect to the parameters of motion at the moment of the introduction of the main instruction; δq_i (i = 1-6) - the deviations in the parameters of motion of a rocket to the moment of the introduction of the main instruction, caused by variation in the pitch angle program $\delta\phi(t)$.

The pitch angle program $\phi(t)$, satisfying condition (6.4), realizes the maximum firing distance. We will carry out the determination of this program under the following assumptions:

1) the effect of the rotation of the earth on the parameters of motion of the rocket in the powered-flight phase is small and it can be disregarded;

2) the aerodynamic forces are equal to zero;

3) the gravitational field in the powered-flight phase is plane-parallel, the acceleration due to gravity is constant (it does not depend on altitude).

The equations of motion of a rocket in the powered-flight phase of its trajectory in a terrestrial coordinate system in this case are written in the form:

$$\left.\begin{aligned}\dot V_x&=\frac{P}{m}\cos\varphi(t);\\ \dot V_y&=\frac{P}{m}\sin\varphi(t)-g;\\ \dot x&=V_x;\\ \dot y&=V_y.\end{aligned}\right\} \tag{6.5}$$

The parameters of motion of a rocket at the moment of the introduction of the main instruction $t_{к}$ can be determined by integrating the equation of (6.5):

$$\left.\begin{aligned}
V_{x_к} &= V_{x0} + \int_0^{t_к} \frac{P}{m} \cos\varphi(t)\,dt;\\
V_{y_к} &= V_{y0} + \int_0^{t_к} \left(\frac{P}{m} \sin\varphi(t) - g\right) dt;\\
x_к &= x_0 + V_{x0} t_к + \int_0^{t_к} \left(\int_0^{t} \frac{P}{m} \cos\varphi(t)\,dt\right) dt =\\
&= x_0 + V_{x0} t_к + \int_0^{t_к} (t_к - t) \frac{P}{m} \cos\varphi(t)\,dt;\\
y_к &= y_0 + V_{y0} t_к + \int_0^{t_к} (t_к - t) \left(\frac{P}{m} \sin\varphi(t) - g\right) dt.
\end{aligned}\right\} \quad (6.6)$$

Time $t_{к}$ in these equations is a constant value. Thus the variations in the parameters of motion at the moment of the main instruction and the variation in the firing range with variation in the pitch angle program will be written in the form:

$$\left.\begin{aligned}
\delta V_{x_к} &= -\int_0^{t_к} \frac{P}{m} \sin\varphi(t)\,\delta\varphi(t)\,dt;\\
\delta V_{y_к} &= \int_0^{t_к} \frac{P}{m} \cos\varphi(t)\,\delta\varphi(t)\,dt;\\
\delta x_к &= -\int_0^{t_к} (t_к - t) \frac{P}{m} \sin\varphi(t)\,\delta\varphi(t)\,dt;
\end{aligned}\right\} \quad (6.7)$$

$$
\left.
\begin{aligned}
&\delta y_{\kappa} = \int_0^{t_{\kappa}} (t_{\kappa} - t)\frac{P}{m}\cos\varphi(t)\,\delta\varphi(t)\,dt;\\
&\delta L = \left(\frac{\partial L}{\partial V_x}\right)_{\kappa}\delta V_{x_{\kappa}} + \left(\frac{\partial L}{\partial V_y}\right)_{\kappa}\delta V_{y_{\kappa}} + \left(\frac{\partial L}{\partial x_{\kappa}}\right)_{\kappa}\delta x_{\kappa} + \left(\frac{\partial L}{\partial y_{\kappa}}\right)_{\kappa}\delta_{y_{\kappa}} =\\
&= \int_0^{t_{\kappa}} \left[-\left(\frac{\partial L}{\partial V_x}\right)_{\kappa}\frac{P}{m}\sin\varphi(t)\,\delta\varphi(t) + \left(\frac{\partial L}{\partial V_y}\right)_{\kappa}\frac{P}{m}\cos\varphi(t)\,\delta\varphi(t) -\right.\\
&\quad - \left(\frac{\partial L}{\partial x}\right)_{\kappa}\frac{P}{m}(t_{\kappa} - t)\sin\varphi(t)\,\delta\varphi(t) +\\
&\quad \left. + \left(\frac{\partial L}{\partial y}\right)_{\kappa}\frac{P}{m}(t_{\kappa} - t)\cos\varphi(t)\,\delta\varphi(t)\right]dt.
\end{aligned}
\right\} \tag{6.7}
$$

Considering the arbitrary nature of variation $\delta\phi(t)$, it is possible to show that the necessary condition of equality to zero variation in range δL is the equality to zero of the integrand at any point of the trajectory of the powered-flight phase:

$$
\begin{aligned}
&-\left[\left(\frac{\partial L}{\partial x}\right)_{\kappa}(t_{\kappa} - t) + \left(\frac{\partial L}{\partial V_x}\right)_{\kappa}\right]\sin\varphi(t) +\\
&+\left[\left(\frac{\partial L}{\partial y}\right)_{\kappa}(t_{\kappa} - t) + \left(\frac{\partial L}{\partial V_y}\right)_{\kappa}\right]\cos\varphi(t) = 0.
\end{aligned} \tag{6.8}
$$

The last equation directly gives the dependence of the optimum pitch angle program on flight time:

$$
\operatorname{tg}\varphi(t) = \frac{\left(\frac{\partial L}{\partial y}\right)_{\kappa}(t_{\kappa} - t) + \left(\frac{\partial L}{\partial V_y}\right)_{\kappa}}{\left(\frac{\partial L}{\partial x}\right)_{\kappa}(t_{\kappa} - t) + \left(\frac{\partial L}{\partial V_x}\right)_{\kappa}}. \tag{6.9}
$$

This formula is suitable in principle for programming the pitch angle of ballistic missiles in firing for any range, if the accepted assumptions are fulfilled. Let us examine these assumptions in more detail.

The first assumption, apparently, has insignificant effect on program $\phi(t)$, since the parameters of motion in the powered-flight phase weakly depend on whether or not the rotation of the earth is taken into account in equations (6.5). Moreover, the effect of the rotation of the earth on the appearance of program $\phi(t)$ with variation in firing range and in the geographical conditions of the rocket launch (the latitude of the launch site and the firing azimuth) is taken into account in formula (6.9) by corresponding variation in the derivatives

$$\frac{\partial L}{\partial V_x}, \frac{\partial L}{\partial V_y}, \frac{\partial L}{\partial x}, \frac{\partial L}{\partial y}.$$

The second assumption can impose substantial restrictions on the possibility of using formula (6.9). For rockets with a firing range of up to 1000-1500 km the powered-flight phase of the trajectory, as a rule, lies in the dense layers of the atmosphere. The determination of the optimum pitch angle program using this formula unavoidably leads to error. For intercontinental missiles the greater part of the powered-flight phase (approximately 60-70%) lies in the rarefied layers of the atmosphere, and in this part of the trajectory the optimum control program is close to program (6.9).

The third assumption is rather well fulfilled in firing for short and intermediate ranges, when the altitude of the powered-flight phase does not exceed 80-100 km, and the range angle at the moment of the main instruction does not exceed several degrees. The condition of the constancy of acceleration due to gravity in this case is fulfilled with an error of 2-3%, and the condition of plane-parallelness the gravitational field – with an error of $1°$-$2°$. For intercontinental rockets the cited errors increased by two or three times.

The errors in determining the pitch angle program due to these errors can be reduced, if the acceleration due to gravity in calculating the powered-flight phase is taken equal to its mean value in this

phase, and the equations of motion (6.5) are written in the coordinate system O'x'y', the origin O' of which is located on the surface of the earth at a distance $x(t_н)/2$ from the launch point in the line of fire; axis O'x' is directed at a tangent to the surface of the earth in the firing plane; axis O'y' – along the vertical.

The equation of the optimum pitch angle program $\phi'(t)$ in this coordinate system takes the form

$$\operatorname{tg}\varphi'(t)=\frac{\left(\frac{\partial L}{\partial y'}\right)_к(t_к-t)+\left(\frac{\partial L}{\partial V_{y'}}\right)_к}{\left(\frac{\partial L}{\partial x'}\right)_к(t_к-t)+\left(\frac{\partial L}{\partial V_{x'}}\right)_к}, \tag{6.10}$$

where $\partial L/\partial x'$, $\partial L/\partial y'$, $\partial L/\partial V_{x'}$, $\partial L/\partial V_{y'}$ – the ballistic range derivatives with respect to the parameters of motion in the coordinate system O'x'y'.

The conversion of the pitch angle program to a coordinate system located at the launch point, can be accomplished using formula

$$\varphi(t)=\varphi'(t)-\chi,$$

where χ – the range angle from the launch point to the origin O' of coordinate system O'x'y'.

It is necessary to note that the determination of the optimum pitch angle program using formula (6.9) is an iterative process, since the partial range derivatives with respect to the parameters of motion of the center of mass of the rocket at the moment of the main instruction are unknown. First the calculation of the trajectory with the pitch angle program selected as a first approximation is carried out and derivatives $(\partial L/\partial x)_н$, $(\partial L/\partial y)_н$, $(\partial L/\partial V_x)_н$, $(\partial L/\partial V_y)_н$ are determined. The ballistic derivatives obtained during the first calculation, are used for determining the second approximation of the pitch angle program already using formula (6.9). Then the calculation of the powered-flight phase is repeated, etc. The rate of

convergence of the iterative process depends on the closeness of the first approximation of program $\phi_1(t)$ to the optimum program.

6.3. THE METHOD OF SELECTING PITCH ANGLE PROGRAMS

General Aspects

As is known, the solution to a variational problem by the direct method consists in the following stages.

1. The construction of the minimizing sequence of functions $y_i(t)$: $y_1(t), y_2(t), \ldots, y_n(t)$, possessing the property:

$$\lim_{n\to\infty} I(y_n(t)) = \min_{y(t)} I(y(t)) = I(y^0(t)), \qquad (6.11)$$

where $I(y(t))$ – the optimizing functional.

Curve $y^0(t)$ is limiting curve of the sequence $\{y_n(t)\}$ and the solution to the variational problem.

2. The proof of existence for the sequence $\{y_n(t)\}$ of limiting curve $y^0(t)$.

3. The proof of the legitimacy of the maximum transfer (6.11).

With respect to the problem of selecting of optimum pitch angle programs it is necessary to find such a sequence $\{\phi_n(t)\}$, which would ensure the extremum of certain functional $I(\phi(t))$, for example the maximum distance $L(\phi(t))$.

The methods of constructing sequence $\{\phi_n\}$ can be rather diverse. However the most expedient is the method based on the representation of sequence $\{\phi_n\}$ in the form of a sequence of families of programs depending on the parameters $\lambda_1, \lambda_2, \ldots, \lambda_n$:

$$\varphi_1(t, \lambda_1), \varphi_2(t, \lambda_1, \lambda_2), \ldots, \varphi_n(t, \lambda_1, \ldots, \lambda_n).$$

If program $\phi_n(t, \lambda_1, \ldots, \lambda_n)$ depends on one unknown parameter $(n = 1)$, then the family of such programs is called uniparametric, with two unknown parameters $(n = 2)$ – biparametric, etc.

Our problem will be to determine such a program of $\phi_n(t, \lambda_1, \ldots, \lambda_n)$, which would approximate function $\phi^0(t)$, realizing the extremum of functional $L(\phi(t))$, i.e., would ensure insignificant difference in the value of functional $L(\phi_n^0(t))$ from the extremum of functional $L(\phi^0(t))$ on the condition that program $\phi_n(t, \lambda_1, \ldots, \lambda_n)$ satisfies all the requirements imposed on the configuration of the trajectory by the rocket design and by its control system, and the expenditures of time on searching for the optimum pitch angle program $\phi_n(t, \lambda_1, \ldots, \lambda_n)$ are acceptable for practice.

The selecting of the sets of programs $\{\phi_n\}$ is to a known extent arbitrary, however, as was already mentioned earlier, the requirements imposed on the configuration of the trajectory and the pitch angle program so narrow this "arbitrariness," that even the initial set of programs for the individual flight phases becomes almost determined. The selection of the sets of programs is limited to a class of continuous functions, since the pitch angle program should be continuous; in the high ram pressure phase the appearance of the set $\{\phi_n\}$ is determined by the condition of small angles of attack; in the region of the main instruction from the radio control system – by the condition of the constancy of the pitch angle, etc.

When selecting the sequence of programs $\{\phi_n\}$ there is no need to carry out the proofs of existence for this sequence of limiting curve $\phi^0(t)$ and for the validity of the maximum transfer (6.11). In fact, the sphere of possible variations in the pitch angle programs taking into account the requirements presented in Section 6.1, is completely limited. The initial set of programs $\phi_1(t, \lambda_1, \ldots, \lambda_1)$ and the optimum pitch program $\phi^0(t)$ lies in this narrow sphere. Thus even without mathematical proof it is obvious, that with an increase in the number of parameters λ_n and when selecting their optimum values from the permissible sphere the program $\phi_n(t, \lambda_1, \ldots, \lambda_n)$ from the examined sequence of sets of continuous functions will approach the

optimum program.

Further, for fulfilling equality (6.11) it is necessary that functional $L(\phi_n(t))$ be continuous in a class of continuous functions $\phi_n(t, \lambda_1, \ldots, \lambda_n)$. When the optimizing functional is the firing range, the continuity of function $L[\phi_n(t, \lambda_1, \ldots, \lambda_n)]$ is obvious.

Finally, it is necessary to note that for practice there is no need for the strict fulfillment of equality (6.11), and the fulfillment of this condition with a certain error ΔL_n, which corresponds to the permissible loss in maximum range:

$$L[\dot{\varphi}^0(t)]-L[\varphi_n(t, \lambda_1, \ldots, \lambda_n)]\leqslant\Delta L_n.$$

Sets of Pitch Angle Programs

The powered-flight trajectory of a rocket can be arbitrarily divided into three phases:

1) the subsonic velocity phase $(0-t_I)$;

2) the trans-and supersonic velocity and the high ($q > 800-1000$ kg/m^2) ram pressure (t_I-t_{II}) phase;

3) the low ram pressure phase or the non-atmospheric phase $(t_{II}-t_\kappa)$.

The 1st Phase. The configuration of the subsonic phase of the trajectory weakly affects maximum range and rocket dispersion. In connection with this the pitch angle program in this phase is selected from the condition of a safe period for the starting of vertical flight t_B, permissible pitch turning of the rocket and ensuring small angles of attack at the point of linking of the 1st and 2nd phases.

The pitch angle program in the 1st phase after vertical flight

($t_a < t < t_I$) can be selected in the form of a cubical parabola (Figure 6.2, a) or two straight lines (see Figure 6.2, b):

$$\varphi_1(t) = at^3 + bt^2 + c \text{ when } t_a < t < t_I \quad (6.12)$$

or

$$\varphi_1(t) = \begin{cases} \frac{\pi}{2} - \dot{\varphi}_1(t - t_a) \text{ when } t_a \leqslant t \leqslant t'; \\ \varphi_1 - \ddot{\varphi}_1(t - t_1) \text{ when } t' \leqslant t \leqslant t_1. \end{cases} \quad (6.13)$$

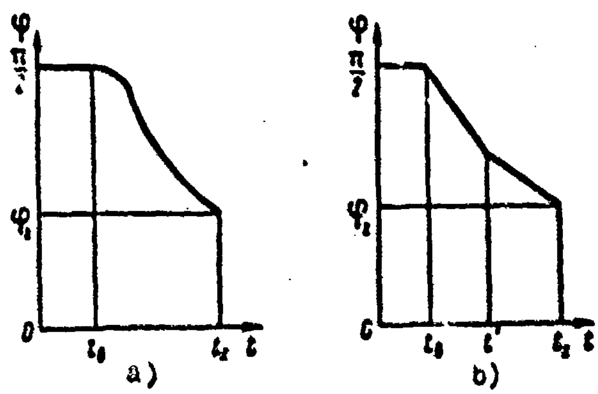


Fig. 6.2. The pitch angle program in the initial phase of the trajectory: a) in the form of a cubical parabola; b) in the form of two segments of straight lines.

Coefficients a, b, c, in the parabola equation and the slope of the straight lines are determined by condition $\alpha = 0$ when $t = t_I$, by the value of pitch angle ϕ_I at the same moment and by the permissible magnitude of the rate of pitch.

The 2nd Phase. The basic requirements, imposed in this phase on the pitch angle program, apart from ensuring maximum range and the assigned dispersion, are the ensuring of limited transverse accelerations, acceptable temperature regimes, rocket controllability and

stage separation reliability. The set of programs in this phase is characterized by the condition of small angles of attack which can be written in the form

$$\dot{\varphi}_2(t) = -\frac{g}{V}\cos\varphi_2(t) \text{ when } (t_1 \leq t \leq t_{II}). \tag{6.14}$$

The integration of equation (6.14) gives the following expression for the pitch angle program in the 2nd phase:

$$\varphi_2(t) = \operatorname{arctg}\left[e^{-\int_{t_1}^{t}\frac{g}{V}dt}\operatorname{tg}\left(\frac{\varphi_1}{2}+\frac{\pi}{4}\right)-\frac{\pi}{2}\right]. \tag{6.15}$$

Thus, the pitch angle program of a rocket ensuring the small values of angles of attack in area of high ram pressures, depends on the initial value of pitch angle ϕ_I and the ratio g/V. Taking into account that ratio g/V practically does not depend on the variations in the flight path within certain limits, it is possible to consider that the pitch angle program of a rocket in the second phase is determined by parameter ϕ_I. The selection of the magnitude of this parameter should be carried out taking into account the above enumerated requirements.

The 3rd Phase. The basic requirements imposed on the pitch angle program in this phase are the ensuring of maximum firing range and the assigned dispersion and in necessary cases the ensuring of the acceptable conditions for radio control system operation.

During flight in a vacuum the maximum range is ensured by the program determined by expression (6.9). This pitch angle program can be accepted as optimum, if no other specific restrictions are imposed on the form of the program in the third flight phase. In accordance with expression (6.9) the pitch angle in this phase decreases roughly with constant angular velocity $\dot{\phi}_3$.

However with such a pitch angle program the ensuring of the

normal conditions for radio control system operation are made difficult.

Furthermore, it is necessary to consider that the variation in the pitch angle of the rocket in the powered-flight phase by 90° (in firing for long ranges) complicates gyro-instrument design [12].

Considering the above cited considerations, it is possible to construct the following sequence of possible pitch angle programs:

1) the configuration of the trajectory is characterized by condition $\alpha = 0$ from $\phi = \phi_I$ up to the end of the powered-flight phase [one-parameter set $\phi(t, \phi_I)$];

2) from $\phi = \phi_I$, to a certain moment of time the program is characterized by the condition $\alpha = 0$, subsequently the pitch angle is constant $\phi = \phi_к$ [biparametric set $\phi(t, \phi_I, \phi_к)$];

3) from the flight phase when $\alpha = 0$ until the attainment of a certain value $\dot{\phi} = \phi_к$ the angular rate of pitch — the maximally permissible $|\dot{\phi}| = \phi_{max}$, subsequently $\phi = \phi_к$ [three-parametric set $\varphi(t, \varphi_I, \varphi_к, \dot{\varphi})$].

Each of the enumerated sets, except the first, rather fully realizes maximum range with assigned dispersion. From the point of view of simplicity of instrument realization the second family is preferable; in this case the replacement of the energetically optimum pitch angle program in the 3rd flight phase by a program with constant angle $\phi = \phi_к$ makes it possible to eliminate the above mentioned deficiencies in the optimum program with insignificant loss in maximum range.

The Selection of the Optimum Parameters of the Pitch Angle Program

In accordance with the direct methods of the calculus of variations the determination of the optimum parameters of the pitch angle program should be carried out by investigating the optimizing functionals (the range, dispersion, etc.) for the extremum. However the

use of the common methods of determining the extremal values of the parameters of the program is not possible as a result of the complex dependence of range and dispersion on the parameters ϕ_I and ϕ_K. In this connection the graphical-analytical method of determining the optimum values of parameters ϕ_I and ϕ_K, which consists in the following, is preferable.

A certain set of curves $\phi(t, \phi_{I1}, \phi_{K1})$, $\phi(t, \phi_{I2}, \phi_{K2})$, ..., $\phi(t, \phi_{I\,m}, \phi_{K\,m})$ is selected, every curve of which should satisfy the additional requirements, imposed on the program. In solving the system of equations of motion of the rocket in the powered-flight and unpowered-flight phases of the trajectory, the dependences of the optimizing functionals $I(\phi(t))$ range, dispersion, etc.), and also of other characteristics of the rocket (for instance, the overloads of the nose section, angles of radio sighting and others) on the parameters of programs ϕ_I, ϕ_K are obtained. The obtained dependences are represented graphically and are used for selecting the optimum values of the parameters of the pitch angle program, which reduces to the following stages.

1. Determining the firing range and the dispersion of the nose sections depending on the parameters of the pitch angle program.

2. Calculating the temperatures and the thicknesses of the body coverings, the "dry" weights of the rocket stages, control element effectiveness, the reliability of stage separation, radio-line-of-sight, overloads acting on the nose section, and also other characteristics for various values of the program parameters.

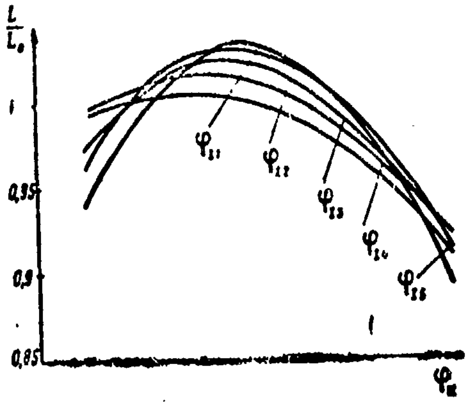
3. Constructing the permissible region of parameter variation for the pitch angle progr[illegible] with respect to rocket and control system design.

4. Selecting the optimum values of the program parameters which correspond to maximum firing range with the assigned dispersion.

As an example let us examine the order of selecting the optimum

parametric values of the pitch angle program for a rocket with assigned firing range L_0. In this case for clarity and simplicity of discussion, from the complete complex of requirements (see Section 6.1) we will consider only the requirements of ensuring radio-line-of-sight (6.1) and (6.2) and permissible overloads.

The approximate dependences of range and dispersion, angles of radio-line-of-sight, angles of departure and reentry of nose section into the atmosphere on the parameters of the pitch angle program are given in Figures 6.3-6.11. From an examination of these graphs it is possible to draw the following conclusions.



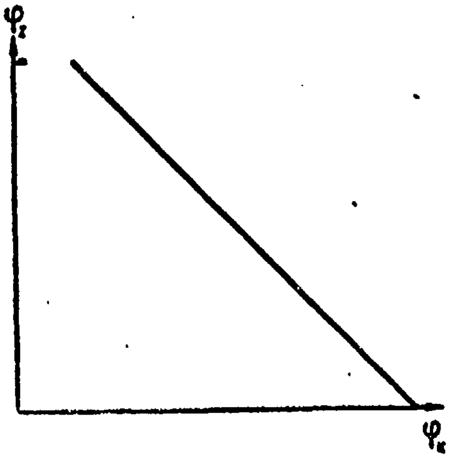


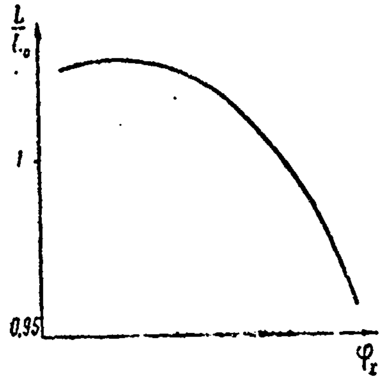
Fig. 6.3. Fig. 6.4.

Fig. 6.3. The dependence of maximum firing range on program parameters ϕ_I and ϕ_κ ($\phi_{I1} < \phi_{I2} <, \ldots\ldots, < \phi_{I5}$)

Fig. 6.4. The connection between the values of program parameters ϕ_I and ϕ_κ, which ensure maximum firing range.

The combinations of program parameters corresponding to maximum range and to minimum dispersion, do not coincide with each other. The angles of departure corresponding to minimum dispersion, are larger than the angles of maximum range; with an increase in the

angle of departure maximum range decreases. Of the conditions of radio-line-of-sight requirement (6.2) is the most essential.



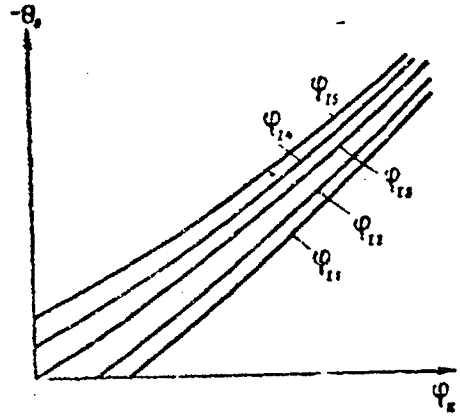


Fig. 6.5.   Fig. 6.6.

Fig. 6.5. The dependence of maximum firing range on program parameter ϕ_I with optimum values of angle ϕ_K.

Fig. 6.6. The dependence of the angle of reentry into the atmosphere Θ_0 on program parameters ϕ_I and ϕ_K.

An example of a permissible region of program parameters obtained taking the dependences given above into account, is shown in Figure 6.12. In this figure the curves, corresponding to equal ranges, are indicated by dot-and-dash lines, and the curves of equal dispersion – by solid lines. The selecting of the optimum values of parameters ϕ_I and ϕ_K from the permissible region reduces to selecting such combinations of these parameters which at the assigned dispersion correspond to maximum range.

The examined method makes it possible to solve the problem of determining the optimum pitch angle program in a complete formulation taking all the technical specifications and limitations into account. Moreover, this method also has a number of other advantages.

In designing it is frequently necessary to solve the question of variation in the optimum pitch angle program with variation in any characteristics of the rocket or the specifications imposed on it.

In this case for selecting a new program the results of the previous investigations can be used. Moreover, by using these results, it is possible to forecast the trend of the variation in the program with variation in the rocket characteristics.

The method makes it possible to find the solution of the problem of optimizing pitch angle program with any accuracy. This fact is very important because at various stages in the design and the development of a rocket it is not always necessary to know the exact solution of the optimization problem; a rough approximation is sometimes sufficient. The presented method in this case is very effective, since it makes it possible to find the solution with the accuracy which is required at the various design stages.

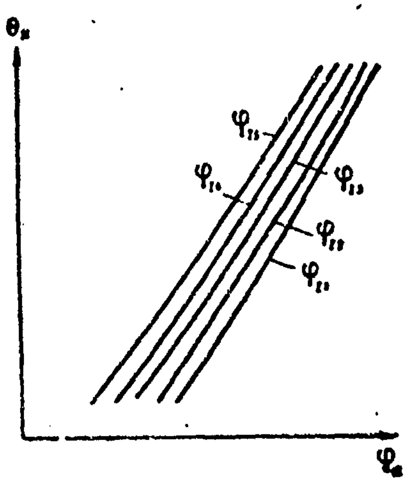


Fig. 6.7.

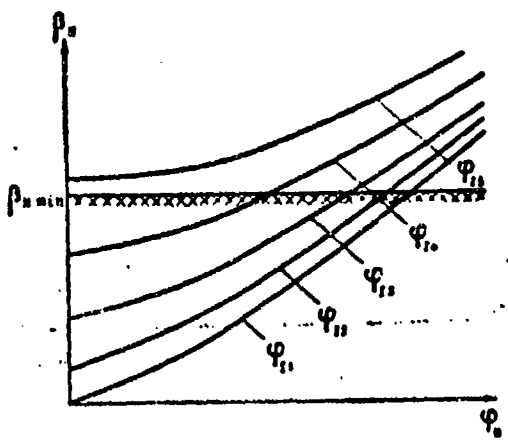


Fig. 6.8.

Fig. 6.7. The dependence of the slope angle of the trajectory at the moment of the main instruction Θ_H on program parameters ϕ_I and ϕ_H.

Fig. 6.8. The dependence of the angle of radio-line-of-sight β_H on program parameters ϕ_I and ϕ_H: β_H – the angle included between the line of radio-line-of-sight and the plane of local horizon at the point of the location of the ground-based antenna; $\beta_{H\ min}$ – minimum permissible angle β_H, determined by the permissible measurement error by the motion parameter radio control system.

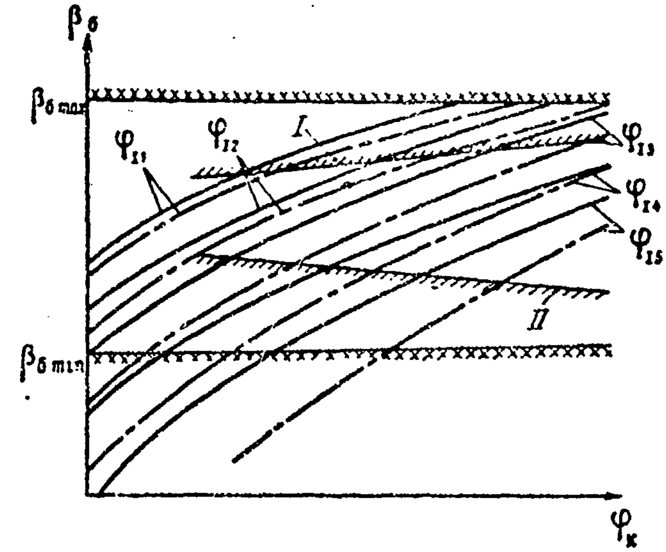


Fig. 6.9. The dependence of onboard angle $\beta_б$ of radio-line-of-sight on program parameters ϕ_I and $\phi_к$: $\beta_б$ – the angle included between the line of radio-line-of-sight and the longitudinal axis of the rocket; $\beta_{б\ max}$, $\beta_{б\ min}$ – the maximum and minimum permissible values of the onboard angle of radio-line of-sight determined by the width of the radiation pattern of the onboard ground-based antennas; I and II – the boundaries of the region of parameters ϕ_I and $\phi_к$, guaranteeing the location of angle $\beta_б$ within the range $\beta_{б\ min} < \beta_б < \beta_{б\ max}$ under all geographical launch conditions: ——— the value of angle $\beta_б$ at the moment of the main instruction: -·-·- the value of angle $\beta_б$ at moment of the begining of operation of the radio control system.

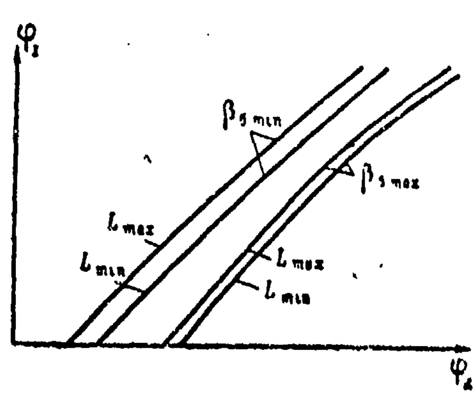


Fig. 6.10. The dependence between program parameters ϕ_I and $\phi_к$, ensuring during firing for maximum and minimum ranges permissible values of the onboard angle of radio-line-of-sight.

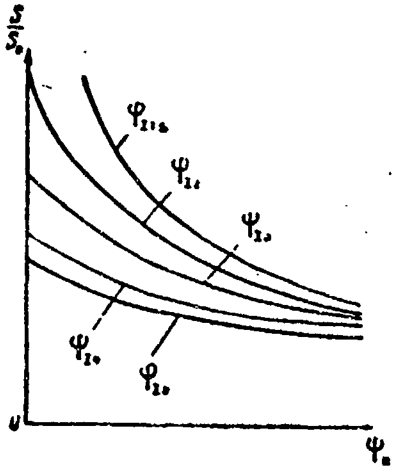


Fig. 6.11. The dependence of range dispersion and lateral dispersion on program parameters ϕ_I and ϕ_H: S_0 – the area of the dispersion ellipse taken as unity.

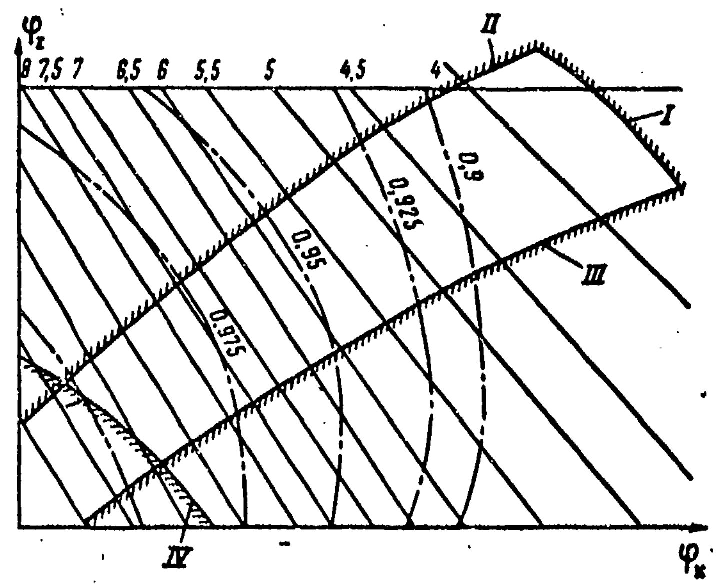


Fig. 6.12. The permissible region of program parameters ϕ_I and ϕ_H: I – boundary of the region with respect to permissible overload; II – boundary of the region with respect to angle $\beta_{\delta\ min}$; III – boundary of the region with respect to angle $\beta_{\delta\ max}$; IV – boundary of the region with respect to angle $\beta_{H\ min}$; —— the line of equal areas of dispersion with respect to area S_0, taken as unity; –·–·– the line of identical firing ranges with respect to range L_0, taken as unity.

CHAPTER VII

SETTING DATA FOR ROCKET LAUNCHING

For launching a guided missile it is necessary to carry out operations connected with preparing the missile systems and the ground equipment for launch. With respect to these operations it is possible to separate into an individual group the operations with regard to preparing the so-called "setting data" of the missile control system and of the ground equipment.

By setting data we understand the data which is intended for prelaunch adjustment of the instruments of the control and aiming systems, ensuring the following of the optimum flight path by the [NS] (ГЧ) to the target. The composition of the setting data is determined by the aiming method, by the type of control system, by the form of the control functional, by the stage separation system, etc.

The present chapter briefly examines the approximate composition of setting data and the methods of determining them for a rocket with an inertial control system.

7.1. METHODS OF PREPARING SETTING DATA

In preparing the initial data for firing long-range missiles the coordinates of the launch and target points are considered given relative to the accepted shape of the earth: latitude ϕ_{Γ}, longitude λ and altitude h. With certain launch coordinates it is sometimes convenient to give the target coordinates by means of the azimuth of

the launch-target line $A_{c\phi}$ and range $L_{c\phi}$, given on a spherical earth.

The basic initial data for firing are the value of the range control functional J, programmed into the flight control system and determining the time of the introduction of the command for the shutdown of the last stage engine, and the firing azimuth A_0, employed for aiming the rocket. These data are also called the basic settings.

The preparing of the data for a rocket launch is not completely culminated by the determining of the basic settings. Depending on the design of the rocket control system (in particular, on the range control principles and of the direction of firing), on the arrangement of the subassemblies and systems mounted on rocket, and also on the characteristics of their operation, besides the basic settings, the auxiliary settings can also be determined, namely:

a) the setting for the preliminary instruction for engine shutdown (in shutting down the engine in two stages);

b) the settings, determining the moments of stage separation (for multistage rockets), for throtting the engine in the case of an increase in overload above permissible overload, etc.

Thus, for example, for a rocket with a range control functional of the type of (5.49)

$$J = w_\lambda + p\int_0^t (w_\mu - w_\mu^*)\,dt$$

for turning the control system devices the following settings are necessary:

a) the value of function J at the moment of the execution of the preliminary and the main commands;

b) the values of angles λ and μ and parameter p, depending on range, geographical conditions, etc.

The initial data for firing (basic and auxiliary settings) are formulated in the form of an appropriate document.

It is necessary to note that the totality of tactico-technical requirements, imposed on one or another type of rocket, and also the actual construction of the flight control system, ensuring the execution of these requirements, determines the variety of methods of preparing the initial data for firing. The detailed development of these methods can be carried out during the designing of a rocket taking its actual characteristics into account. Thus only general concepts of the methods of calculating setting data for firing are given below.

It is possible to distinguish two methods of obtaining basic settings:

1) calculating the "falling" trajectory for the given coordinates of the launch point and the target (integrating the equations of motion of the rocket);

2) calculating the basic settings with the aid of firing tables (with final formulas).

The first method of determining basic settings can be used in the early preparation of initial data. With rather reliable and operationally convenient special electronic computers intended for calculating "falling" trajectory, it can also be used in preparing a rocket for launch.

The second method makes it possible with rather simple calculations, usually carried out by "manual calculation" methods, to obtain the basic settings both in advance and during the preparation of a missile for launch. The reliability of the obtained results is ensured by checking the calculations. It should be noted that for rockets with long firing ranges the use of firing tables becomes difficult due to their great volume.

Of great interest is the preparing of initial data for firing with the aid of special-purpose electronic computers [EC] (ЭВМ).

It is possible to present the following operating principles of such machines.

1. From the given coordinates of the launch point and the target site rocket flight control programs (pitch program, apparent velocity program and others) are selected. The selection of the programmed functions of the control system can be carried out by an earlier worked out algorithm, for example, with the use of methods presented in Chapter VI. For the selected control programs by calculating the "falling" trajectory the necessary settings of the rocket flight control devices are determined.

2. For the possible firing conditions the set of rocket flight control programs selected earlier is input into the machine "memory." In preparing the data it is necessary depending on the firing conditions from the given set to select the corresponding rocket flight control programs and by calculating the "falling" trajectory to determine the setting of the control devices.

3. The setting of the rocket flight control devices, and also the control programs are calculated earlier and depending on the launch conditions are input into the machine "memory" in the form of coefficients of certain known functions. The machine carries out the calculation of all the necessary settings with comparatively simple final formulas (similar to calculating settings in compiling firing tables).

The enumerated operating principles of special-purpose electronic computers have their advantages and disadvantages.

The first method is the most universal. But, on the other hand, the volume of computational operations with this method is so great, that for carrying it out a special-purpose electronic computer will

be required possessing the features of a general-purpose permanently fixed console computer.

The second method is based on calculating "falling" trajectory with selected control programs and requires solving of a complex system of equations of motion. For solving such a problem the special-purpose electronic computer should also possess all the possibilities of a general-purpose fixed console computer. It is not possible to make such a machine simultaneously satisfying the requirements making it suitable for military application (compactness, transportability, servicing ease and others).

The third method makes it possible to carry out the preparation of the data for firing with final formulas with a comparatively small number of simple operations. This method is the most acceptable for the use of special-purpose electronic computers.

The final data on basic settings, obtained with the aid of firing tables (or upon solving the "falling" trajectory), are formulated in the form of an appropriate document, the approximate form of which is the following.

I. Initial Data

1. Spherical range $L_{c\phi}$...

2. Spherical azimuth of the launch-target line $A_{c\phi}$...

II. Data for Launch

1. Aiming Azimuth A_0...

2. Value of the control functional J...

7.2. CALCULATING "FALLING" TRAJECTORY

The calculation of "falling" trajectory is carried out for the purpose of determining aiming azimuth and time of introduction of the

preliminary and main instructions for shutting down the engine ensuring the transmission (with assigned accuracy) of optimum trajectory from the launch point to the target. Since the moments of time of the introduction of the preliminary and main instructions are usually connected by the relationship

$$t_{г.к}=t_{п.к}+\Delta t,$$

where Δt = const (for optimum trajectory), the problem reduces to determining only the azimuth and the time of the introduction of the main instruction.

The basic initial data for calculating "falling" trajectory are the following:

1) the characteristics of standard atmosphere, the gravitational field of the earth and the shape of earth;

2) the aerodynamic, geometric, centering [c.g.] and weight characteristics of the rocket;

3) the engine characteristics;

4) the control system characteristics (including delays in activation of the instruments);

5) the engine operation time schedule;

6) the program of variation in pitch angle and apparent velocity with respect to flight time;

7) the geodetic coordinates of the launch and target points.

The system of differential equations describing the motion of the center of mass of the rocket in the powered-flight and unpowered-flight phases of the trajectory is composed taking into account the

requirements for the permissible magnitude of errors in determining data for launch.

The calculation of "falling" trajectory is based on the numerical integration of the system of equations of motion of the rocket in the powered-flight and unpowered-flight phases of the trajectory. The ensuring of impact on the assigned target is accomplished by selecting the optimum time (and the value of functional J corresponding to it) of engine shutdown and aiming azimuth.

The calculation of falling trajectory can be carried out in two stages in the following manner:

1. Given: the approximate aiming azimuth A_0', the coordinate of the launch point (T_0) and the range L. It is necessary to determine the time of introduction of the main instruction, necessary for obtaining assigned range L with the assigned aiming azimuth A_0'.

2. The coordinates of the launch and target points (T_0, T_u) and the time of the introduction of the main instruction determined in the first stage are given. It is necessary to determine the precise azimuth, and then the final value of the time of the introduction of the main instruction.

In carrying out the calculations in the first stage for the assigned launch point, the accepted approximate value of the firing azimuth and the two randomly selected values of the time of the introduction of the main instruction t_1 and t_2 the coordinates of the impact points (T_1 and T_2) are determined. From the assigned coordinates of the launch point and the obtained coordinates of the impact points geodetic ranges L_1 and L_2 are determined. Subsequently the time of the main instruction is made more precise by linear interpolation

$$t_3 = t_1 + \frac{t_2 - t_1}{L_2 - L_1}(L - L_1). \qquad (7.1)$$

The calculation of the powered-and unpowered-flight phases is carried out for obtained time t_3.

The time of the introduction of the main instruction is again made more precise using the formula given above. In this case it is assumed, that

$$t_1 = t_3;\ t_2 = \bar{t},$$

where $\bar{t}$ – that one of the moments of time t_1 and t_2, for which ΔL_i, equal to

$$\Delta L_i = |L - L_i| \quad (i = 1;\ 2), \tag{7.2}$$

will be less

This definitizing is carried out until the following inequality is satisfied

$$\Delta L_i \leqslant \varepsilon(L), \tag{7.3}$$

where $\varepsilon(L)$ – the assigned accuracy of "falling" trajectory calculation (for range).

The calculations in the second stage are carried out for the purpose of determining aiming azimuth. For the two values of the time of the introduction of the main instruction t_1 and t_2, calculated under condition (7.3), and the given azimuth A_0, the coordinates of the impact points $T_1(\phi_{r1}, \lambda_1)$ and $T_2(\phi_{r2}, \lambda_2)$ and the distance D between the target point and a line, passing through points T_1 and T_2 are determined. The definitizing of the initial value of azimuth is carried out with formula

$$A_0 = A_0' + \frac{D}{\partial D/\partial A}. \tag{7.4}$$

where $\partial D/\partial A$ is calculated by the method of numerical differentiation.

The selection of aiming azimuth is finalized upon satisfying condition

$$|D| \leqslant \varepsilon(A), \tag{7.5}$$

where $\varepsilon(A)$ – the assigned accuracy of the calculation of "falling" trajectory with respect to azimuth.

After selecting the aiming azimuth the calculation of "falling" trajectory is carried out using the value of this azimuth for the final determination of the time of the introduction of the main instruction, i.e., the checking of condition (7.3).

7.3. COMPILING FIRING TABLES

Tabular and Actual Conditions of Rocket Launch

The principle of compiling firing tables consists in the following. The idealized conditions of rocket launch – the so-called tabular conditions, for which the determination of data for firing does not present any specific difficulties, are examined. The difference in the actual launch conditions from the tabular conditions is taken into account by calculating corrections. The determining of the corrections and their tabulation is the content of the problem of compiling firing tables.

Usually they try to make rocket control systems so that the possible deviations in the actual launch conditions from the tabular conditions (let us call these deviations perturbing factors) are compensated for by the operation of the control system. However a control system can have systematic errors dependent on the magnitude of the perturbing factors. Thus, one of the problems in compiling firing tables is determining the effect of the systematic errors of the control system on the deviation in the impact points from the target depending on the magnitude of the perturbing factors and when necessary the calculation of the corrections to the table settings corresponding to the tabular conditions.

Since it is not expedient (and also sometimes not possible) to compensate for all the perturbing factors by the control system, another problem in compiling firing tables is the study of the effect of certain perturbing factors on the deviations in the NS impact points from the target and the compensation for them by corrections to the basic settings corresponding to the tabular conditions.

The most important of these factors are the geophysical conditions of rocket launch. The basic problem in compiling firing tables is the obtaining of corrections in the setting data, taking the geophysical conditions of launch into account.

Moreover, in compiling firing tables the necessity of introducing corrections can arise taking into account control loop instrumental errors, deviations in the launch weight of the rocket and in specific thrust, anomalies in the gravitational field and other perturbing factors. All these corrections in the basic setting data (with the exception of the geophysical conditions of launch) are small values (within the limits of a few kilometers) and their introduction into the firing tables can be due only to an attempt to increase firing accuracy. In connection with the fact that the calculation of small corrections does not present any fundamental difficulties and is not always necessary, we will examine the questions connected with the calculation of these corrections.

By tabular it is possible to understand the following conditions:

a) The earth is a non-rotating sphere with radius $R = 6371$ km;

b) acceleration due to gravity is directed toward the center of the earth and in magnitude is inversely proportional to the square of distance from the center of the earth, i.e., it obeys Newton's law;

c) the acceleration due to gravity on the surface of the earth is equal to the normal value $g_0 = 9.81$ m/s^2;

d) the launch and target points are located on the surface of the earth;

e) wind is absent, and the remaining meteorological elements correspond to standard atmosphere;

f) all the basic parameters of a rocket correspond to rated values;

g) anomalies in the gravitational field are absent.

Under tabular conditions the basic settings depend only on the firing range (firing azimuth is determined from purely geometrical conditions and is equal to the launch-target line).

Actual firing conditions differ considerably from tabular conditions. The effect of these differences on firing accuracy (with the exception of the geophysical conditions of launch) and the solution of question of the necessity of taking them into account in the basic settings depend on the control system characteristics. In general in calculating basic settings depending on geophysical conditions it is expedient to take into account the rotation of the earth, the non-sphericity of the earth and the non-centrality of the gravitational field.

Calculating basic settings

Since the method of determining setting data can be worked out only by taking into account the actual characteristics of a rocket, then for example let us examine a rocket equipped with a longitudinal acceleration integrator, the axis of sensitivity of which is directed at a certain constant angle λ to the initial horizon at the launch point, by the system regulating apparent velocity and by the normal and lateral stabilization systems. The instruments of the normal and lateral stabilization systems and the sensing heads of the longitudinal acceleration integrators are mounted on a gyrostabilized platform. The programs of variation in pitch and apparent velocity

are considered given and do not depend on the firing conditions. The range control functional is determined by the magnitude of apparent velocity w_λ in direction λ: $J = w_\lambda$:

Let us examine the calculation of the basic settings J and A_0 depending on geophysical conditions.

The problem consists in determining the range control functional J and the firing azimuth A_0 from the known geodetic coordinates of the launch point $\phi_{г0}$, λ_0 and the target $\phi_{гц}$, $\lambda_ц$.

Firing tables are rather convenient for practical use, if the spherical firing range $L_{cф}$, the spherical azimuth of the launch-target line $A_{cф}$ and latitude $\phi_{г0}$ in them are taken as input values.

Values $L_{cф}$ and $A_{cф}$ from known $\phi_{г0}$, λ_0 and $\phi_{гц}$, $\lambda_ц$ can be determined by formulas:

$$\left.\begin{aligned} L_{c\phi} &= R\arccos[\sin\varphi_{ц0}\sin\varphi_{ц.ц}+\cos\varphi_{ц0}\cos\varphi_{ц.ц}\cos(\lambda_ц-\lambda_0)];\\ A_{c\phi} &= \arcsin\left[\sin(\lambda_ц-\lambda_0)\frac{\cos\varphi_{ц.ц}}{\sin\frac{L_{c\phi}}{R}}\right], \end{aligned}\right\} \quad (7.6)$$

in which the geocentric latitude of the launch point or the target point is connected with the geodetic latitude of the corresponding point by formula

$$\varphi_ц = \operatorname{arctg}\left(\frac{b^2}{a^2}\operatorname{tg}\varphi_г\right), \quad (7.7)$$

where a and b – respectively the semimajor and the semiminor axes of the terrestrial ellipsoid.

The dependence of settings J and A_0 on $A_{cф}$ and $\phi_{г0}$ for assigned $L_{cф}$ can be represented in the form of the following series:

$$\left.\begin{array}{l} J = J_0 + J_1\Phi_1 + J_2\Phi_2 + \ldots + J_N\Phi_N; \\ A_0 = A_{c\varphi} + K_1\Psi_1 + K_2\Psi_2 + \ldots + K_M\Psi_M, \end{array}\right\} \quad (7.8)$$

where N and M – the number of the selected members of the expansion; Φ_i and Ψ_j – the spherical functions from ϕ_{r0}, $A_{c\phi}$.

The coefficients of the expansions $J_1, \ldots, J_N, K_1, \ldots, K_M$ are calculated according to the appropriate formulas on the basis of data, obtained by numerical integration of the series of the reference trajectories. As a result the values of coefficients $J_i(L_{c\phi})$ and $K_i(L_{c\phi})$ are obtained for all reference ranges of the range interesting us.

The values of magnitudes J_i and K_i for the intermediate (between reference ranges) ranges placed in the firing tables, are determined by quadratic interpolation of dependences $J_i(L_{c\phi})$ and $K_i(L_{c\phi})$.

The interval of the inserted ranges is selected so that the error due to linear interpolation of the tabular ranges in calculating the basic settings does not exceed the permissible.

For the final solution to the question of the number of the members of the expansion inserted into the firing tables the effect on firing range and lateral deflection of the last members of the expansion is investigated.

Evaluation of the necessary number of members of the expansions is carried out proceeding from the required accuracy in obtaining magnitudes J and A_0 on the basis of analyzing of the remaining members of the expansions, defined as the difference in the corresponding values of functional $J'(\phi_{r0}, A_0)$ and the deviation in direction $\Delta A'(\phi_{r0}, A_0)$, calculated by numerical integration, and of the values of functional $J''(\phi_{r0}, A_0)$ and the deviation in direction $\Delta A''(\phi_{r0}, A_0)$. calculated on the basis of the firing tables.

As a criterion of accuracy it is possible to take the condition

when the systematic errors in determining the impact points of nose sections due to the rejection of terms of the expansions of higher orders do not exceed certain permissible values.

In conclusion a number of check computations of the basic settings is carried out with respect to the firing tables and by solving the equations of motion of the rocket. A comparison of the data obtained in this way, makes it possible to judge the correctness and the accuracy of the compilation of the firing tables.

System of Calculating and Formulating Firing Tables

1. The calculation of coefficients q_{κ}^{i} and p_{κ}^{i} of the expansions of the parameters of the end of the powered-flight phase q_{κ} and p_{κ} depending on the geodetic conditions is carried out using formulas:

$$\left.\begin{aligned} q_{\kappa} &= q_{\kappa}^{(0)} + \sum_{i=1}^{N-1} q_{\kappa}^{i} \Phi_{i}(\varphi_{r0}, A_0); \\ p_{\kappa} &= \sum_{i=1}^{M} p_{\kappa}^{i} \Psi_{i}(\varphi_{r0}, A_0), \end{aligned}\right\} \tag{7.9}$$

where $q_{\kappa} = (x_{\kappa}, y_{\kappa}, V_{x\kappa}, V_{y\kappa})$; $p_{\kappa} = (z_{\kappa}, V_{z\kappa})$.

The appropriate formulas and results of the solution of the differential equations of motion of the rocket in the powered-flight phase the trajectory are used for a number of combinations((ϕ_{r0}, A_0).

The results of the calculations of each parameter q_{κ}^{i} and p_{κ}^{i} in argument t_{κ} (from $t_{\kappa\ min}$ to $t_{\kappa\ max}$) are formulated in the form of a table.

2. Calculation of the parameters of the end of the powered-flight phase for the selected combinations (ϕ_{r0}, A_0). Formulas (7.9) and $J = w_{\lambda}$, the results of calculations of q_{κ}^{i} and p_{κ}^{i}, the

recommended combinations ($\phi_{\Gamma 0}$, A_0) are used. The results of the calculations are formulated depending on argument $t_к$ in the form of Table 7.1.

Table 7.1.

$t_к$	$(A_0, \varphi_{\Gamma 0})_2$	$x_к$	$y_к$	$V_{xк}$	$V_{yк}$	J	$z_к$	$V_{zк}$
$t_{к1}^{min}$	$(A_0, \varphi_{\Gamma 0})_1$							
	....							
	$(A_0, \varphi_{\Gamma 0})_m$							
...	....							
$t_{кn}^{max}$	$(A_0, \varphi_{\Gamma 0})_1$							
	....							
	$(A_0, \varphi_{\Gamma 0})_m$							

3. Calculation of spherical range $L_{c\phi}$, the spherical azimuth of the launch-target line $A_{c\phi}$ and of the correction in spherical azimuth $\Delta A_{c\phi} = A_0 - A_{c\phi}$.

Formula (7.6) and the results of the calculation of the parameters of the end of the powered-flight phase ($x_к$, $y_к$, $V_{x\ к}$, $V_{y\ к}$, J, $z_к$, $V_{z\ к}$) are used.

The coordinates of the impact point $\phi_{\Gamma ц}$, $\lambda_ц$ are determined by solving the differential equations of motion of the rocket in the unpowered-flight phase for all m of the initial conditions (the parameters of the end of the powered flight phase). In this case longitude λ_0 can be arbitrarily assigned and can be taken constant for all calculations. The results of the calculations are formulated in the form of Table 7.2.

4. The formulation of firing tables. For each reference value of range $L_{c\phi l}^{оп}$, entered in the table, coefficients are determined.

These coefficients for ranges lying between values J_i, K_i, are determined by quadratic interpolation. The results of the calculations can be formulated in the form of Table 7.3.

Table 7.2

$t_к$	$(A_0, \varphi_{r0})_1$				. . .	$(A_0, \varphi_{r0})_m$			
	$A_{сф}$	$L_{сф}$	$\Delta A_{сф}$	J		$A_{сф}$	$L_{сф}$	$\Delta A_{сф}$	J
$t_{к1}^{min}$									
$t_{к2}$									
. . . .									
. . . .									
. . . .									
$t_{кn}^{max}$									

Table 7.3

$L_{сф}$	$\frac{\partial J}{\partial L_{сф}}$	J_0	J_1	. . .	J_N	K_1	K_2	. . .	K_M
км	$\frac{м}{км \cdot сек}$ s	$\frac{м}{сек}$ s	$\frac{м}{сек}$ s		$\frac{м}{сек}$ s	мин min	мин min		мин min
$L_{сф1}^{min}$									
. . . . . .									
. . . . . .									
$L_{сфn}^{max}$									

For each range (or a certain scope of ranges) the necessity of taking into account all N coefficients of J_i and M coefficients of K_i on the basis calculating the remaining members is estimated. When possible a portion of the coefficients is not taken into account.

The final number of coefficients depending on range are inserted into the firing tables in a given form. Moreover, coefficients characterizing the effect of small perturbing factors on the basic settings

are also placed in the firing tables.

From the above examined system of compiling firing tables it follows that the basic settings are not directly their output values. To obtain basic settings it is necessary to carry out a number of calculations whose results depend upon geophysical conditions, and also can depend on certain official rocket data (for instance, launch weight), meteorological conditions (for instance, the temperatures of the propellant components) and others. The greater part of the calculations (the calculation of geophysical conditions, official rocket data) can be carried out beforehand with a certain problem and certain rockets intended for its execution.

CHAPTER VIII

MAXIMUM FIRING RANGE

8.1. THE CONCEPTS OF MAXIMUM FIRING RANGE AND GUARANTEED PROPELLANT RESERVES

The contemporary long-range ballistic missiles are intended for delivering a nose section over long distances, including intercontinental distances. Because of this the questions connected with ensuring the necessary firing range, are of significant interest.

Each actual type of rocket is designed for a definite scope of ranges, the boundary points of which are respectively called minimum L_{min} and maximum L_{max} firing ranges. In firing for ranges going beyond the boundaries of this scope, a rocket can not satisfy these or other technical specifications. Thus, for instance, in firing for ranges exceeding maximum range, the following abnormalities can arise:

- deficiency of propellant components;

- loss of strength by the individual structural elements due to an increase in the overloads at the end of the powered-flight phase in the atmospheric part of the unpowered-flight phase of the trajectory;

- loss of strength by the nose section due to an increase in aerodynamic heating in the atmospheric phase of the trajectory;

- an increase in the dispersion of the nose section impact points above the permissible values, etc.

It is evident that for an ideally designed missile limitations of this type should simultaneously set a limit to a subsequent increase in range. For instance, there is no sense in having a reserve of heat-shield covering, if it cannot be used due to a deficiency of propellant or loss of rocket strength.

Subsequently we will examine only those limitations on firing range which are due to the possibility of a deficiency of propellant components. Thus let us assume that the optimum trajectory and the basic design parameters of a rocket are already selected so as to ensure the greatest firing range.

Maximum firing range is a complex functional due to the parameters and functions characterizing the design of the rocket, the engine, the control system, the control program, the conditions of launch and flight, etc. The maximum possible firing range of an individual rocket, corresponding to the actual conditions of launch and flight, we customarily call rated flight range $L_{расч}$. Rated range cannot serve as design characteristic of a rocket ensuring a given spread of firing ranges, since under actual production and operating conditions the basic parameters of a rocket and the conditions of rocket launch and flight (firing conditions) are not stable, but vary within certain limits.

The design parameters of a rocket and the firing conditions accepted in calculations as standard parameters, we will call optimum (usually they correspond to their average-statistical characteristics). The flight range, corresponding to optimum rocket characteristics and firing conditions, we will call optimum $L_{ном}$. Deviations in the characteristics of a rocket and in firing conditions from optimum values, determining the deviations in rated range from optimum, we will call perturbing factors.

In accordance with the random character of perturbing factors the region of their variation and the range of the variation in rated

range can be determined only with a certain probability $1 - \varepsilon$. Apparently, for given probability ε two boundaries of variation in rated ranges exist: maximum $L_{\text{расч}}^{\max}$, corresponding to probability $P\left(L \geqslant L_{\text{расч}}^{\max}\right) = \frac{\varepsilon}{2}$, and minimum $L_{\text{расч}}^{\min}$, corresponding to probability $P\left(L \leqslant L_{\text{расч}}^{\min}\right) = \frac{\varepsilon}{2}$, (Fig. 8.1). Since the designer should reliably ensure the firing of a rocket in the given spread of ranges, including the attaining of maximum range L_{max}, then in designing a rocket it is necessary to proceed from condition $L_{\text{расч}}^{\min} = L_{\max}$. In this case for an entire ensemble of rockets of a specific class the reserves of propellant ensure the shutting down of the engine upon command from the control system with a probability of $P = 1 - \frac{\varepsilon}{2}$, i.e., the achievement of the assigned maximum range L_{max} is ensured with a given probability. It is evident, that with probability $P < 1 - \frac{\varepsilon}{2}$ in rocket launches for L_{max} depending on the actual combination of the perturbing factors the running out of propellant or of one of its components can occur, which leads to spontaneous engine shutdown. This fact is impermissible because of the considerable increase in NS impact point dispersion. Thus the shutting down of the engine of single-stage rockets and of the last stages of multistage rockets is always carried out upon instruction from the control system.

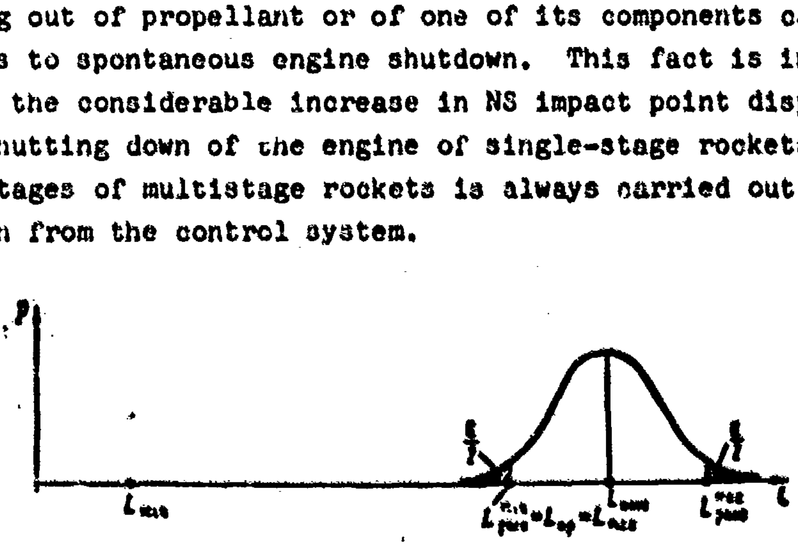


Fig. 8.1. For determining maximum firing range by the method of statistical testing.

Hence, maximum range $L_{\text{пр}}$ is the minimum value of rated range $L_{\text{расч}}^{\min}$, which can be attained by a rocket, if the propellant reserves available on it ensure with an assigned reliability $P = 1 - \frac{\varepsilon}{2}$ shutting down of the engine upon instruction from the control system (i.e., upon achievement of a specific value of the control functional). Accordingly, the maximum value of the control functional $J_{\text{пр}}$ is this greatest value of the functional whose achievement is ensured by the available

propellant reserves with an assigned level of reliability.

According to thid definition by the maximum firing range of a multistage rocket it is necessary to understand that maximum range which can be attained, if the propellant reserves available on each stage ensure with an assigned reliability the shutting down of the engine of the last stage upon instruction from the control system.

Let us examine in more detail the connection between maximum firing range and the amount of propellant which can be used for the operation of the engine system. Let us visualize a rocket flight for maximum range under optimum conditions. In this flight for the creation of thrust certain amounts of the propellant components will be used up which we will call the optimum working reserves of the propellant components. In actual flight, in order to attain that same range, an additional amount of propellant can be required to compensate for the effect of perturbing factors, for example for overcoming the additional drag, caused by head wind. So that maximum firing range is guaranteed by the shutting down of the engine upon control command with a reliability of $1-\frac{\varepsilon}{2}$, it is necessary to have on board a rocket, additional (above working) propellant reserves, necessary to compensate for the effect of the perturbing factors with the same level of reliability. These additional reserves are called guaranteed propellant component reserves.

Let us examine in more detail the concept of guaranteed propellant reserves. Let the firing of a series of rockets for one and the same maximum range L_{max} be carried out. In order to attain this range under diverse firing conditions with the effect of various random factors, it is necessary to expend in each flight a different amount of propellant. The propellant reserves, necessary for flight over an assigned range, are the random variables, subordinate to a certain low of probability distribution (Fig. 8.2). In order that the assigned flight range L_{max} is attained with a reliability of $1-\frac{\varepsilon}{2}$ (i.e., in order that $L_{max} = L_{np}$), it is necessary to fuel the rocket with a certain amount of propellant $G^{\text{зп}}_{\text{топл}}$, ensuring this level of reliability.

Of the total propellant reserve $G^{\text{зп}}_{\text{топл}}$ let us distinguish the nonworking reserves $G^{\text{нер}}_{\text{топл}}$ and the optimum working reserve $G^{\text{р}}_{\text{топл}}$.

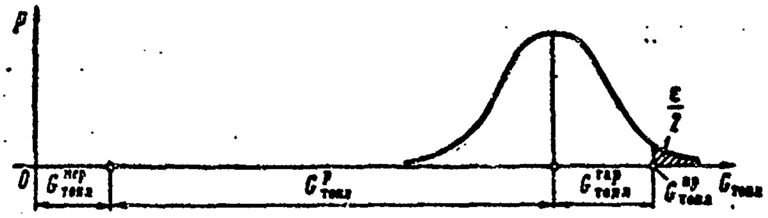


Fig. 8.2. Tankage breakdown with respect to propellant function classification.

Working reserves are used as the working medium of the engine for ensuring maximum firing range during flight under optimum conditions.

Non-working reserves include the propellant which cannot be used in flight as the working medium of the engine, but must be loaded on the rocket to ensure normal functioning of the engine system (propellant used for engine priming; propellant necessary cavitationless operation of the pumps; the residual non-intake propellant; propellant expended before launch, etc.).

The difference

$$G^{пр}_{топл}-(G^{нер}_{топл}+G^{р}_{топл})=G^{гар}_{топл}$$

will represent the guaranteed propellant reserves.

During optimum flight for range $L = L_{пр}$ the guaranteed propellant reserves remain in the rocket tanks. Hence a simple method emerges for determining the value of maximum range - from the calculation of optimum trajectory of a rocket carried out with the unused guaranteed reserves. The entire complexity of the problem of estimating maximum range by this method is transferred to the calculation of the guaranteed reserves. The degree of reliability in ensuring maximum range is equal in this case to the degree of reliability accepted in calculating the guaranteed reserves.

The guaranteed propellant reserves are ballast reducing the maximum firing range. Actually the most important parameter determining rocket flight range, is the ratio of propellant weight burned in the

powered-flight phase (i.e., the working propellant reserve), to the gross weight of the rocket. This ratio is greater the smaller is the unpowered-flight phase weight of the rocket, which is the sum of the weights of the airframe and the engines, the nose section, the control system and the guaranteed propellant reserves. Other things being equal the maximum flight range will be shorter the greater are the guaranteed propellant reserves.

For this reason it is inexpedient to consider the effect of all the perturbing factors only under the designation of guaranteed reserves. The greater number of factors examined as random perturbations and taken into account in calculating guaranteed reserves, the greater is the weight of the rocket with assigned maximum firing range. A rocket, in which the guaranteed reserves would ensure one and the same maximum range when firing both eastward and westward, as well as in winter on the coldest days and in summer on the hottest days, etc., would be extremely overweight and inconvenient in operation. Thus it is advisable to determine the portion of perturbing factors with sufficient accuracy before rocket launch and to take them into account in calculating the tankage breakdown by a corresponding selection of the working propellant reserves. Thus, for instance, in calculating tankage breakdown it is possible to consider the temperature (the specific gravities) of the propellant components and the official[1] value of parameter K - the ratio of the weight per-second expenditures of propellant components.

Although the taking into account of the perturbing factors in calculating the tankage breakdown of the rocket propellant components makes it possible to more completely take advantage of the energy capabilities of the rocket; it is far from expedient to take all the known factors into account in calculating tankage breakdown. It is evident that the greater the number of factors taken into account in preparing a rocket for launch, the more difficult it is to ensure the operational simplicity of the rocket and its high combat readiness.

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[1]The value of parameter K, obtained from the data of engine bench tests.

Thus a number of perturbing factors is taken into account by introducing corrections either into the value of maximum firing range (for instance, the effect of geodetic launch conditions), or into the degree of reliability of ensuring maximum range.

In this way, the effect of perturbing factors in ensuring maximum firing range can be taken into account:

- by designating the guaranteed propellant reserves;

- by selecting the tankage breakdown of the working propellant reserves;

- by introducing corrections into the value of maximum firing range or into the degree of its reliability.

In accordance with what has been said it is expedient to divide the system of perturbing factors affecting maximum firing range into two groups. Included in the first group are certain perturbing factors whose effect on maximum range is taken into account before launch (in the tankage breakdown of the rocket propellant, in the corrections for the value of maximum range and in an appropriate selection of the target). Included in the other group are the random factors whose effect is compensated for by the presence on the rocket of guaranteed propellant reserves.

Maximum range can be found by calculation during designing. The value of maximum range obtained by calculation, we will call the calculated maximum firing range.

It is expedient to subdivide the problem of determining calculated maximum range into two problems: the calculation of maximum range for certain optimum firing conditions and the taking into account of the effect of firing conditions on maximum range.

It is possible to accept, for example, the following conditions as optimum:

- the geodetic latitude of the launch point $\phi_{\Gamma 0} = 45°$

- the aximuth of aiming direction $A_0 = 0°$

- the standard atmosphere parameters;

- the temperature of the propellant components, equal to the temperature accepted for optimum in selecting the control programs.

The final determination of maximum firing range can be carried out from the results of the agreement of calculated maximum range with the data of flight tests, which also makes it possible to take into account, in addition to propellant deficiency, the other limitations on maximum firing range.

The calculation of maximum range in the general case can be carried out by using the method of statistical testing. For this n calculations of rated range for random combinations of random variables of perturbing factors are carried out and the value of maximum range with assigned degree of reliability P is determined by statistical processing of n values of ranges corresponding to the complete propellant burnup (see Fig. 8.1).

Since the method of statistical testing requires considerable expenditures of time (including machine time), it is much simpler and more convenient to determine the value of maximum range from a calculation of an unperturbed rocket trajectory carried out with unused guaranteed reserves. The degree of reliability in ensuring maximum range in this case is equal to the degree of reliability accepted in calculating guaranteed reserves.

The initial data for calculating maximum range with this method is the launch and final weights of each rocket stage.

The launch weight G_{0i} of each i-th stage under optimum firing conditions is determined with the formula

$$G_{0i} = G_{\text{сух}\,i} + G^{\text{р}}_{\text{топл}\,i} + G^{\text{гар}}_{\text{топл}\,i} + G^{\text{нер}}_{\text{топл}\,i} - G^{\text{зост}}_{\text{топл}\,i} + G_{\text{надд}\,i} + G_{0,i+1}, \quad (8.1)$$

where - $G^{p}_{\text{топл}\,i}$ - the weight of the optimum working propellant reserve; $G^{\text{гар}}_{\text{топл}\,i}$ - the weight of the guaranteed reserves; $G^{\text{неp}}_{\text{топл}\,i}$ - the weight of the non-working propellant reserves; $G^{\text{дост}}_{\text{топл}\,i}$ - the propellant expended before launch; $G_{\text{надд}\,i}$ - the weight of the working pressurizing medium; $G_{\text{сух}\,i}$ - the "dry" weight of the separating part of the i-th stage; $G_{0,i+1}$ - the initial weight of the filled (i + 1)-th stage;

From this expression the values of the final weights corresponding to the complete burnup of the working propellant reserves and to the unused guaranteed reserves are determined. Then taking into account the values of the launch and final weights of each stage of the rocket the powered-and unpowered-flight phases of the trajectory are calculated under optimum flight conditions. The value obtained as a result of calculating the value of range is taken as the maximum firing range.

As is evident, the examined method is suitable only for maximum range verifying calculations, when the guaranteed propellant reserves are already known. Thus calculation methods are of interest which make it possible to determine maximum range and the guaranteed propellant reserves corresponding to it, depending on the perturbing factors.

One of these methods is based on calculating perturbed missile trajectories. This method is methodically simple, but it requires definite expenditures of machine time for the multiple solving of a system of differential equations of motion of the rocket. Another approximate method is based on expanding the expression for firing range into a series for the parameters characterizing the motion of the center of mass of the rocket, with the rejection of terms higher than the first (or second) orders of smallness.

The characteristics of the determination of the guaranteed propellant reserves, the determinations of the tankage breakdown of propellant components and the calculations of the effect of firing conditions on maximum range will be examined below. A rational solution of these interdependent questions taking into account the design fetures of the rocket and the control system and the operating

conditions of the rocket is a necessary condition for ensuring required maximum firing range.

8.2 GUARANTEED PROPELLANT RESERVES

The problem of evaluating guaranteed propellant consists in determining the amount of propellant which is necessary for compensating for the effect of the worst combination of perturbing factors. In calculating guaranteed propellant reserves it is necessary to determine:

- the composition maximum values and the probable characteristics of the perturbing factors, besides the factors which are arguments of tankage breakdown or are taken into account by introducing corrections into maximum firing range;

- the effect of each perturbing factor on maximum firing range (for evaluating the significance of a perturbing factor);

- the amount of propellant, necessary for compensating for the total effect of perturbing factors with their worst combination on maximum firing range, i.e., guaranteed propellant reserve.

The basic perturbing factors taken into account in determining guaranteed propellant, reserves, can be subdivided into the following groups, compiled according to general criteria:

- deviations in "dry" weight;

- deviations in propellant weight;

- deviations in engine parameters;

- deviations in atmospheric parameters;

- control system errors;

- deviations in the initial parameters of motion.

The actual list of perturbing factors depends on the design features of the rocket, its operating conditions, etc.

Equations of Motion of the Rocket

For evaluating guaranteed propellant reserves let us examine a simplified system of equations of motion of the center of mass of a rocket in the firing plane during the powered-flight phase of the trajectory:

$$\left.\begin{aligned} \frac{dV}{dt} &= g\frac{P-X}{G} - g\sin\theta; \\ \frac{d\theta}{dt} &= g\frac{P\alpha+Y}{GV} - \frac{g}{V}\cos\theta; \\ \frac{dx_3}{dt} &= V\cos\theta; \\ \frac{dy_3}{dt} &= V\sin\theta, \end{aligned}\right\} \tag{8.2}$$

which let us supplement with the simplified system of control equations - with the equations of the regulation of apparent velocity and pitch angle:

$$\left.\begin{aligned} &V + \int_0^t g\sin\theta\,dt = w_{x1}^{*}(t); \\ &\alpha + \theta = \varphi^{*}(t) \end{aligned}\right\} \tag{8.3}$$

and with the equation of variation in weight

$$G = G_0 - \int_0^t \dot{G}\,dt. \tag{8.4}$$

In these equations lift is calculated taking the lift of the control elements into account with the aid balancing equation $M_z = 0$, i.e., with formula

$$Y_{\text{бал}} = Y\frac{x_{\text{дв}} - x_{\text{д}}}{x_{\text{дв}} - x_{\text{т}}}.$$

The aerodynamic forces are determined taking wind into account[1]

$$X = ic_x\left(\alpha_W, \frac{V_W}{a}\right) q_W S;$$
$$Y = c_y\left(\alpha_W, \frac{V_W}{a}\right) q_W S.$$

Parameters λ_i characterizing rocket design, the engine system the control system and the flight conditions are introduced into the right sides of the equation. Thus, for instance, it is possible to write

$$G = \lambda_1 - \int_0^t \lambda_2 dt;\quad P = \lambda_2\lambda_3 - \lambda_5 \frac{p}{p_0};$$
$$X = \lambda_4 c_x \frac{\varrho}{\varrho_0} V_W^2,$$

where $\lambda_1 = G_0$ - the launch weight; $\lambda_2 = \dot{G}$ - flow rate per second; $\lambda_3 = P_{\text{уд.п}}$ - specific thrust in a vacuum; $\lambda_4 = \frac{i\varrho_0 S}{2}$ - coefficient in the formula for drag; $\lambda_5 = S_a p_0$ - the coefficient of the altitude performance of the engine.

The deviations in parameters λ_i from their optimum values λ_i^*;

$$\Delta\lambda_i = \lambda_i - \lambda_i^*,$$

and also in the initial conditions V_0, Θ_0, x_0, y_0 from the optimum values V_0^*, Θ_0^*, x_0^*, y_0^*:

$$\Delta V_0 = V_0 - V_0^*, \ldots, \Delta y_0 = y_0 - y_0^*$$

are the disturbing factors.

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[1]Here i - the coefficient taking rocket deformation into account.

The Effect of Small Perturbing Factors on Maximum Firing and the Components of Guaranteed Propellant Reserves

A Rocket without a RKS System

For evaluating the effect of perturbing factors on maximum firing range let us make use of the known expression for deviation ΔL in range L from its optimum value L*:

$$\Delta L = L - L^{*} = \frac{\partial L}{\partial V}\Delta V + \frac{\partial L}{d\Theta}\Delta\Theta + \frac{\partial L}{\partial x}\Delta x + \frac{\partial L}{\partial y}\Delta y. \qquad (8.5)$$

Here $L^{*} = L(V_{\kappa}^{*}, \Theta_{\kappa}^{*}, x_{\kappa}^{*}, y_{\kappa}^{*})$, i.e., the optimum flight range, is determined by the optimum values of the parameters of motion at the calculated moment of time of NS separation t_{κ}^{*}; $\frac{\partial L}{\partial V}, \ldots, \frac{\partial L}{\partial y}$ - the partial derivatives determined either in the quadratures of [2], or by numerical methods for optimum trajectory and the calculated moment of time t_{κ}^{*}; $\Delta V, \ldots, \Delta y$ - deviations in the parameters of motion $V(t_{\kappa}), \ldots, y(t_{\kappa})$ at moment t_{κ} of NS separation from their calculated values

$$V^{*}(t_{\kappa}^{*}), \ldots, y^{*}(t_{\kappa}^{*}).$$

Perturbations $\Delta V, \ldots, \Delta y$ arise as a result of the effect of perturbing factors $\Delta\lambda_{i}$. Certain perturbing factors also vary the moment t_{κ} of NS separation, determined by control equation $J(t_{\kappa}) = J^{*}$. However we will not examine the effect of this fact on maximum range, assuming $\frac{\partial L}{\partial t}\Delta t_{\kappa} \approx 0$. Expression (8.5) makes it possible to determine as a first approximation the variation in maximum range with the variation in the parameters of motion at the moment of NS separation due to the effect of perturbations $\Delta\lambda_{i}$.

Assuming values $\Delta\lambda_{i}$ to be small and being limited to linear terms of the expansion, we obtain

$$\Delta V_{i} = \frac{\partial V}{\partial\lambda_{i}}\Delta\lambda_{i}, \ldots, \Delta y_{i} = \frac{\partial y}{\partial\lambda_{i}}\Delta\lambda_{i}, \qquad (8.6)$$

where the values of derivatives $\frac{\partial V}{\partial\lambda_{i}}, \frac{\partial y}{\partial\lambda_{i}}, \frac{\partial x}{\partial\lambda_{i}}, \frac{\partial\Theta}{\partial\lambda_{i}}$ correspond to the optimum trajectory and to calculated moment of time t_{κ}^{*}. We

will not dwell on the method of determining these derivatives, since it is examined in book [2].

Substituting the dependences of (8.6) the expression for the deviation in maximum range (8.5), we obtain

$$\Delta L_i = \frac{\partial L}{\partial \lambda_i} \Delta \lambda_i, \qquad (8.7)$$

where

$$\frac{\partial L}{\partial \lambda_i} = \frac{\partial L}{\partial V} \frac{\partial V}{\partial \lambda_i} + \ldots + \frac{\partial L}{\partial y} \frac{\partial y}{\partial \lambda_i}. \qquad (8.8)$$

The approximate variation in certain of the coefficients $dL/d\lambda_i$ with respect to rocket flight time is shown in Fig. 8.3.

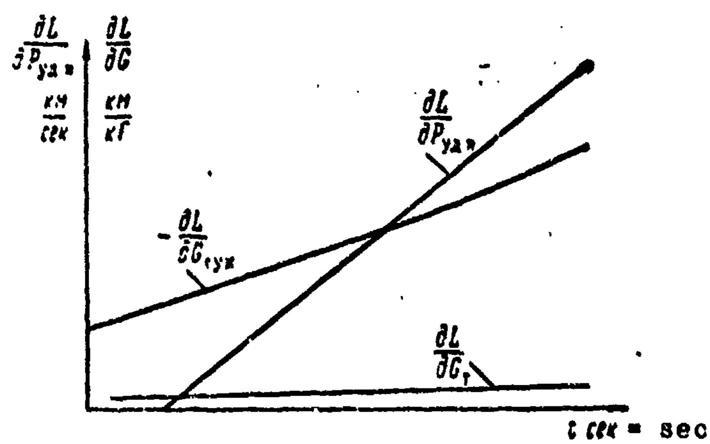

Fig. 8.3 Derivatives $\partial L/\partial \lambda_к$ depending on rocket flight time in the powered-flight phase of the trajectory.
Designations: $\frac{км}{сек} = \frac{km}{sec}$, $\frac{км}{кг} = \frac{km}{kgf}$

The loss in distance Δl_i due to the effect of perturbing factor $\Delta\lambda_i$ can be compensated for by the operation of the engine system during the course of time $\Delta t_{кi}$, equal to

$$\Delta t_{кi} = \frac{\Delta L_i}{\dot{L}}. \qquad (8.9)$$

where

$$\dot{L}=\frac{dL}{dt_к}=\frac{\partial L}{\partial V}\dot{V}+\frac{\partial L}{\partial \theta}\dot{\theta}+\frac{\partial L}{\partial x}\dot{x}+\frac{\partial L}{\partial y}\dot{y} \tag{8.10}$$

- the derivative, which takes into account the variation in firing range with variation in engine operating time (in a small vicinity of moment $t_к$*), and the values of all the magnitude in the expression for this derivative are determined for the optimum parameters of motion $V^*(t_к^*),\ldots,y^*(t_к^*)$.

The propellant expended during the additional time $\Delta t_{кi}$:

$$\Delta G_{топл\,i}^{гар}=\dot{G}\Delta t_{к\,i}, \tag{8.11}$$

where $\dot{G} = \dot{G}^*(t_к^*)$, is the component of the guaranteed propellant reserve which compensates for perturbation $\Delta\lambda_i$ for ensuring optimum maximum firing range L*.

The components of the guaranteed propellant component reserves are distributed in accordance with the formulas:

$$\left.\begin{aligned}\Delta G_{ок\,i}^{гар}&=\frac{K}{K+1}\Delta G_{топл\,i}^{гар};\\ \Delta G_{г\,i}^{гар}&=\frac{1}{K+1}\Delta G_{топл\,i}^{гар}.\end{aligned}\right\} \tag{8.12}$$

A Rocket with a RKS System

In this case the effect on the motion of a rocket of the overwhelming majority of perturbing factors is compensated for by a corresponding variation in engine thrust by varying the propellant consumption per second $\dot{G}$. The indicated fact makes it possible to substantially simplify the determination the components of the guaranteed propellant reserves, having placed at the basis for the calculation the equation of the ideal operation of the RKS system:

$$P-X_1=\frac{G}{g}\dot{w}_{x1}. \tag{8.13}$$

This one equation already makes it possible to determine the components of the guaranteed propellant reserve $\Delta G^{rap}_{\text{топл} i}$ as perturbations in the final weight of the rocket $\Delta G_{\kappa i}$, brought about by perturbations in the thrust and drag forces due to the effect of perturbing factors $\Delta\lambda_i$.

The effect of the perturbing factors on the losses in maximum range (when a guaranteed reserve does not exist,) can then be determined on the basis of the following considerations. An overexpenditure of propellant by magnitude $\Delta G_{\kappa i}(t^*_\kappa)$ due to the effect of perturbation $\Delta\lambda_i$ leads to premature engine shutdown (due to burnout) at moment $t_{\kappa i}=t_\kappa^*-\Delta t_{\kappa i}$ (Fig. 8.4) and to a loss in range

$$\Delta L_i=\dot{L}\Delta t_{\kappa i}=\dot{L}\frac{\Delta G_{\kappa i}}{\dot{G}}=\frac{\partial L}{\partial G}\Delta G_{\kappa i}, \tag{8.14}$$

where $\frac{\partial L}{\partial G}=\frac{\dot{L}}{\dot{G}}$ - the derivative determined for the optimum trajectory and t^*_κ.

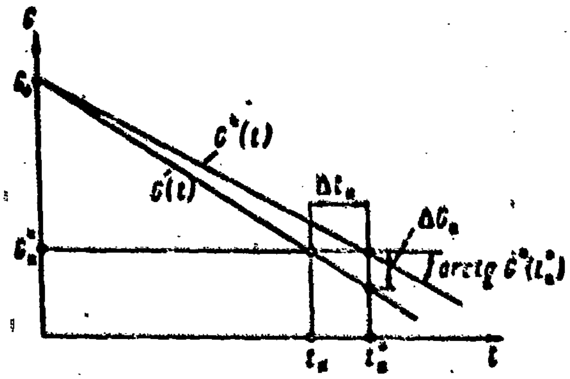

Fig. 8.4. For determining the guaranteed propellant reserve for a rocket with a RKS system.

The component of guaranteed propellant reserve $\Delta G^{rap}_{\text{топл} i}$, necessary in order to ensure engine shutdown upon the signal from the control system, will be

$$\Delta G^{rap}_{\text{топл} i}=\frac{\partial G_\kappa}{\partial\lambda_i}\Delta\lambda_i. \tag{8.15}$$

Let us examine $\partial G_{K}/\partial\lambda_i$ using as an example the method of determining derivatives. Let us consider that an engine system consists of the main and the controlling engines; the regulating of apparent velocity is brought about by varying the flow rate per second of the main engine.

Let us investigate the effect on the final weight of a rocket of the following basic design parameters:

$\lambda_1=G_0$ - the launch weight of the rocket;

$\lambda_2=P^{\text{осн}}_{\text{уд.п}}$ - the specific thrust of the main engine;

$\lambda_3=P^{\text{у}}_{\text{уд.п}}$ - the specific thrust of the controlling engines;

$\lambda_4=\dot{G}^{\text{у}}$ - the propellant flow rate per second through the controlling engines.

Let us represent the expression for engine system thrust (1.35) in the form

$$P=P^{\text{осн}}_{\text{уд.п}}\dot{G}^{\text{осн}}+P^{\text{у}}_{\text{уд.п}}\dot{G}^{\text{у}}-(S_a p_0)^{\text{осн}}\frac{p}{p_0}-(S_a p_0)^{\text{у}}\frac{p}{p_0}. \qquad (8.16)$$

Taking into account that

$$\frac{dG}{dt}=-\dot{G}^{\text{осн}}-\dot{G}^{\text{у}}, \qquad (8.17)$$

we will obtain

$$P=-P^{\text{осн}}_{\text{уд.п}}\frac{dG}{dt}-P^{\text{осн}}_{\text{уд.п}}\dot{G}^{\text{у}}+P^{\text{у}}_{\text{уд.п}}\dot{G}^{\text{у}}-(S_a p_0)^{\text{осн}}\frac{p}{p_0}-(S_a p_0)^{\text{у}}\frac{p}{p_0}. \qquad (8.18)$$

Differentiating equation (8.13) for parameters λ_1, we have

$$\frac{\partial P}{\partial\lambda_i}-\frac{\partial G}{\partial\lambda_i}\frac{\dot{w}^{*}_{x1}}{g}=0. \qquad (8.19)$$

Derivatives $\partial P/\partial\lambda_{K}$ are determined by differentiating the expression for thrust (8.18). Thus, for instance, for $\lambda_1=G_0$ we have

$$\frac{\partial P}{\partial G_0}=-P^{\text{осн}}_{\text{уд.п}}\frac{\partial}{\partial G_0}\left(\frac{dG}{dt}\right).$$

Carrying out analogous operations for other λ_i and varying the order of differentiation, we obtain:

$$\left.\begin{aligned}
&\frac{\partial P}{\partial G_0}=-P^{\text{осн}}_{\text{уд.п}}\frac{d}{dt}\left(\frac{\partial G}{\partial G_0}\right);\\
&\frac{\partial P}{\partial P^{\text{осн}}_{\text{уд.п}}}=-P^{\text{осн}}_{\text{уд.п}}\frac{d}{dt}\left(\frac{\partial G}{\partial P^{\text{осн}}_{\text{уд.п}}}\right)+\dot{G}^{\text{осн}};\\
&\frac{\partial P}{\partial P^{\text{у}}_{\text{уд.п}}}=-P^{\text{осн}}_{\text{уд.п}}\frac{d}{dt}\left(\frac{\partial G}{\partial P^{\text{у}}_{\text{уд.п}}}\right)+\dot{G}^{\text{у}};\\
&\frac{\partial P}{\partial \dot{G}^{\text{у}}}=-P^{\text{осн}}_{\text{уд.п}}\frac{d}{dt}\left(\frac{\partial G}{\partial \dot{G}^{\text{у}}}\right)-P^{\text{осн}}_{\text{уд.п}}+P^{\text{у}}_{\text{уд.п}}.
\end{aligned}\right\} \tag{8.20}$$

Substituting the obtained expressions of the derivatives of (8.20) into equation (8.19), we obtain the differential equations for determining the derivatives $\partial G/\partial\lambda_i$ in the form:

$$\left.\begin{aligned}
&\frac{d}{dt}\left(\frac{\partial G}{\partial G_0}\right)+\frac{\dot{w}^{*}_{x1}}{gP^{\text{осн}}_{\text{уд.п}}}\frac{\partial G}{\partial G_0}=0;\\
&\frac{d}{dt}\left(\frac{\partial G}{\partial P^{\text{осн}}_{\text{уд.п}}}\right)+\frac{\dot{w}^{*}_{x1}}{gP^{\text{осн}}_{\text{уд.п}}}\frac{\partial G}{\partial P^{\text{осн}}_{\text{уд.п}}}=\frac{\dot{G}^{\text{осн}}}{P^{\text{осн}}_{\text{уд.п}}};\\
&\frac{d}{dt}\left(\frac{\partial G}{\partial P^{\text{у}}_{\text{уд.п}}}\right)+\frac{\dot{w}^{*}_{x1}}{gP^{\text{осн}}_{\text{уд.п}}}\frac{\partial G}{\partial P^{\text{у}}_{\text{уд.п}}}=\frac{\dot{G}^{\text{у}}}{P^{\text{осн}}_{\text{уд.п}}};\\
&\frac{d}{dt}\left(\frac{\partial \dot{G}}{\partial \dot{G}^{\text{у}}}\right)+\frac{\dot{w}_{x1}}{gP^{\text{осн}}_{\text{уд.п}}}\frac{\partial G}{\partial \dot{G}^{\text{у}}}=\frac{P^{\text{у}}_{\text{уд.п}}}{P^{\text{осн}}_{\text{уд.п}}}-1.
\end{aligned}\right\} \tag{8.21}$$

The initial conditions for solving the differential equations of (8.21) are the following:

$$\left.\begin{aligned}
&\left(\frac{\partial G}{\partial\lambda_i}\right)_{t=0}=0 \text{ where } i=2\div 4;\\
&\left(\frac{\partial G}{\partial\lambda_i}\right)_{t=0}=1 \text{ where } i=1.
\end{aligned}\right\} \tag{8.22}$$

Each of the equations of (8.21) is a linear differential equation of the 1st order

$$\frac{d}{dt}\left(\frac{\partial G}{\partial\lambda_i}\right)+\frac{\dot{w}^{*}_{x1}}{gP^{\text{осн}}_{\text{уд.п}}}\frac{\partial G}{\partial\lambda_i}=\vartheta_i, \tag{8.23}$$

the solution of which gives the desired derivatives of the final weight with respect to the design parameters of the rocket

$$\frac{\partial G_{\kappa}}{\partial \lambda_{l}}=e^{-\alpha}\left[\int_{0}^{t_{\kappa}^{*}} \beta_{l} e^{\alpha} dt+\left(\frac{\partial G}{\partial \lambda_{l}}\right)_{t=0}\right], \qquad (8.24)$$

where

$$\alpha=\frac{w_{x1}^{*}(t_{\kappa}^{*})}{g P_{\text{уд.п}}^{\text{осн}}}. \qquad (8.25)$$

Using Tsiolkovsky's formula

$$w_{x1}^{*}(t_{\kappa}^{*})=-P_{\text{уд.п}}^{\text{д.у}} g \ln \mu_{\kappa}, \qquad (8.26)$$

where

$$\mu_{\kappa}=\frac{G(t_{\kappa}^{*})}{G_{0}}=\frac{G_{\kappa}}{G_{0}}, \qquad (8.27)$$

we will obtain approximate expressions for $\partial G_{\kappa}/\partial \lambda_{i}$ in the final form:

$$\left.\begin{aligned}
&\frac{\partial G_{\kappa}}{\partial G_{0}}=\mu_{\kappa}^{n};\\
&\frac{\partial G_{\kappa}}{\partial P_{\text{уд.п}}^{\text{осн}}}=-\frac{\dot{G}^{\text{осн}} G_{0}}{\dot{G}^{\text{у}}\left(P_{\text{уд.п}}^{\text{осн}}-P_{\text{уд.п}}^{\text{у}}\right)}\left(\mu_{\kappa}-\mu_{\kappa}^{n}\right);\\
&\frac{\partial G_{\kappa}}{\partial P_{\text{уд.п}}^{\text{у}}}=-\frac{G_{0}}{P_{\text{уд.п}}^{\text{осн}}-P_{\text{уд.п}}^{\text{у}}}\left(\mu_{\kappa}-\mu_{\kappa}^{n}\right);\\
&\frac{\partial G_{\kappa}}{\partial \dot{G}^{\text{у}}}=\frac{G_{0}\left(\mu_{\kappa}-\mu_{\kappa}^{n}\right)}{\dot{G}^{\text{у}}},
\end{aligned}\right\} \qquad (8.28)$$

where $n=\frac{P_{\text{уд.п}}^{\text{д.у}}}{P_{\text{уд.п}}^{\text{осн}}}$ - the ratio of the specific thrusts of the engine system as a whole and of the main engines.

If controlling engines are absent, then

$$\frac{\partial G_{\kappa}}{\partial P_{\text{уд.п}}}=-\frac{G_{\kappa}}{P_{\text{уд.п}}} \ln \mu_{\kappa}=-\frac{G_{0}}{P_{\text{уд.п}}} \mu_{\kappa} \ln \mu_{\kappa}. \qquad (8.29)$$

In Fig. 8.5 the approximate variation in derivatives $\frac{\partial G}{\partial G_0}$ and $\frac{\partial G}{\partial P_{\text{уд.п}}}$ during the time of flight $t = t_{\text{к}}^{*}$ is shown.

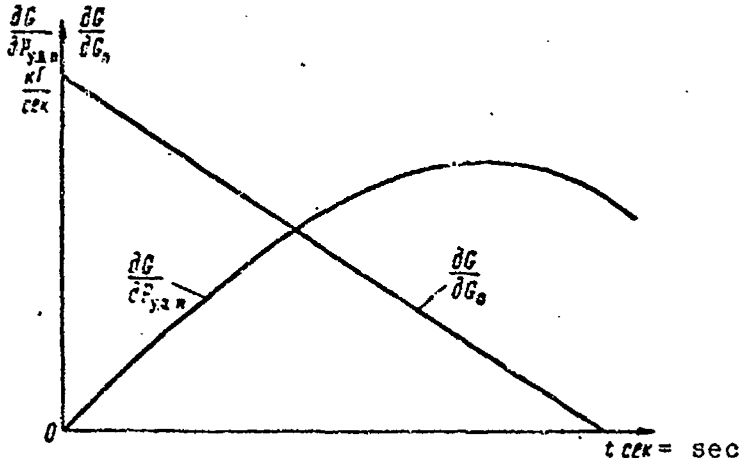

Fig. 8.5. Derivatives $\partial G/\partial \lambda_к$ depending on rocket flight time in the powered-flight phase of the trajectory.
Designation: $\frac{кг}{сек} = \frac{kgf}{sec}$

Expressions (8.28) and (8.29) are approximate due to the use of Tsiolkovsky's formula (8.26). With an increase in range the accuracy in determining the guaranteed reserves is increased with the aid of these expressions. The most precise results of expression (8.28) and (8.29) are given for the second and all the subsequent rocket stages whose flight occurs practically in airless space.

Total Guaranteed Reserves

The determining of total guaranteed propellant reserves is carried out taking the following circumstances into account (for generality we will examine multistage rockets).

1. The guaranteed reserves of the i-th stage are calculated on the basis of an assigned reliability of ensuring maximum range (for single-stage rockets) or of an assigned reliability of engine shut-down upon the command of the control system (for multistage rockets).

2. The distribution of the guaranteed reserves with respect to the rocket stages is carried out so that the losses in range will be minimal.

3. It is assumed that the degrees of reliability in designating the guaranteed reserves of each of the propellant components are equal

to each other and are equal to the degree of reliability of engine shutdown upon instruction from the control system.

4. It is assumed that the perturbing factors obey the normal distribution law, with the exception of certain perturbing factors, the probability nature of which is more accurately characterized by the law of uniform probability density distribution.

Included in these, for example, are:-

- the deviations in the specific gravities of the oxidizer and fuel due to variation in the chemical composition of the fuel during its storage;

- the deviation in parameter K due to the difference in the temperature of the propellant components from the optimum.

5. Of the totality of possible values of a perturbing factor characterized by the distributive law, only those random values are examined which adversely affect flight range.

6. The maximum values of the perturbing factors and of the components of the guaranteed reserves correspond to the degree of reliability of ensuring maximum firing range B. Thus, with probability B

$$\left.\begin{aligned} |\Delta\lambda_i| &\leqslant u\sigma_{\lambda i}; \\ \Delta G^{\text{гар}}_{\text{топл}\, i} &\leqslant u\sigma^{\text{гар}}_{i}, \end{aligned}\right\} \tag{8.30}$$

where u - the argument of Laplace function $\Phi(u)$, corresponding to probability $\Phi = B$; $\sigma_{\lambda i}$ and $\sigma_i^{\text{гар}}$ - the standard deviations in the i-th perturbing factor and the corresponding component of the guaranteed reserve.

Taking the indicated circumstances into account the value of the total guaranteed reserve of propellant component (oxidizer of fuel) of a given rocket stage is determined by the formula

$$G_j^{\text{гар}} = \sqrt{\sum_i (\Delta G_{ji}^{\text{гар}})^2}, \tag{8.31}$$

or

$$G_j^{\text{гар}} = u\sqrt{\sum_i (\sigma_{ji}^{\text{гар}})^2} = u\sigma_{j\Sigma}^{\text{гар}}, \tag{8.32}$$

where j - the subscript of the propellant component (ок [ox] or г [f]); $\sigma_{j\Sigma}$ - the standard deviation in the total reserve of the j-th propellant component.

Let us note that formula (8.31) for determining the total guaranteed reserves of oxidizer and fuel is approximate, since it is based on the assumption, that all the components $\Delta G_i^{\text{гар}}$ obey the normal distribution laws. If in the total guaranteed reserves the portion of components, obeying the law of uniform density, is significant, it is more correct to carry out the composition of the various distribution laws in accordance with the methods of the probability theory. The reliability of the series of premises, on which the calculations of the guaranteed reserves are based, cannot be reliably checked in the design stage. In particular, it can turn out, that the assumption concerning the normal distribution laws of such perturbing factors, as variance in parameter K, errors in propellant loading and others, is only approximately satisfied: the values of the perturbing factors accepted in the calculations are insufficiently reliable; the principle of summing such components of the guaranteed reserves, as the deviation in parameter K, with the remaining components will be subject to additional substantiation, etc. As a result of this the calculated values of the guaranteed reserves and the calculated maximum range corresponding to them cannot be highly reliable. Thus it is advisable to somewhat increase the calculated values of the guaranteed reserves.

The Distribution of the Guaranteed Reserves Among Rocket Stages

The Shutting Down of the Engine System of a Separating Stage Upon Command from the Control System

On multistage rockets the shutting down of the engine systems of separating stages can be carried out upon instruction from the control system upon the achievement by the rocket of the prescribed value of the controlling functional.

The shutting down of an engine system in such a manner assumes the presence in the separating stages of guaranteed reserves, necessary for compensating for the effect of perturbing factors (deviations in specific thrust, weight and aerodynamic characteristics and others) and ensuring with the required reliability the shutting down of the engine systems of the separating stages with assigned values of the controlling functional. The effect of the perturbing factors acting on the rocket after stage separation, is compensated for by the guaranteed propellant reserves which are located on the subsequent stage.

For a multistage rocket the degree of reliability of ensuring maximum firing range is

$$B = B_1 B_2 \ldots B_l \ldots B_n, \qquad (8.33)$$

where l - the number of the stage; n - the number of stages; B_l - the reliability of ensuring the maximum value of the control functional of the l-th stage on condition that the (l - 1)-th stage has attained the maximum value of the functional.

Let us examine the problem of determining for the assigned value B of such values of B_l which, satisfying condition (8.33), ensure the greatest maximum range. For solving this problem we will examine the deviation in maximum range from the value corresponding to that case, when the guaranteed propellant reserves at each stage ensure the achieving of a control functional with a level of reliability $\Phi(u^*) = B$. With values of the degrees of reliability $B_l \neq B$ the guaranteed

propellant reserves of the l-th stage vary by value

$$\left.\begin{aligned} \Delta G_l^{\text{гар}} &= \sigma_{\Sigma l}^{\text{гар}} \Delta u; \\ \Delta u &= u(B_l) - u(B), \end{aligned}\right\} \tag{8.34}$$

where $u(B_l)$ - the argument of the Laplace function corresponding to degree of reliability B_l; $\sigma_{\Sigma l}^{\text{гар}}$ - the standard deviation in the guaranteed reserve of the l-th stage, determined in accordance with formula (8.32).

The variation in maximum range due to the variation in the guaranteed reserves of each stage by value $\Delta G_l^{\text{гар}}$ is determined by expression

$$\Delta L = \sum_{l=1}^{n} \left\{ \left(\frac{\partial L}{\partial G_\text{к}} \right)_l [u(B_l) - u(B)]\, \sigma_{\Sigma l}^{\text{гар}} \right\}, \tag{8.35}$$

where $\left(\frac{\partial L}{\partial G_\text{к}} \right)_l$ - the derivative of maximum range with respect to the final weight of the l-th stage.

Relationship (8.35) expresses the dependence of the increase in maximum range on the degrees of reliability of ensuring the control functionals of all the rocket stages, and taking into account expression (8.33) and on the degree of reliability of ensuring maximum range.

In view of the small number of stages (l = 2-3) the investigation of expression (8.35) for the extremum can be readily carried out by numerical methods. Figures 8.6 and 8.7 show the approximate variation in the maximum range of a two-stage missile depending on the degree of reliability of ensuring it.

The Shutting Down of the Engine System of a Separating Stage upon Propellant Burnout

For reducing the guaranteed propellant reserves on multistage rockets the method of shutting down the engine systems of the separating stages (excluding the last one) upon complete propellant burnout (or of one of the propellant components) can be used. The shutting

down of the engine systems upon burnout makes it possible to use the quantities of the guaranteed propellant reserves in the separating stages (excluding the last one) as working reserves. In this case the compensating for the effect of the perturbing factors acting on the rocket during flight in the powered-flight phase of the trajectory, is carried out in the last rocket stage. The placing of guaranteed reserves only on the last stage makes it possible to more rationally use the propellant reserves and due to this to considerably improve the energy possibilities of the rocket (to increase maximum firing range or the payload weight, etc.).

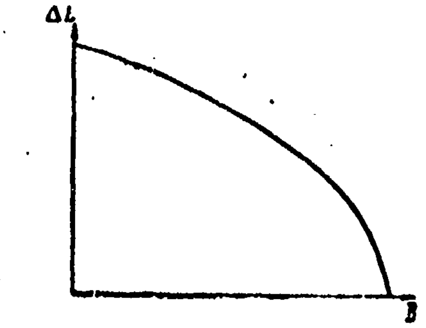

Fig. 8.6. Variation in maximum firing range depending on the reliability of ensuring it.

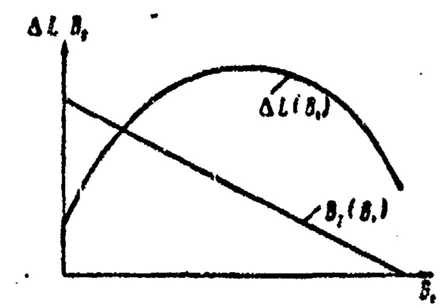

Fig. 8.7. Variation in maximum firing range depending on the reliability of ensuring the controlling functionals with the reliability of ensuring the controlling functionals with the reliability of ensuring maximum range B*.

The shutting down of the engine systems of separating stages can be carried out with respect to the first burned out component upon a

signal from the fuel residue sensors or from the sensors of the simultaneous tank emptying system.

An evaluation of the advisability of shutting down the engine systems of rocket stages upon propellant burnout can be carried out by comparing the maximum firing ranges ensured by various means of engine system shutdown.

Reducing the Guaranteed Reserves by Regulating the Expenditures of the Propellant Components

An increase in maximum firing range can be attained by installing a special propellant component metering system on the rocket.

If special measures are not taken, then during the operation of the rocket engines deviation from the optimum value of parameter K always appears (i.e., the ratio of oxidizer and fuel expenditures), as a result of which the situation can arise, when one of the propellant components will be completely consumed before the moment of the introduction of the command for shutting down the engines during firing for maximum range. To eliminate this situation it is necessary to have onboard the rocket an additional amount of the first and second components. This additional amount of propellant constitutes a great part of the guaranteed reserve and, in order to decrease it, it is expedient to use some system for regulating the propellant component expenditures ensuring the required ratio of propellant components.

For the purpose of more rational use in flight of the propellant reserve available onboard the rocket, simultaneous tank emptying systems are finding broad application.

This system is intended for regulating the relative volumetric propellant component expenditure. It includes propellant component level sensors, installed in each of the tanks, a computer which works out the instructions, and actuating elements.

The installation onboard a rocket of a simultaneous tank emptying system makes it possible to:

- substantially decrease the guaranteed propellant reserves and thus, to increase the maximum firing range;

- reduce the requirements imposed for the accuracy of adjustment of the engine system with respect to parameter K and for the accuracy of filling the rocket with propellant components;

- simplify the preparation of the initial data for filling the rocket with the propellant components.

As a result of installating onboard the rocket of a propellant component flow regulating system the maximum firing range is changed, which is due to:

- the reduction in the guaranteed propellant reserves;

- the increase in the "dry" weight of the rocket due to the installation of the control system;

- the reduction in the specific thrust of the engine system.

A reduction in the guaranteed propellant reserves occurs due to the component of guaranteed reserves intended for compensating for the variance in parameter K. With the installing of a simultaneous tank emptying system this component is eliminated, and the incomplete expenditure of one of the components is due only to an error in the control system.

For evaluating the advisability of installing one or another propellant proponent flow regulation system the total gain in firing range can be determined by the formula

$$\Delta L = \frac{\partial L}{\partial G_{\text{сух}}}\left(G_{\text{с.р}} + G_{\text{с.р}}^{\text{гар}} - G^{\text{гар}}\right) + \frac{\partial L}{\partial G_{\text{топл}}}\left(G^{\text{гар}} - G_{\text{с.р}}^{\text{гар}}\right) + \frac{\partial L}{\partial P_{\text{уд.п}}}\left(P_{\text{уд.п}}^{\text{с.р}} - P_{\text{уд.п}}\right), \tag{8.36}$$

where $G_{\text{с.р}}$ - the weight of the control system; $G_{\text{с.р}}^{\text{гар}}$, $G^{\text{гар}}$ - the weight

of the guaranteed propellant reserves with the presence onboard the rocket of a control system and with its absence; $P^{\text{с.р}}_{\text{уд.п}} - P_{\text{уд.п}}$ - the difference between specific thrust of the engine with a control system exists, and without it; $\frac{\partial L}{\partial G_{\text{сух}}}$, $\frac{\partial L}{\partial G_{\text{топл}}}$ - the partial range derivatives with respect to "dry" weight and propellant weight respectively.

Despite the fact that the weight of the rocket is increased due to the weight of the control system, the total passive weight of the rocket is decreased due to the significant reduction in the guaranteed propellant reserves, as a result of which the maximum firing range is substantially increased.

8.3. DETERMINING TANKAGE BREAKDOWN

By filling method we will understand the principle of determining the breakdown of the propellant components loaded onboard the rocket, and not the method of the technical realization of this principle. The filling method influences not only the energy characteristics of the rocket and its operational convenience, but also the design of the control system and the rocket as a whole, for example for a number of control programs due to the permissilbe limits of boosting and throttling of the engine and by varying the launch weight of the rocket with the presence of the apparent velocity control system.

In connection with this an important problem arises with respect to selecting the method of determining tankage breakdown, in the very best manner satisfying the requirements of reliable ensuring maximum firing range and operational convenience of the rocket.

Methods of Determining Tankage Breakdown and the Requirements, Imposed on It

It is possible to formulate the following basic requirements, taking into account which should be selected for the method of determining tankage breakdown:

- the most complete utilization of the energy capabilities of the rocket;

- convenience and operational simplicity of the rocket under field conditions, reducing to a minimum the time, necessary for determining tankage breakdown in the prelaunch period;

- maximum simplification of the design of the rocket, control system and engine system.

Besides these requirements, when selecting the method of calculating tankage breakdown another series of initial conditions is assumed which is due to the design characteristics of the rocket and its subassemblies, and also to the specifics of field operation. Thus, for instance, for all filling methods the following conditions are usually taken as initial:

- the launch weight of the rocket should not go beyond the permissible limits which are determined, for example, with the presence of a RKS system, by the maximum permissible filling of this system or in the absence of such a system, with the maximum permissible value of flight range reduction;

- the guaranteed and non-working propellant component reserves are determined for their optimum temperature and are assumed constant in weight (or in volume) in the operating range of the propellant component temperature variation;

- the maximum propellant component filling doses should not exceed the available volumes of the propellant systems (tanks and conduits) under all operating conditions;

- the working reserves of oxidizer and fuel should be in a ratio which makes it possible to completely use them up:

- the propellant component filling doses are selected for maximum firing range and remain constant in the taken firing range spread.

Besides the indicated conditions, it is possible to name additional requirements, only characteristic of an actual given method of

filling and of a type of rocket. These additional conditions will be examined below with the characteristics of the basic methods of filling.

Among the number of possible methods of filling it is possible to distinguish the following basic ones:

1) "individual" filling;

2) filling with weight doses;

3) filling with volume doses.

Let us briefly examine the characteristics of these methods.

With the individual method the filling doses are determined by taking into account certain actual characteristics of individual rockets, the propellant components and the firing conditions so that it is possible to more completely use the volumes of the fuel systems and to obtain a gain in flight range. It is evident that it is also possible to propose a large number of possible individual filling methods differing from eachother in the number and the makeup of the perturbing factors being considered. In principle it is possible to work out such an individual filling method which will make it possible to take into account everything known at the moment of the onset of the deviation in the parameters affecting the energetic characteristics of the rocket (the temperature of the oxidizer and fuel, value of parameter K of the engine, the volumes of the fuel systems, the dry weight of the rocket, etc.). However the considering of a large number of perturbing factors with an individual filling method requires the execution of a considerable volume of computational operations in the prelaunch period and increases the time for preparing the missile for launch. Moreover, the individual filling method also has the following deficiency connected with the operation of the rocket. Since with a given method maximum utilization of the available tank volumes is assumed, the time for positioning the filled missile on the launch pad is limited because the variation in ambient temperature can lead to an increase or a decrease in the temperature of the propellant components and to a variation in their volumes and, as

consequence, to the overfilling of the fuel systems or to an uncalculated operating regime of the engine system in flight. For the indicated reasons the use of the individual filling method is advantageous only for carrier rockets.

From the point of view of operation and the readiness of a rocket for launch most expedient are the methods of servicing a rocket with working propellant reserves, not dependent with respect to weight or volume on certain perturbing factors.

With the weight method of servicing the weight of the propellant components loaded into the rocket is constant in the given temperature range; the distribution of the fueling doses between oxidizer and fuel is carried out in such a manner, so as to ensure equivalent losses in range with possible variations in temperature up to the boundaries of a given range. (Variation in temperature affects the weight ratio of the per-second propellant component expenditures K and thereby value of the unused remainders of oxidizer and fuel).

However the application of such a servicing method gives rise to the necessity during the designing of a rocket of specifying for additional tank volumes providing for the possibility of the expansion of the volumes of the loaded propellant components in a given temperature range. Such a method of servicing is especially undesirable for ground-based launches, when the range of possible temperature variations of the propellant components is rather broad (about 100°C), and a system of thermostatic control is not specified.

For rockets, equipped with RKS systems, the advantage of the weight fueling method is the fact that the necessary limits of engine boosting and throttling in view of the absence of perturbations in launch weight are narrower than in the case, when the weights of the fueling doses depend on the temperature of the propellant components.

With the volume method rocket fueling with propellant components is carried out with constant volume doses of oxidizer and fuel under all rocket operating conditions; the ratio of the volumes of oxidizer and fuel is selected from the condition of complete expenditure of the

working propellant reserves under optimum conditions.

The fueling method affects the necessary volumes of the propellant systems and through them the rocket design. The interconnection between rocket characteristics and the engine system, the system of perturbing factors and the fueling method can be traced using as an example the determination of the necessary volumes of the propellant systems.

Determining the Necessary Volumes of Propellant Systems

Let us carry out a determination of the necessary volumes of propellant systems proceeding from a given value of the optimum launch weight of a rocket (or stage) G_{01}. Using formula (8.1), let us find the weight of the optimum working propellant reserve $G^{р}_{т\,\max}$, after all the other components of launch weight G_0 have been determined.

The weight fueling doses of oxidizer and fuel are determined by formulas:

$$\left.\begin{aligned} G_{ок}&=\frac{K}{K+1}G^{р}_{т\,\max}+G^{нзп}_{ок}+G^{гар}_{ок};\\ G_{г}&=\frac{1}{K+1}G^{р}_{т\,\max}+G^{нзп}_{г}+G^{гар}_{г}.\end{aligned}\right\}\tag{8.37}$$

where K - the ratio of the weight per-second propellant component expenditures under optimum conditions.

The maximum possible volume fueling doses $V^{\max}_{ок}$ and $V^{\max}_{г}$ are calculated taking the fueling method into account. Since the weight method provides the loading of weight doses of oxidizer and fuel independent of their temperature, the maximum possible fueling volumes are determined for the maximum temperature of a given range:

$$V^{\max}_{j}=\frac{G_j}{\gamma_j T^{\max}}.\tag{8.38}$$

where j - the propellant component subscript (ок [ox] or г [f]).

With the volume fueling method the maximum volume fueling doses are equal to the optimum:

$$V_j^{\max} = \frac{G_j}{\gamma_j T^*}. \qquad (8.39)$$

To the maximum possible volume fueling doses it is necessary to add the minimally permissible free volumes in the tanks, necessary for normal operation of the pressurization system $V^{\text{св}}_{\min j}$, and also the volumes ΔV_j, necessary for variations in the fueling doses due to random factors - errors in fueling (metering) the propellant components and errors in manufacturing the propellant tanks.

Finally the necessary volumes of propellant systems are determined by the formula

$$V_j^{\text{потр}} = V_j^{\max} + V^{\min}_{\text{св}j} + \Delta V_j. \qquad (8.40)$$

Determining Fueling Doses with Assigned Propellant System Volumes

To increase maximum firing range it is necessary to as completely as possible use the volumes of the propellant tanks. Let us examine how it is possible to solve this problem with weight and volume methods of fueling a rocket with propellant components.

For the working range of temperature variation in the propellant components $T_j^{\min} \leqslant T_j \leqslant T_j^{\max}$ let us determine the rated volumes of propellant systems (let us disregard the effect of temperature on the volumes of the main propellant lines V_{Mj}):

$$V_j^{\text{расп}}(T_{\text{ок}}) = V_{бj}(T_{\text{ок}}) + V_{мj} - V^{\min}_{\text{св}j} - \Delta V_j, \qquad (8.41)$$

where $V_{бj}(T_{\text{ок}})$ - the volume of the propellant tanks depending on temperature:

$$V_{бj}(T_{\text{ок}}) = V_{бj}^{\text{ном}}\left[1 + 3\alpha_j\left(T_{\text{ок}} - T_{\text{ок}}^{\text{ном}}\right)\right];$$

α_j - the coefficient of linear expansion of the tank material.

Let us find the ratio of the working reserves of the propellant

components $K_б = G^р_{ок}/G^р_г$, which is mainly determined by the rated volumes of the propellant systems:

$$K_б(T_{ок}) = \frac{V^{расп}_{ок}(T_{ок})\,\gamma_{ок}(T_{ок}) - G^{нер}_{ок}\dfrac{\gamma_{ок}(T_{ок})}{\gamma^{ном}_{ок}} - G^{гар}_{ок}\dfrac{\gamma_{ок}(T_{ок})}{\gamma^{ном}_{ок}}}{V^{расп}_{г}(T_{ок})\,\gamma_{г}(T_{ок}) - G^{нер}_{г}\dfrac{\gamma_{г}(T_{ок})}{\gamma^{ном}_{г}} - G^{гар}_{г}\dfrac{\gamma_{г}(T_{ок})}{\gamma^{ном}_{г}}}. \qquad (8.42)$$

Parameter $K_б(T_{ок})$ is necessary in order to determine, which of the propellant component tanks is limiting depending on propellant temperature. The temperature range of the propellant components, in which the rated volume of the oxidizer system is limiting, is determined from condition $K(T_{ок}) \geqslant K_б(T_{ок})$. If $K(T_{ок}) < K_б(T_{ок})$, then the volume of the fuel system is limiting.

Let us determine the loading of propellant component doses with the weight method in the following manner.

Using the formula (8.41) we find the rated volumes of propellant systems at maximum possible temperature $V^{расп}_j(T_{max})$. The ratio of working propellant component reserves determined by these rated volumes, is equal to

$$K_б(T_{max}) = \frac{V^{расп}_{ок}(T_{max})\,\gamma_{ок}(T_{max}) - G^{нер}_{ок} - G^{гар}_{ок}}{V^{расп}_{г}(T_{max})\,\gamma_{г}(T_{max}) - G^{нер}_{г} - G^{гар}_{г}}. \qquad (8.43)$$

It is natural to assume the non-working and the guaranteed propellant reserves with the weight fueling method in the working temperature range is constant in weight.

Let us compare value $K_б(T_{max})$ with the optimum value of parameter K. For the case, when $K^{ном} \geqslant K_б(T_{max})$, we compute the weight fueling doses of propellant components in the following manner:

the weight fueling dose of oxidizer

$$G_{ок}(T) = V^{расп}_{ок}(T_{max})\,\gamma_{ок}(T_{max}); \qquad (8.44)$$

the weight of the working oxidizer reserve

$$G^{р}_{ок}(T_j) = G_{ок}(T) - G^{нер}_{ок} - G^{гар}_{ок}; \qquad (8.45)$$

the weight of the working fuel reserve

$$G_{\text{г}}^{\text{р}}(T)=\frac{G_{\text{ок}}^{\text{р}}(T)}{K^{\text{ном}}};\tag{8.46}$$

the weight fueling dose of fuel

$$G_{\text{г}}(T)=G_{\text{г}}^{\text{р}}(T)+G_{\text{г}}^{\text{нер}}+G_{\text{г}}^{\text{гар}}.\tag{8.47}$$

For the case, when $K^{\text{ном}}\leqslant K_{\text{б}}(T_{max})$, we calculate the weight fueling doses in a similar manner, first determining the fueling dose, then the working fuel reserve and finally the working oxidizer reserve and the oxidizer fueling dose.

With the volume method the determination of the fueling doses is carried out in the following manner. The rated volumes of the propellant systems and the ratio of the rated working component reserves are calculated by formulas (8.41) and (8.42).

When $K^{\text{ном}}\geqslant K_{\text{б}}(T^{\text{ном}})$ the weight fueling doses of oxidizer and fuel are determined by formulas:

$$G_{\text{ок}}(T)=V_{\text{ок}}^{\text{расч}}(T)\gamma_{\text{ок}}(T);\tag{8.48}$$

$$G_{\text{г}}(T)=\frac{G_{\text{ок}}(T)-G_{\text{ок}}^{\text{нер}}-G_{\text{ок}}^{\text{гар}}}{K(T)}+G_{\text{г}}^{\text{нер}}+G_{\text{г}}^{\text{гар}}.\tag{8.49}$$

When $K^{\text{ном}}\leqslant K_{\text{б}}(T^{\text{ном}})$ the determination of the weight fueling doses is carried out in a similar manner, with the exception of the order of calculating the doses: first the fuel dose is calculated proceeding from the rated volume of the fuel system, and after that the oxidizer dose.

With the volume method of calculating the fueling doses with a temperature increase the weight of the propellant being loaded decreases, which gives rise to a certain reduction in maximum range. However the volume method of fueling makes it possible to use a tank simultaneous emptying system, which increases the maximum firing range.

8.4. THE EFFECT OF FIRING CONDITIONS ON MAXIMUM RANGE

As was noted above, certain perturbing factors are not considered in calculating the fueling doses and the guaranteed propellant reserves. The effect on maximum range of such factors, as geodetic and meteorological conditions can be considered by introducing appropriate corrections into the value of maximum range. For this reason maximum range is a function of certain firing conditions.

Most frequently the arguments of maximum range are the temperatures of the propellant components of the time of rocket launch and the geodetic launch conditions.

The effect of propellant temperature on maximum range (or on the maximum control functional) is manifested in two ways: through the variation in the engine parameters as well as through the variation in the engine parameters as well as through the variation in the components of the fueling doses, and it can be approximately evaluated with the aid of the expressions given below.

$$\Delta S=\left(\frac{\partial S}{\partial P_{\text{уд.п}}}\frac{\partial P_{\text{уд.п}}}{\partial T}+\frac{\partial S}{\partial \dot{G}}\frac{\partial \dot{G}}{\partial T}+\frac{\partial S}{\partial G_0}\frac{\partial G_0}{\partial T}+\frac{\partial S}{\partial G_{\text{к}}}\frac{\partial G_{\text{к}}}{\partial T}\right)(T-T_{\text{ном}}). \quad (8.50)$$

With the presence of an apparent velocity control system

$$\Delta S=\frac{\partial S}{\partial G_{\text{к}}}\left(\frac{\partial G_{\text{к}}}{\partial G_0}\frac{\partial G_0}{\partial T}+\frac{\partial G_{\text{к}}}{\partial T}+\frac{\partial G_{\text{к}}}{\partial P_{\text{уд.п}}}\frac{\partial P_{\text{уд.п}}}{\partial T}+\right.$$
$$\left.+\frac{\partial G_{\text{к}}}{\partial \dot{G}^{\text{у}}}\frac{\partial \dot{G}^{\text{у}}}{\partial T}\right)(T-T_{\text{ном}}), \quad (8.51)$$

where S - the firing distance L or the value of the control functional J; $\frac{\partial P_{\text{уд.п}}}{\partial T}$; $\frac{\partial \dot{G}}{\partial T}$; $\frac{\partial \dot{G}^{\text{у}}}{\partial T}$ - the derivatives of specific thrust and of the per-second expenditures of the engine system and the controlling engines with respect to propellant temperature; $\frac{\partial G_0}{\partial T}$; $\frac{\partial G_{\text{к}}}{\partial T}$ - derivatives of the launch and final weights of the rocket (stage) with respect to propellant temperature.

With large deviations in the launch and final weights more

accurate results, than with the use of formulas (8.50) and (8.51), can be obtained by mathematical modeling of rocket motion on a digital computer.

A substantial effect on maximum range is rendered by the geodetic launch conditions which are characterized by the values of the geodetic latitude of the launch point $\phi_{\Gamma 0}$ and the azimuth of the aiming direction A_0. The dependence of maximum range on the geodetic launch conditions $L_{\text{пр}}(\varphi_{\text{г0}}, A_0)$ is used for determining rocket operational zone, within the limits of which the reaching of targets is possible. This zone can be found as a result of calculating the trajectories of the powered- and unpowered-flight phases with various values $\phi_{\Gamma 0}$ and A_0, and also using the final formulas of elliptical theory. In the latter case it is necessary to disregard the variation in the parameters of the trajectory in the powered-flight phase because of the rotation of the earth.

The characteristic dependence of the variations in maximum range on the geodetic firing conditions for rocket conditions is represented in Fig. 8.8.

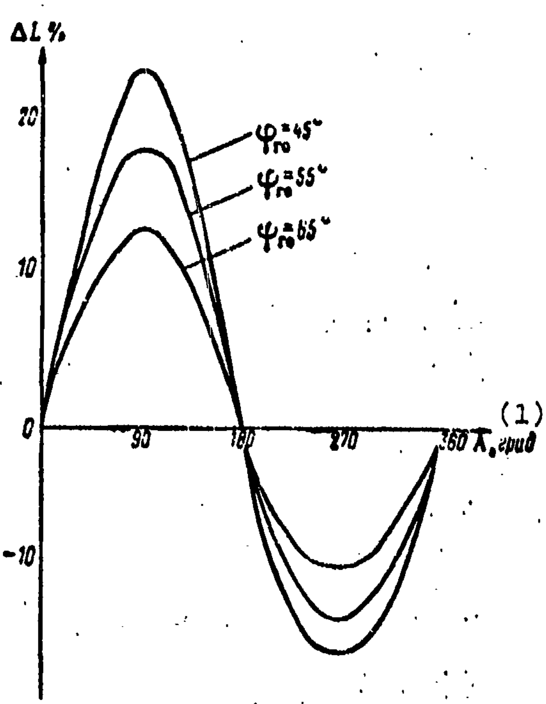

Fig. 8.8. Variation in maximum firing range under various geodetic conditions.
Key: (1) grid.

In the case of rocket launches under various geodetic conditions variation in the values of the guaranteed propellant component reserves is also possible, in order not to permit substantial variations in the maximum firing range and in the reliability of ensuring it. Thus, for instance, for rockets, not having RKS systems, during launches in a westward direction the value of guaranteed reserves should be increased, and during launches eastwards - decreased with respect to the value of the guaranteed reserves calculated for launches in a northerly direction. Another approach is also possible, when for constant maximum range the value of the guaranteed propellant reserves is maintained constant under all geodetic firing conditions and the variation in the reliability of ensuring maximum range is considered.

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